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FRACTAL IMAGE MAGNIFICATION BASED ON COMPANION IMAGES

Qiang Wang
College of Information Science and Technology
Dalian Maritime University
No. 1, Linghai Road, Dalian 116026, P. R. China
wangqiang2011@dlmu.edu.cn

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Abstract. *Based on the resolution independence of fractal coding, a novel image magnification method is proposed in this paper. Firstly, fractal coding is a lossy image coding method and the lost information in fractal encoding process will lead to the distortion of decoded images. Then, the companion image is introduced to extend the domain block pool and reduce the lost information in the fractal encoding process. Furthermore, an accelerated method is also proposed to help us select the optimal companion image. Experiments show that compared with other similar methods, the proposed method can provide the magnified images with better quality.*
Keywords: Fractal coding, Image magnification, Companion image

1. **Introduction.** Since fractal image coding itself has many attractive characteristics, such as the novel idea, fast decoding, resolution independence and the potential high compression ratio [1,2], fractal image coding has attracted much attention of the researchers worldwide in the past two decades. Except image compression [3,4], image magnification is another successful application for fractal image coding. Firstly, fractal image coding is a lossy image compression method and the fractal encoding itself will result in some lossy information. Then, the image magnification is realized in the fractal decoding process. The information lost in the encoding process will lead to distortion of decoded images. In order to overcome this problem, Chung et al. proposed an enhancement layer [5]. Lai et al. proposed pixel averaging and error compensation and incorporated them into an improved range block partitioning strategy [6]. All the operations adapted above are to reduce the lost information in the fractal encoding process. In order to accelerate the fractal encoding process furthermore, Wee and Shin incorporated the no-search fractal coding method into image magnification [7]. Wang et al. introduced a lossless no-search method [8]. In addition, Zhang et al. proposed a constraint model which can improve the quality of magnified images after the fractal decoding process [9]. In our research, we propose another way to reduce the collage errors in the encoding process. Firstly, we verify that for one input image, if we encode it with another image which is named as the companion image simultaneously, there will be a larger domain block pool and this can help to reduce the matching errors of range blocks. According to the collage theorem [10], we know that the lost information in the encoding process can be reduced. In addition, we need to select an optimal companion image. Since the complexity of fractal encoding is high, selecting the optimal companion image will bring a large amount of computations. In order to overcome this problem, an accelerated method is also proposed to shorten this process. Since the amount of lost information is proportional to collage errors, a lower limit of the percentage of accumulated collage error (LLPACE) is proposed to estimate the collage errors of all range blocks. While LLPACE reaches a large proportion, such as 90%, the collage errors of all range blocks can be calculated. Finally, in order to verify

the effectiveness of our method, the corresponding experimental procedures and results will be given in detail.

This paper is organized as follows. The conventional image magnification method based on fractal coding is reviewed in Section 2. In Section 3, the proposed method will be analyzed and represented in detail. In Section 4, the procedures of our method will be described and the performance comparison between the proposed method and the other four methods will be also given. Finally, the conclusion will be given in Section 5.

2. Review of Conventional Fractal Image Magnification.

Fractal image magnification consists of encoding process and decoding process. Firstly, the fractal encoding process is to establish an iterated functional system (IFS) whose fixed point can approximate the input image well. The procedures of the fractal encoding process can be described as follows.

Step 1: Partition the input image into non-overlapping range blocks. Domain blocks can be obtained by sliding a window over the input image.

Step 2: Establish a domain block pool by performing eight isometric transformations and an affine transformation on the domain blocks.

Step 3: For each range block, search the best matched domain block and calculate the affine transformation coefficients by minimizing the following equation

$$Error = \min_{s,o} \|\boldsymbol{R} - s\boldsymbol{D} - o\boldsymbol{I}\|^2 \tag{1}$$

where $\boldsymbol{R}$ and $\boldsymbol{D}$ denote the range block and the contracted domain block, respectively. $\boldsymbol{I}$ denotes a matrix whose elements are all ones. s and o are the coefficients for the affine transformation. $Error$ is the collage error between $\boldsymbol{R}$ and $\boldsymbol{D}$.

At the fractal decoding phase, the fractal decoding process aims to reconstruct an image with high resolution. We first select an initial image whose size is two times larger than the input image. The sizes of range blocks and domain blocks are also set to be two times larger than those in the encoding process. Then, by making use of the information stored in the fractal encoding process, the corresponding transformations are recursively performed on the initial image. After about ten iterations, the decoding process will converge to the final magnified image. In the fractal decoding process, the quality of decoded images in our research is measured by peak signal to noise ratio (PSNR) as follows:

$$PSNR = 10\log_{10}\left(255^2 \Big/ \left(\frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(\boldsymbol{I}_{ij} - \boldsymbol{I}^*_{ij})^2\right)\right) \tag{2}$$

where M and N denote the width and the height of the input image. $\boldsymbol{I}$ and $\boldsymbol{I}^*$ represent the original image and the distorted image, respectively.

