## ADAPTIVE NEURAL SPEED REGULATION CONTROL FOR INDUCTION MOTORS STOCHASTIC NONLINEAR SYSTEMS

LICHAO LIU, YUMEI MA, JINPENG YU, WEI LI AND XIAOLING WANG

College of Automation Engineering Qingdao University No. 308, Ningxia Road, Qingdao 266071, P. R. China yjp1109@hotmail.com

Received December 2015; accepted March 2016

ABSTRACT. This paper studies the problem of speed regulation control for induction motors stochastic nonlinear system based on neural networks and backstepping technique. During the controller design, neural networks are used to approximate the unknown nonlinearities, and backstepping technique is employed to construct controllers. The proposed controller ensures that all signals of the closed-loop system remain bounded in probability, and the tracking error converges to an arbitrarily small neighborhood around the origin. Simulation results demonstrate the effectiveness of the proposed approach.

**Keywords:** Induction motor, Neural networks, Backstepping, Stochastic nonlinear systems

1. Introduction. Due to the inherent advantages of induction motors as simplicity of design, high reliability, low cost and minimum maintenance, they are widely used in industrial applications. However, the machine parameters and load characteristics are not perfectly known [1] and they can sensibly vary during motor operation [2]; what is more, stochastic disturbance frequently exists in engineering applications. Nevertheless, up to now, few results on adaptive control have been developed for induction motors stochastic nonlinear systems. Thus, it is more realistic to design controllers based on the induction motors system where the stochastic nonlinearities are taken into consideration. Many research results on backstepping technique obtained from deterministic nonlinear systems [3, 4] have been successfully extended to the stochastic nonlinear systems. However, the backstepping technique suffers from the problem of "explosion of terms" [5]. Various schemes have been proposed for solving such a problem. Approximation-based adaptive neural control schemes have been found to be particularly useful for the control of highly uncertain, nonlinear, and complex systems [6, 7]; different from the classical adaptive backstepping control approach, adaptive neural control provides a systematic methodology to control and design a class of strict-feedback nonlinear systems with unknown smooth nonlinear functions, where neural networks are used to approximate the unknown smooth functions. Thus, the major problems with the traditional backstepping are cured.

In this paper, radial basis function (RBF) neural networks [8] are used to approximate packaged unknown nonlinearities, and then a novel adaptive neural scheme is developed via backstepping technique for induction motor stochastic derive systems. The proposed adaptive controller guarantees that all the signals in the closed-loop system remain bounded in probability and the tracking error eventually converges to a small area around the origin in the sense of mean quartic value. The simulation results illustrate the effectiveness of the proposed control scheme.

The remainder of the paper is organized as follows. In Section 2, the mathematical model of IM stochastic drive system is described. Then, the adaptive neural control for

IM stochastic system is proposed in Section 3. The simulation results are given in Section 4. Finally, some conclusions are presented.

2. Mathematical Model of IM Stochastic Nonlinear System. The dynamic model of IM system under (d - q) coordinate axis is expressed as follows

$$\begin{cases} \frac{d\omega}{dt} = \frac{n_p L_m}{L_r J} \psi_d i_q - \frac{T_L}{J} \\ \frac{di_q}{dt} = -\frac{L_m^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2} i_q - \frac{n_p L_m}{\sigma L_s L_r} \omega \psi_d - n_p \omega i_d - \frac{L_m R_r}{L_r} \frac{i_q i_d}{\psi_d} + \frac{1}{\sigma L_s} u_q \\ \frac{d\psi_d}{dt} = -\frac{R_r}{L_r} \psi_d + \frac{L_m R_r}{L_r} i_d \\ \frac{di_d}{dt} = -\frac{L_m^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2} i_d + \frac{L_m R_r}{\sigma L_s L_r^2} \psi_d + n_p \omega i_q + \frac{L_m R_r}{L_r} \frac{i_q^2}{\psi_d} + \frac{1}{\sigma L_s} u_d \end{cases}$$
(1)

