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ADAPTIVE FEEDBACK LINEARIZATION TOGETHER WITH A COMPUTED TORQUE CONTROLLER FOR THE PATH TRACKING OF A NON-HOLONOMIC MOBILE ROBOT

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Abstract. *Mobile robots have attracted considerable attention for their common applications in a wide field. In any application, the high motion accuracy is essential to guarantee the security and availability for these robots. Many studies have focused on the development of control law for these robots to address the non-holonomic issues based on the kinematic model. However, the control inputs designed based on the kinematics are just the velocities which are not the forces for the actual drivers. In this paper, to improve the motion accuracy, we propose a double closed-loop control system for the robot based on the kinematics and dynamics together. Firstly, to deal with the unknown disturbances considering the non-holonomic issues, an adaptive feedback linearization controller is proposed based on kinematic model. The kinematics based controller can generate the given velocities for the robot to achieve precise steering. Then, a computed torque controller is proposed based on the dynamic model to generate the torques for the actual drivers to accomplish the tracking of given velocities. Finally, path and trajectory tracking simulations are conducted to demonstrate the effectiveness of the proposed controller.*
Keywords: Mobile robot, Non-holonomic constraint, Adaptive feedback linearization, Computed torque controller, Unknown disturbances

1. **Introduction.** Mobile robots with two independently driven wheels suitable for a variety of applications have been studied for many years [1]. These applied mobile robots include the autonomous guided vehicle (AGV) robots which are most used to transport materials by following markers or wires on the floor [2,3], a helpmate robot which is a robotic materials transport system designed for hospitals and nursing homes [4], the minerva which is a talking robot designed to accommodate people in public spaces [5], the wheelesley which is a robotic wheelchair system developed in MIT [6], and floor-cleaning robots [7]. All of these developed robots need the high-precision movement to track a predefined path or follow up some marks on the floor to finish their tasks. Therefore, to ensure their security and availability, the high path and trajectory tracking performance is essential for these robots to precisely track the path moving from the starting location to the target location.

It is well known that this type of robot is a non-holonomic system which is a challenge for the development of control law. Thus, the motion control of these robots has attracted considerable attention in recent years. The backstepping methodology is a famous way to address the problem of non-holonomic constraints [8,9], such as the adaptive tracking controller based on the backstepping approach in [10,11], and the robust adaptive visual stabilizing controller with the utilization of backstepping technique proposed in [12]. In addition, a control method incorporating neural-dynamic optimized model predictive approach is proposed in [13] to address the issues of non-holonomic. Furthermore, authors in [14] developed a control method by using non-autonomous chaotic system to deal with the non-holonomic problem. However, these controllers are all designed based on the kinematic model, and the control inputs obtained by kinematics based controller are just the linear and angular velocities which have no direct relationship with the actual control variables, which are in general the forces or torques. Therefore, to further improve the motion performances, a controller to generate the actual torques for the robot based on dynamic model and the linear and angular velocities is necessary. A method of optimal control at energy performance index considering the dynamics of mobile robots was proposed in [15]. However, this method cannot deal with the non-holonomic problem. A nonlinear control for tracking and obstacle avoidance of this kind of robot was developed in [16]. Although considering the dynamics, this controller focuses on the special issues of the obstacle avoidance. Therefore, in this paper, we proposed a double closed-loop control system that enables the integration of kinematics and dynamics to improve the path tracking accuracy of the robot. Firstly, to address the unknown disturbances issues, a novel adaptive feedback linearization control law based on the kinematics was proposed to determine the given linear and angular velocities. This controller can ensure the asymptotical stability of the position and orientation for the robot to repress the unknown disturbances and can actively address the non-holonomic problem by the feedback linearization. Then, a computed torque controller was proposed based on the dynamics to design the actual torques for the robot to ensure the tracking accuracy of velocities. Finally, path tracking simulations were conducted to verify the effectiveness of the proposed control method.

This paper is organized as follows. Section 2 describes the structure of the mobile robot and derives the kinematic and dynamic model. A double closed-loop controller based on the kinematics and dynamics is proposed for the robot in Section 3. Section 4 shows the simulations of the proposed control method and the results show that the proposed control method is feasible and effective. A conclusion is given in Section 5.