3. The Proposed Method.

Fractal image coding is a lossy image compression method. According to the collage theorem, the collage errors in the encoding process provide the upper limit of the reconstructed errors in the decoding process. Reducing the collage errors will help to improve the quality of decoded images. Since the collage errors come from the dismatch between the range blocks and their corresponding best matched domain blocks, extending the domain block pool is an effective way to solve this problem. Firstly, we take out another image which is named as the companion image. Then, we incorporate it and the input image into one image and encode it. Since the companion image can provide more domain blocks which will increase the possibility of better block matching, the collage errors will be reduced and more useful information will be reserved. In Table 1, we select three 256×256 images, Bridge, Lena and Camera, as the companion images and encode them with the input image, Bird, respectively. We can clearly see that the companion images can effectively help to improve the quality of the decoded images.

Table 1. Illustration of the PSNR improvements while encoding the input image with three companion images respectively

Images	Bird	Bird+Bridge	Bird+Lena	Bird+Camera
PSNR(dB)	38.36	38.69	39.22	39.01

We can also see that the input image with different companion images will result in the decoded images with different quality. In order to obtain the optimal companion image conveniently, an improved selection method is also proposed in our research.

In order to minimize (1), we set its derivatives with respect to s and o to be zeros, respectively and can get

$$s = \left\langle \boldsymbol{R} - \bar{r}\boldsymbol{I}, \boldsymbol{D} - \bar{d}\boldsymbol{I} \right\rangle / \|\boldsymbol{D} - \bar{d}\boldsymbol{I}\|^2, \quad o = \bar{r} - s\bar{d} \tag{3}$$

Substituting (3) back to (1) yields

$$Error = \|\boldsymbol{R} - s\boldsymbol{D} - o\boldsymbol{I}\|^2 = \|\boldsymbol{R} - r\boldsymbol{I}\|^2 - s^2\|\boldsymbol{D} - d\boldsymbol{I}\|^2 \leq \|\boldsymbol{R} - r\boldsymbol{I}\|^2 \tag{4}$$

From (4), we know that the collage error of one range block is smaller than its variance. For arbitrary image, we first partition it into range blocks and sort them by their variances from the largest to the smallest. Then, a range block sequence is established and the range blocks are encoded one by one. From (4), we know that the accumulated variances (AV) are always larger than the accumulated collage errors (ACE). By making use of the relationship between AC and ACE, we can get the following inequality

$$\underset{\text{(LLPACE)}}{\frac{\text{ACE}_{\text{Encoded}}}{\text{ACE}_{\text{Encoded}} + \text{AV}_{\text{The rest}}}} \leq \underset{\text{(APACE)}}{\frac{\text{ACE}_{\text{Encoded}}}{\text{ACE}_{\text{Encoded}} + \text{ACE}_{\text{The rest}}}} \tag{5}$$

The right part of (5) represents the actual percentage of ACE, and we call it APACE. Since AV is larger than the corresponding ACE, and the left part of (5) is smaller than its right part and provides the lower limit of the percentage of ACE, we call it LLPACE. LLPACE provides us an effective way to estimate APACE. In summary, the procedures of the above method can be described as follows.

Step 1: For one input image, partition it into range blocks. Sort them by their variances from the largest to the smallest.

Step 2: With the same order of variances, take out one range block and encode it.

Step 3: Calculate LLPACE. If LLPACE reaches 90%, APACE will be limited in 90%-100% and go to Step 4; if not return to Step 2.

Step 4: By making use of ACE, calculate the estimated average collage error (ACER) with (6).

$$\text{ACER} \approx \frac{\text{ACE}}{\text{Num}} \Big/ \left(\frac{90\% + 100\%}{2}\right) \tag{6}$$

where Num denotes the total number of range blocks and is used to convert ACE into ACER. $(90\% + 100\%)/2$ denotes the estimated APACE.

4. **Experiments.** Four 256×256 images, Lena, Couple, Camera and Zelda, are used in the experiment. Correspondingly, the contracted version of the above four images can be obtained by averaging every four neighboring pixels and considered as test images. In addition, the size of range blocks is 4×4. The size of domain blocks and the search step are both set to be 8.