where  $\sigma = 1 - \frac{L_m^2}{L_r L_s}$ .  $\omega$ ,  $L_m$ ,  $n_p$ , J,  $T_L$  and  $\psi_d$  denote the rotor position, rotor angular velocity, mutual inductance, pole pairs, inertia, load torque and rotor flux linkage.  $i_d$  and  $i_q$  stand for the d - q axis currents.  $u_d$  and  $u_q$  are the d - q axis voltages.  $R_s$  and  $L_s$  mean the resistance and inductance of the stator.  $R_r$  and  $L_r$  denote the resistance and inductance of the stator.  $R_r$  and  $L_r$  denote the resistance and inductance of the rotor. For simplicity, the following notations are introduced:  $x_1 = \omega$ ,  $x_2 = i_q$ ,  $x_3 = \psi_d$ ,  $x_4 = i_d$ ,  $a_1 = \frac{n_p L_m}{L_r}$ ,  $b_1 = -\frac{L_m^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2}$ ,  $b_2 = -\frac{n_p L_m}{\sigma L_s L_r}$ ,  $b_3 = n_p$ ,  $b_4 = \frac{L_m R_r}{L_r}$ ,  $b_5 = \frac{1}{\sigma L_s}$ ,  $c_1 = -\frac{R_r}{L_r}$ , and  $d_2 = \frac{L_m R_r}{\sigma L_s L_r^2}$ . Then, the IM stochastic system can be described in the following form:

$$\begin{pmatrix}
dx_1 = \left(\frac{a_1}{J}x_2x_3 - \frac{T_L}{J}\right)dt + \psi_1^T dw \\
dx_2 = \left(b_1x_2 + b_2x_1x_3 - b_3x_1x_4 - b_4\frac{x_2x_4}{x_3} + b_5u_q\right)dt + \psi_2^T dw \\
dx_3 = \left(c_1x_3 + b_4x_4\right)dt + \psi_3^T dw \\
dx_4 = \left(b_1x_4 + d_2x_3 + b_3x_1x_2 + b_4\frac{x_2^2}{x_3} + b_5u_d\right)dt + \psi_4^T dw
\end{cases}$$
(2)

For stochastic control system dx = f(x) dt + h(x) dw, where f(.) and h(.) are locally Lipchitz functions, and f(0) = 0, h(0) = 0. The following concepts are proposed:

**Definition 2.1.** For any given V(x), define the differential operator L as follows:

$$LV = \frac{\partial V}{\partial x}f + \frac{1}{2}Tr\left\{h^T\frac{\partial^2 V}{\partial x^2}h\right\}$$

**Assumption 2.1.** The sign of  $g_i$  which is defined as the coefficient of  $x_i$  does not change, so there exists constants  $b_m$  and  $b_M$  such that for  $1 \le i \le 4$ ,  $0 < b_m \le g_i \le b_M < \infty$ .

In this paper, RBF neural networks will be used to approximate continuous function, which are used as the form  $f(Z) = W^T S(Z)$ , within which  $Z \in \Omega_Z$  is input vector,  $W = [w_1, w_2, \ldots, w_l]^T$  is weight vector, l > 1 is neural networks node number, and  $S(Z) = [s_1(Z), s_2(Z), \ldots, s_l(Z)]^T$  means basis function vector with  $s_i(Z)$  being used as Gaussian function as follows:  $s_i(Z) = \exp\left[-\frac{(Z-\mu_i)^T(Z-\mu_i)}{n_i^2}\right]$ ,  $i = 1, 2, \ldots, l$ , where  $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{iq}]^T$  is the center of the receptive field, and  $\eta_i$  is the width of Gaussian function. In [9], it has been shown that for f(Z) over a compact set  $\Omega_Z$  with sufficiently large l, for any  $\varepsilon > 0$ , there exists an RBF neural network  $W^T S(Z)$  such as  $f(Z) = W^T S(Z) + \delta(Z), \forall Z \in \Omega_Z$ , where W is ideal weight vector, and  $\delta(Z)$  is approximation error and satisfies  $|\delta(Z)| \leq \varepsilon$ .