2. **Wheeled Mobile Robot and Modeling.** In our laboratory, a mobile robot with two independently driven wheels shown in Figure 1 was developed [17]. It consists of two driving wheels, and two free omni-directional wheels at the front and back of the vehicle. The robot can only move in the direction normal to the axis of the driving wheels and this feature is well known as the non-holonomic constraint.

To derive the control law for the wheeled mobile robot, a schematic structure of the mobile robot at the 2-D plane was set up shown in Figure 2.

The parameters and coordinate systems were defined as follows. $\Sigma(x_c, y_c, O)$: absolute coordinate system; G: position of the center of gravity considering the effect of loads; C: position of the geometric center; d: distance between the center of gravity and the geometrical center; v_c: linear velocity at point C; ϕ: mobile robot orientation defined as

FIGURE 1. Wheeled mobile robot

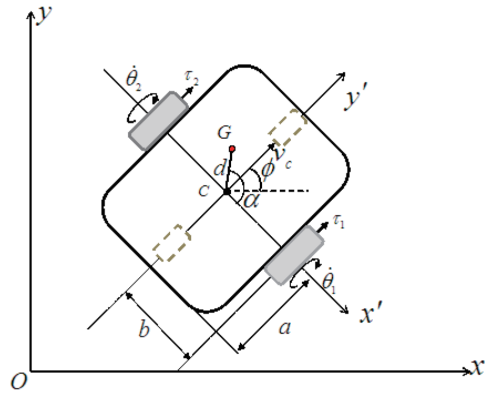

FIGURE 2. Structural model of the mobile robot

the angle between the direction of v_c and the x-axis; ω: angular velocity of the robot; τ_1, τ_2: driving torques of the two driving wheels; α: angle between Cx' and CG; $2b$: distance between the driving wheels; $2a$: length of the robot; $\dot{\theta}_1$, $\dot{\theta}_2$: angular speeds of the two driving wheels.

First, a kinematic analysis of the robot nonlinear system was carried out based on the 2-D coordinate system shown in Figure 2, considering the input uncertainties case, its kinematics model is shown as Equation (1).

$$\begin{cases} \dot{x}_c = (v_c + \xi_v)\cos\phi \\ \dot{y}_c = (v_c + \xi_v)\sin\phi \\ \dot{\phi} = \omega + \xi_\omega \end{cases} \tag{1}$$

where, ξ_v and ξ_ω are the unknown input disturbances. The dynamic equations for the mobile robot based on the Lagrange formalism were derived as follows.

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \boldsymbol{M} \begin{bmatrix} \dot{v}_c \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \dfrac{m_{11}+m_{12}}{r} & \dfrac{m_{11}-m_{12}}{r}b \\ \dfrac{m_{21}+m_{22}}{r} & \dfrac{m_{21}-m_{22}}{r}b \end{bmatrix} \begin{bmatrix} \dot{v}_c \\ \dot{\omega} \end{bmatrix} \tag{2}$$

where

$$\begin{aligned} m_{11} &= \frac{r^2(M+m)}{4b^2}(b+d\cos\alpha)^2 + \frac{r^2[I+d^2(M+m)]}{4b^2} + (J_\omega + k^2 J_0) \\ m_{12} &= \frac{r^2(M+m)}{4b^2}(b+d\cos\alpha)(b-d\cos\alpha) - \frac{r^2[I+d^2(M+m)]}{4b^2} \\ m_{21} &= m_{12} \\ m_{22} &= \frac{r^2(M+m)}{4b^2}(b-d\cos\alpha)^2 + \frac{r^2[I+d^2(M+m)]}{4b^2} + (J_\omega + k^2 J_0) \end{aligned}$$

where, M is the robot mass and m is the mass of the applied load. I is the moment of inertia of the robot, J_ω is the moment of inertia of the driving wheels, J_0 is the moment of inertia of the DC motor, r is the radius of driving wheels, and k is the gear ratio.