4.1. **Selection of optimal companion image.** In this part, the contracted Lena is taken as the input image, and the other three contracted images are considered as the candidate companion images. When we incorporate the input image and arbitrary candidate companion image into one image, by making use of the method in Section 3, the one that can provide smallest collage error will be considered as the optimal companion image. Table 2 illustrates the comparison between the conventional fractal encoding and the accelerated method by ACER and the amount of computations. We can see that the same as the actual ACER, the estimated ACER can also help us to select the optimal companion image, the contracted Camera, and reduce about one third of total computations.

Table 2. Comparison between the conventional fractal encoding and the accelerated method

Input image (contracted)	Candidate optimal companion images (contracted)	Conventional fractal encoding		Accelerated method	
		ACER	Amount of computations	Estimated ACER	Amount of computations
Lena	Bird	57.78	100%	60.40	69.43%
	Camera	57.03	100%	59.54	67.48%
	Zelda	60.97	100%	63.69	67.09%

4.2. **Performance of the proposed method.** We will compare the proposed method with the other four image magnification methods. The procedures of the proposed method are listed as follows.

Step 1: For one image, the original image and its contracted version are regarded as the reference image and the input image respectively.

Step 2: Select the optimal image with the method in Section 4.1.

Step 3: Incorporate the input image and the optimal companion image into one image. By performing the fractal encoding and decoding methods on the synthetic image as Section 2, we can get the magnified image.

Step 4: Compare the reference image and the magnified image.

Table 3 illustrates the experimental results while selecting different input images. From the results, we can observe that compared with the conventional image magnification methods, such as the bilinear method and the bicubic method, and the other two methods in [5] and [6], the proposed method can improve the quality of decoded images obviously.

5. **Conclusion.** In this paper, a fractal image magnification method based on companion images is proposed. Firstly, the companion image can help to reduce the lost information in the fractal encoding process. This can effectively improve the quality of the magnified

Table 3. Performance comparison between the proposed method and the other four methods

Input images (contracted)	Bilinear	Bicubic	Method in [6]	Method in [5]	Proposed method	
					Optimal companion image (contracted)	PSNR (dB)
Lena	28.51	29.75	30.46	30.40	Camera	30.76
Couple	31.56	32.64	32.94	32.93	Lena	33.02
Camera	25.66	26.53	27.28	27.13	Lena	27.42
Zelda	32.71	34.02	34.22	34.27	Lena	34.49

image. Then, since the computational complexity of selecting the optimal companion image is high, an accelerated method is also introduced. Finally, simulations show that our method can provide better performance than two conventional methods and the other two similar methods. In future research, we will attempt to develop more effective methods to select the optimal companion image.

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REFERENCES

[1] A. E. Jacquin, Fractal image coding: A review, *Proc. of the IEEE*, vol.81, no.10, pp.1451-1456, 1993.
[2] B. Wohlberg and G. Jager, A review of the fractal image coding literature, *IEEE Trans. Image Processing*, vol.8, no.12, pp.1716-1729, 1999.
[3] S. L. Du, Y. P. Yan and Y. D. Ma, Quantum-accelerated fractal image compression: An interdisciplinary approach, *IEEE Signal Processing Letters*, vol.22, no.4, pp.499-503, 2015.
[4] Y. Y. Sun, R. D. Xu, L. N. Chen, R. Q. Kong and X. P. Hu, A novel fractal coding method based on M-J sets, *Plos One*, vol.9, no.7, pp.1-11, 2014.
[5] K. H. Chung, Y. H. Fung and Y. H. Chan, Image enlargement using fractal, *Proc. of IEEE International Conference on Acoustics, Speech and Signal Processing*, Hong Kong, pp.273-276, 2003.
[6] C. M. Lai, K. M. Lam, Y. H. Chan and W. C. Siu, An efficient fractal based algorithm for image magnification, *Proc. of International Symposium on Intelligent Multimedia, Video and Speech Processing*, Hong Kong, pp.571-574, 2004.
[7] Y. C. Wee and H. J. Shin, A novel fast fractal super resolution technique, *IEEE Trans. Consumer Electronics*, vol.56, no.3, pp.1537-1541, 2010.
[8] Q. Wang, D. Q. Liang and S. Bi, Image magnification based on lossless no-search fractal coding, *ICIC Express Letters, Part B: Applications*, vol.4, no.1, pp.75-79, 2013.
[9] X. L. Zhang, L. S. Shen and K. M. Lam, Image magnification based on fractal codes and model constraint, *Acta Electronica Sinica*, vol.34, no.3, pp.433-436, 2006.
[10] Y. Fisher, *Fractal Image Compression: Theory and Application*, Springer-Verlag, New York, 1994.