3. Adaptive Neural Control for IM Stochastic Nonlinear Systems via Backstepping. In this section, we will present an adaptive neural control for induction motors stochastic nonlinear systems based on backstepping. Step 1: For the reference signal  $y_{d1}$ , define the tracking error variable as  $z_1 = x_1 - y_{d1}$ . From the first differential equation of (2), one has  $\dot{z}_1 = \dot{x}_1 - \dot{y}_{d1}$ . The unknown constant  $\theta$  is specified as  $\theta = \max\left\{\frac{1}{b_m} ||W_i||^2; i = 1, 2, 3, 4\right\}$ .  $\hat{\theta}$  is the estimation of  $\theta$ , and  $\tilde{\theta} = \theta - \hat{\theta}$ . Consider Lyapunov function candidate as  $V_1 = \frac{1}{4}z_1^4 + \frac{1}{2\lambda}b_m\tilde{\theta}^2$ . By (2), one has

$$LV_{1} \leq z_{1}^{3} \left( \frac{a_{1}}{J} x_{2} x_{3} + \bar{f}_{1} \left( Z_{1} \right) \right) - \frac{3}{4} z_{1}^{4} + \frac{3}{4} l_{1}^{2} - \lambda^{-1} b_{m} \tilde{\theta} \hat{\theta}$$

$$\tag{3}$$

where  $l_1$  is a designed positive constant and  $\bar{f}_1(Z_1) = -\frac{T_L}{J} - \dot{y}_{d1} + \frac{3}{4}z_1 + \frac{3}{4}l_1^{-2}z_1 \|\psi_1\|^4$ . According to the universal approximation property of RBF neural networks, for any given  $\varepsilon_1 > 0$ , there exists a neural network  $W_1^T S_1(Z_1)$  such that  $\bar{f}_1(Z_1) = W_1^T S_1(Z_1) + \delta_1(Z_1)$ ,  $|\delta_1(Z_1)| \leq \varepsilon_1$  with  $\delta_1(Z_1)$  being the approximation error. Furthermore, it follows from Young's inequality [10] that

$$z_1^3 \bar{f}_1(Z_1) \le \frac{b_m}{2r_1^2} z_1^6 \theta S_1^T S_1 + \frac{1}{2} r_1^2 + \frac{3}{4} z_1^4 + \frac{1}{4} \varepsilon_1^4$$
(4)

Define  $z_2 = x_2 - \alpha_1$ . Choose the virtual control law  $\alpha_1 = -k_1 z_1 - \frac{1}{2r_1^2} z_1^3 \hat{\theta} S_1^T S_1$ . Using Young's inequality and (4), we can get

$$LV_{1} \leq -v_{1}z_{1}^{4} + \frac{1}{4}\frac{a_{1}x_{3}}{J}z_{2}^{4} + \frac{1}{2}r_{1}^{2} + \frac{3}{4}l_{1}^{2} + \frac{1}{4}\varepsilon_{1}^{4} + \frac{b_{m}}{\lambda}\tilde{\theta}\left(\frac{\lambda}{2r_{1}^{2}}z_{1}^{6}S_{1}^{T}S_{1} - \hat{\theta}\right)$$
(5)

with  $v_1 = \left(k_1 - \frac{3}{4}\right)b_m > 0.$ 

**Step 2:** Now choose the Lyapunov function candidate as  $V_2 = V_1 + \frac{1}{4}z_2^4$ . By Equation (5), one has

$$LV_{2} \leq -v_{1}z_{1}^{4} + \frac{1}{2}r_{1}^{2} + \frac{3}{4}\sum_{i=1}^{2}l_{i}^{2} + \frac{1}{4}\varepsilon_{1}^{4} + \frac{b_{m}}{\lambda}\tilde{\theta}\left(\frac{\lambda}{2r_{1}^{2}}z_{1}^{6}S_{1}^{T}S_{1} - \hat{\theta}\right) + z_{2}^{3}\left(b_{5}u_{q} + \bar{f}_{2}\left(Z_{2}\right)\right) - \frac{3}{4}z_{2}^{4} + z_{2}^{3}\left(\varphi_{2}\left(Z_{2}\right) - \frac{\partial\alpha_{1}}{\partial\hat{\theta}}\hat{\theta}\right)$$

$$(6)$$

where  $\bar{f}_2(Z_2) = \frac{1}{4} \frac{a_1}{J} x_3 z_2 + b_1 x_2 + b_2 x_1 x_3 - b_3 x_1 x_4 - b_4 \frac{x_2 x_4}{x_3} - \frac{\partial \alpha_1}{\partial x_1} \left( \frac{a_1}{J} x_2 x_3 - \frac{T_L}{J} \right) - \sum_{i=0}^{1} \frac{\partial \alpha_1}{\partial y_d^{(i)}} y_d^{(i+1)} - \frac{\partial^2 \alpha_1}{\partial x_2} \psi_1^T \psi_1 + \frac{3}{4} l_2^{-2} z_2 \left\| \psi_2 - \frac{\partial \alpha_1}{\partial x_2} \psi_1 \right\|^4 + \frac{3}{4} z_2 - \omega_2 \left( Z_2 \right) = W_2^T S_2 \left( Z_2 \right) + \delta_2 \left( Z_2 \right) \text{ and } l_2 > 0$ 

$$-\frac{\partial^2 \alpha_1}{2\partial x_1^2} \psi_1^T \psi_1 + \frac{3}{4} l_2^{-2} z_2 \left\| \psi_2 - \frac{\partial \alpha_1}{\partial x_1} \psi_1 \right\| + \frac{3}{4} z_2 - \varphi_2 \left( Z_2 \right) = W_2^T S_2 \left( Z_2 \right) + \delta_2 \left( Z_2 \right) \text{ and } l_2 > 0.$$
  
The smooth function is defined as  $\langle \varphi_2 \left( Z_2 \right) = -k_2 \frac{\partial \alpha_1}{\partial t} \hat{\theta} + \frac{\partial \alpha_1}{\partial t} \frac{\lambda z_1^6}{\delta} S_1^T S_1 - \frac{\lambda S_2^T S_2}{\delta t} z_1^3 \left| z_1^3 \frac{\partial \alpha_1}{\delta t} \right|^2$ 

The smooth function is defined as  $\varphi_2(Z_2) = -k_0 \frac{\partial \alpha_1}{\partial \hat{\theta}} \hat{\theta} + \frac{\partial \alpha_1}{\partial \hat{\theta}} \frac{\lambda z_1^6}{2r_1^2} S_1^T S_1 - \frac{\lambda S_2^T S_2}{2r_2^2} z_2^3 \left| z_2^3 \frac{\partial \alpha_1}{\partial \hat{\theta}} \right|.$ Similarly, for given  $\varepsilon_2 > 0$ , we can get  $z_2^3 \bar{f}_2(Z_2) \leq \frac{b_m}{2r_2^2} z_2^6 \theta S_2^T S_2 + \frac{1}{2}r_2^2 + \frac{3}{4}z_2^4 + \frac{1}{4}\varepsilon_2^4$ . The control law  $u_q$  is designed as  $u_q = -k_2 z_2 - \frac{1}{2r_2^2} z_2^3 \hat{\theta} S_2^T S_2$ . Furthermore, we can obtain

$$LV_{2} \leq \sum_{i=1}^{2} \left( -v_{i}z_{i}^{4} + \frac{1}{2}r_{i}^{2} + \frac{3}{4}l_{i}^{2} + \frac{1}{4}\varepsilon_{i}^{2} \right) + \frac{b_{m}}{\lambda}\tilde{\theta} \left( \sum_{i=1}^{2} \frac{\lambda}{2r_{i}^{2}} z_{i}^{6}S_{i}^{T}S_{i} - \overset{\cdot}{\theta} \right) + z_{2}^{3} \left( \varphi_{2}\left(Z_{2}\right) - \frac{\partial\alpha_{1}}{\partial\hat{\theta}}\overset{\cdot}{\theta} \right)$$

$$(7)$$

with  $v_2 = \left(k_2 - \frac{3}{4}\right)b_m > 0.$ 

**Step 3:** For the reference signal  $y_{d2}$ , define the tracking error variable as  $z_3 = x_3 - y_{d2}$ . Choose the Lyapunov function as  $V_3 = V_2 + \frac{1}{4}z_3^4$ . Similarly, we can obtain

$$LV_{3} \leq \sum_{i=1}^{2} \left( -v_{i}z_{i}^{4} + \frac{1}{2}r_{i}^{2} + \frac{3}{4}l_{i}^{2} + \frac{1}{4}\varepsilon_{i}^{2} \right) + \frac{3}{4}l_{3}^{2} + \frac{b_{m}}{\lambda}\tilde{\theta}\left(\sum_{i=1}^{2}\frac{\lambda}{2r_{i}^{2}}z_{i}^{6}S_{i}^{T}S_{i} - \hat{\theta}\right)$$

L. LIU, Y. MA, J. YU, W. LI AND X. WANG

$$+z_2^3\left(\varphi_2\left(Z_2\right) - \frac{\partial\alpha_1}{\partial\hat{\theta}}\hat{\theta}\right) + z_3^3\left(b_4x_4 + \bar{f}_3\left(Z_3\right)\right) - \frac{3}{4}z_3^4\tag{8}$$

where  $l_3$  is a positive designed constant, and  $\bar{f}_3(Z_3) = c_1 x_3 - \dot{y}_{d2} + \frac{3}{4} z_3 + \frac{3}{4} l_3^{-2} z_3 ||\psi_3||^4 = W_3^T S_3(Z_3) + \delta_3(Z_3)$ . Define  $z_4 = x_4 - \alpha_3$ . Construct the virtual control law  $\alpha_3$  as  $\alpha_3 = -k_3 z_3 - \frac{1}{2r_2^2} z_3^3 \hat{\theta} S_3^T S_3$ . Similarly, for given  $\varepsilon_3 > 0$ , we can obtain

$$LV_{3} \leq \sum_{i=1}^{3} \left( -v_{i}z_{i}^{4} + \frac{1}{2}r_{i}^{2} + \frac{3}{4}l_{i}^{2} + \frac{1}{4}\varepsilon_{i}^{2} \right) + \frac{b_{m}}{\lambda}\tilde{\theta} \left( \sum_{i=1}^{3} \frac{\lambda}{2r_{i}^{2}} z_{i}^{6}S_{i}^{T}S_{i} - \overset{\cdot}{\hat{\theta}} \right)$$
$$+ z_{2}^{3} \left( \varphi_{2}\left(Z_{2}\right) - \frac{\partial\alpha_{1}}{\partial\hat{\theta}}\overset{\cdot}{\hat{\theta}} \right) + \frac{1}{4}b_{4}z_{4}^{4}$$
(9)

with  $v_3 = (k_3 - \frac{3}{4}) b_m > 0.$ 

**Step 4:** At this step, we will construct the control law  $u_d$ . Choose  $V_4 = V_3 + \frac{1}{4}z_4^4$ . Then, we have

$$LV_{4} \leq \sum_{i=1}^{3} \left( -v_{i}z_{i}^{4} + \frac{1}{2}r_{i}^{2} + \frac{3}{4}l_{i}^{2} + \frac{1}{4}\varepsilon_{i}^{2} \right) + \frac{3}{4}l_{4}^{2} + \frac{b_{m}}{\lambda}\tilde{\theta}\left(\sum_{i=1}^{3}\frac{\lambda}{2r_{i}^{2}}z_{i}^{6}S_{i}^{T}S_{i} - \overset{\cdot}{\hat{\theta}}\right) - \frac{3}{4}z_{4}^{4} + z_{2}^{3}\left(\varphi_{2}\left(Z_{2}\right) - \frac{\partial\alpha_{1}}{\partial\hat{\theta}}\overset{\cdot}{\hat{\theta}}\right) + z_{4}^{3}\left(b_{5}u_{d} + \bar{f}_{4}\left(Z_{4}\right)\right) + z_{4}^{3}\left(\varphi_{4}\left(Z_{4}\right) - \frac{\partial\alpha_{3}}{\partial\hat{\theta}}\overset{\cdot}{\hat{\theta}}\right)$$
(10)

where  $l_4 > 0$ , and  $\bar{f}_4(Z_4) = -\frac{\partial \alpha_3}{\partial x_2} \left( b_1 x_4 + d_2 x_3 + b_3 x_1 x_2 + b_4 \frac{x_2^2}{x_3} + b_5 u_d \right) b_1 x_4 + d_2 x_3 + b_3 x_1 x_2 + b_4 \frac{x_2^2}{x_3} = W_4^T S_4(Z_4) + \delta_4(Z_4)$ . For given  $\varepsilon_4 > 0$ ,  $z_4^3 \bar{f}_4(Z_4) \le \frac{b_m}{2r_4^2} z_4^6 \theta S_4^T S_4 + \frac{1}{2}r_4^2 + \frac{3}{4}z_4^4 + \frac{1}{4}\varepsilon_4^4$ . We design  $u_d$  as  $u_d = -k_4 z_4 - \frac{1}{2r_4^2} z_4^3 \theta S_4^T S_4$ . It can be verified easily that

$$LV_{4} \leq \sum_{i=1}^{4} \left( -v_{i}z_{i}^{4} + \frac{1}{2}r_{i}^{2} + \frac{3}{4}l_{i}^{2} + \frac{1}{4}\varepsilon_{i}^{2} \right) + \frac{b_{m}}{\lambda}\tilde{\theta} \left( \sum_{i=1}^{4} \frac{\lambda}{2r_{i}^{2}} z_{i}^{6}S_{i}^{T}S_{i} - \hat{\theta} \right)$$
$$+ z_{2}^{3} \left( \varphi_{2}\left(Z_{2}\right) - \frac{\partial\alpha_{1}}{\partial\hat{\theta}}\hat{\theta} \right) + z_{4}^{3} \left( \varphi_{4}\left(Z_{4}\right) - \frac{\partial\alpha_{3}}{\partial\hat{\theta}}\hat{\theta} \right)$$
(11)

with  $v_4 = k_4 b_m > 0$ . The adaptive law is chosen as

$$\dot{\hat{\theta}} = \sum_{i=1}^{4} \frac{\lambda}{2r_i^2} z_i^6 S_i^T S_i - k_0 \hat{\theta}$$
(12)

where  $k_0$ ,  $\lambda$ ,  $k_i$  and  $r_i$  are positive design parameters for i = 1, 2, 3, 4.

**Theorem 3.1.** Consider the system (2) and the reference signals  $y_{d1}$  and  $y_{d2}$ . Then under the action of the adaptive neural controllers  $u_q$ ,  $u_d$  and the adaptive law (12), the tracking error of the closed-loop controlled system will converge to a small neighborhood of the origin and all the closed-loop signals are bounded.

**Proof:** For the stability analysis of the closed-loop system, choose the following stochastic Lyapunov function as  $V = V_4$ . It follows from (11) and (12) that

$$LV \leq \sum_{i=1}^{4} \left( -v_i z_i^4 + \frac{1}{2} r_i^2 + \frac{3}{4} l_i^2 + \frac{1}{4} \varepsilon_i^2 \right) + \frac{k_0 b_m}{\lambda} \tilde{\theta} \hat{\theta}$$
$$+ z_2^3 \left( \varphi_2 \left( Z_2 \right) - \frac{\partial \alpha_1}{\partial \hat{\theta}} \hat{\theta} \right) + z_4^3 \left( \varphi_4 \left( Z_4 \right) - \frac{\partial \alpha_3}{\partial \hat{\theta}} \hat{\theta} \right)$$
(13)

For the term  $\frac{k_0 b_m}{\lambda} \tilde{\theta} \hat{\theta}$ , the following inequality is obvious.

$$\frac{k_0 b_m}{\lambda} \tilde{\theta} \hat{\theta} = -\frac{k_0 b_m}{\lambda} \tilde{\theta}^2 + \frac{k_0 b_m}{\lambda} \tilde{\theta} \theta \le -\frac{k_0 b_m}{2\lambda} \tilde{\theta}^2 + \frac{k_0 b_m}{2\lambda} \theta^2 \tag{14}$$

1750

By using (12) and  $\varphi_2(Z_2)$ , we have

$$-z_2^3 \frac{\partial \alpha_1}{\partial \hat{\theta}} \hat{\theta} \le k_0 z_2^3 \frac{\partial \alpha_1}{\partial \hat{\theta}} \hat{\theta} - z_2^3 \frac{\partial \alpha_1}{\partial \hat{\theta}} \frac{\lambda}{2r_1^2} z_1^6 S_1^T S_1 + \sum_{i=2}^3 \frac{\lambda}{2r_i^2} z_i^6 S_i^T S_i \left| z_2^3 \frac{\partial \alpha_1}{\partial \hat{\theta}} \right| = -z_2^3 \varphi_2 \left( Z_2 \right)$$

which implies that the term  $z_2^3 \left( \varphi_2(Z_2) - \frac{\partial \alpha_1}{\partial \hat{\theta}} \hat{\theta} \right)$  is negative. Similarly, we can get the result that the term  $z_4^3 \left( \varphi_4(Z_4) - \frac{\partial \alpha_3}{\partial \hat{\theta}} \hat{\theta} \right)$  is also negative. Furthermore, we can obtain

$$LV \le \sum_{i=1}^{4} \left( -v_i z_i^4 + \frac{1}{2} r_i^2 + \frac{3}{4} l_i^2 + \frac{1}{4} \varepsilon_i^2 \right) - \frac{k_0 b_m}{2\lambda} \tilde{\theta}^2 + \frac{k_0 b_m}{2\lambda} \theta^2$$
(15)

Next, let  $a_0 = \min \{4v_i, k_0, i = 1, 2, 3, 4\}$ , and  $b_0 = \frac{1}{2} \sum_{i=1}^4 r_i^2 + \frac{3}{4} \sum_{i=1}^4 l_i^2 + \frac{1}{4} \sum_{i=1}^4 \varepsilon_i^2 + \frac{k_0 b_m}{2\lambda} \theta^2$ , then (15) can be rewritten in the following form

$$LV \le -a_0 V + b_0, \quad t \ge 0.$$
 (16)

Therefore,  $z_i$  and  $\hat{\theta}_i$  are bounded in probability.  $\alpha_i$  is also bounded in probability because  $||S_i|| \leq s$ . Consequently, all the signals in the closed-loop system remain bounded in the sense of probability. Furthermore, (16) and [11] imply that  $\frac{dE[V(t)]}{dt} \leq -a_0 E[V(t)] + b_0$ .

Thus, to guarantee that the tracking error converges to a small neighborhood around the origin, we can properly adjust the parameters  $a_0$  and  $b_0$ .

4. Simulation Results. In order to illustrate the effectiveness of the proposed results, the simulation is run for the induction motor with the parameters:  $J = 0.0586 \text{Kgm}^2$ ,  $R_s = 0.1\Omega$ ,  $R_r = 0.15\Omega$ ,  $L_s = L_r = 0.0699 \text{H}$ ,  $L_m = 0.068 \text{H}$ , and  $n_p = 1$ . The simulation is carried out under the initial condition  $x_1 = 0$ ,  $x_2 = 0$ ,  $x_3 = 1$  and  $x_4 = 0$ . The reference signals are taken as  $y_{d1} = 0.5 (\sin(t) + \sin(0.5t))$ , and  $y_{d2} = 1$ . And load torque  $T_L$  is chosen as  $T_L = \begin{cases} 0.5, & 0 \le t \le 5, \\ 1.0, & t \ge 5. \end{cases}$ 

The RBF NNs are chosen in the following way. The NNs contain eleven nodes with centers spaced evenly in the interval [-9, 9] and widths being equal to 2, respectively. The proposed adaptive neural controllers are used to control the induction motor. In order to get the proper  $a_0$  and  $b_0$ , the control parameters are chosen as:  $k_1 = 60$ ,  $k_2 = 100$ ,  $k_3 = 200$ ,  $k_4 = 120$ ,  $r_1 = r_2 = 2$ ,  $r_3 = r_4 = 4$ ,  $\lambda = 1$ , and  $k_0 = 0.01$ . Figure 1 shows the reference signal  $x_1$  and  $y_{d1}$  and Figure 2 shows the reference signal  $x_3$  and  $y_{d2}$ . It can be observed from Figure 1 and Figure 2 that the system can track the given reference signal well. Figure 3 and Figure 4 show the trajectories of  $u_q$  and  $u_d$ . It can be seen that the controllers are bounded. From the above simulation results, it is clearly seen that the proposed controllers can track the reference signal quite well even under parameter uncertainties and load torque disturbance.

5. **Conclusions.** Based on backstepping technique, an adaptive NN control method is developed to control the induction motors stochastic drive system. The designed controllers guarantee the speed tracking error can converge to a small neighborhood of the origin. The simulation results show that the proposed adaptive neural network speed controllers can overcome the influences of nonlinearities and make sure that the proposed approach track the given tracking signal well. In the future work, we will focus on the practical application of the proposed control algorithm.



FIGURE 1. Trajectories of  $x_1$  and  $y_{d1}$ 



FIGURE 2. Trajectories of  $x_3$  and  $y_{d2}$ 



FIGURE 3. Trajectory of the control law  $u_q$ 



FIGURE 4. Trajectory of the control law  $u_d$ 

Acknowledgment. This work was partially supported by the Natural Science Foundation of China (61573204, 61573203, 61501276), Shandong Province Outstanding Youth Fund (ZR2015JL022) and the China Postdoctoral Science Foundation (2014T70620, 2013 M541881, 201303062) and Qingdao Postdoctoral Application Research Project.

## REFERENCES

- F. Shi, Y. Ma, J. Yu, H. Yu and Z. Qu, Neural network-based adaptive dynamic surface control for induction motors, *ICIC Express Letters, Part B: Applications*, vol.6, no.10, pp.2869-2875, 2015.
- [2] C. M. Kwan and F. L. Lewis, Robust backstepping control of induction motors using neural networks, *IEEE Trans. Neural Networks*, vol.11, no.5, pp.1178-1187, 2000.
- [3] B. Chen, X. Liu, K. Liu and C. Lin, Direct adaptive fuzzy control of nonlinear strict-feedback systems, *Automatica*, vol.45, no.6, pp.1530-1535, 2009.
- [4] J. Yu, Y. Ma, B. Chen and H. Yu, Adaptive fuzzy tracking control for induction motors via backstepping, *ICIC Express Letters*, vol.5, no.2, pp.425-431, 2011.
- [5] A. Stotsky, J. K. Hedrick and P. P. Yip, The use of sliding modes to simplify the backstepping control method, *Proc. of the American Control Conference*, vol.3, no.7, pp.1703-1708, 1997.
- [6] L. A. Zadeh, Fuzzy sets, Inform. Contr., vol.8, no.3, pp.338-353, 1965.
- [7] J. Yu, P. Shi, W. Dong, B. Chen and C. Lin, Neural network-based adaptive dynamic surface control for permanent magnet synchronous motors, *IEEE Trans. Neural Netw. Learn. Syst.*, DOI: 10.1109/TNNLS.2014.2316289, 2014.
- [8] X. Luan, F. Liu and P. Shi, Neural network based stochastic optimal control for nonlinear Markov jump systems, *International Journal of Innovative Computing*, *Information and Control*, vol.6, no.8, pp.3715-3723, 2010.
- [9] R. M. Sanner and J. E. Slotine, Gaussian networks for direct adaptive control, *IEEE Trans. Neural Networks*, vol.3, no.6, pp.837-863, 1992.
- [10] Z. Yu and H. Du, Adaptive neural tracking control for stochastic nonlinear systems with time-varying delay, *Journal of Control Theory and Applications*, vol.28, no.2, pp.1808-1812, 2011.
- [11] H. Deng, M. Krstic and R. Willians, Stabilization of stochastic nonlinear systems driven by noise of unknown covariance, *IEEE Trans. Automatic Control*, vol.46, no.8, pp.1237-1253, 2001.