3. **Double Closed-Loop Control System.** Based on the kinematic model (1) and dynamic model (2), a double closed-loop control system was proposed for the mobile robot, which consists of the inner loop of dynamics based feedback controller and the outer loop of the kinematics based controller as shown in Figure 3. The outer loop is used to generate the given velocity for the dynamic controller from desired trajectory, and the inner loop is used to generate the necessary torque for the robot to precisely track the given velocity.

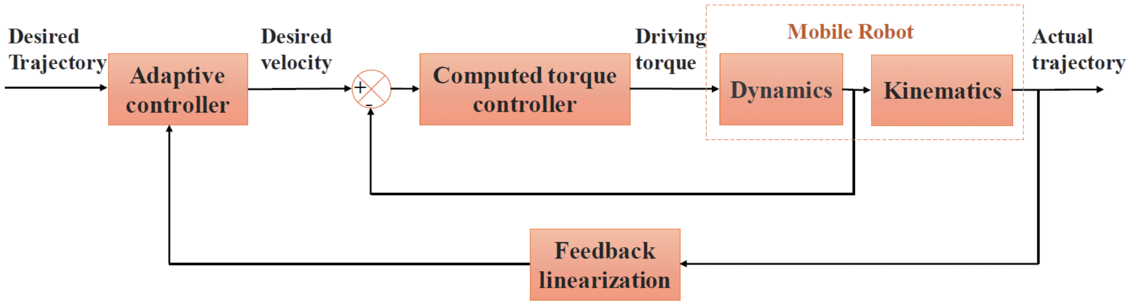

FIGURE 3. Double closed-loop control system of the mobile robot

3.1. **Adaptive feedback linearization controller.** To design the control law for the robot, firstly, kinematic model considering the input disturbances was transformed into the following third-order error chained form:

$$\begin{cases} \dot{\widetilde{x}}_1 = \widetilde{u}_1 + \widetilde{\xi}_\omega \\ \dot{\widetilde{x}}_2 = \widetilde{u}_2 + \widetilde{\xi}_v - \widetilde{x}_3\widetilde{\xi}_\omega \\ \dot{\widetilde{x}}_3 = \widetilde{x}_2(\omega_d - \widetilde{u}_1) + \widetilde{x}_2\widetilde{\xi}_\omega \end{cases} \tag{3}$$

using the following coordinate transformations and input

$$\begin{cases} \widetilde{x}_1 = e_\phi \\ \widetilde{x}_2 = e_x\cos\phi + e_y\sin\phi \\ \widetilde{x}_3 = e_x\sin\phi - e_y\cos\phi \end{cases}$$

and

$$\begin{cases} \widetilde{u}_1 = e_\omega \\ \widetilde{u}_2 = e_v - x_3 e_\omega \end{cases}$$

where, e_x, e_y and e_ϕ are the tracking errors of x, y position and orientation angle ϕ respectively. e_v and e_ω are the tracking error of the linear and angular velocities. $\widetilde{\xi}_v$ and $\widetilde{\xi}_\omega$ are the estimated errors of the unknown inputs disturbances.

Theorem 3.1. *Consider the error chained form (3) with a feedback linearization controller and adaptive control law are given as*

$$\begin{cases} \widetilde{u}_1 = -k_1\widetilde{x}_1 \\ \widetilde{u}_2 = -k_1\widetilde{x}_1\widetilde{x}_3 - k_2\widetilde{x}_2 - \omega_d\widetilde{x}_3 \end{cases} \tag{4}$$

$$\begin{cases} \dot{\widetilde{\xi}}_\omega = -\Gamma_1\widetilde{x}_1 \\ \dot{\widetilde{\xi}}_v = -\Gamma_3\widetilde{x}_2 \end{cases} \tag{5}$$

where k_1, k_2, Γ_1 and Γ_3 are positive constant numbers, and then the error state system is table. Then, the stability analysis of the position tracking is given as follows.

Proof: Define the Lyapunov function as

$$V = \frac{1}{2}\left(\widetilde{x}_1^2 + \frac{1}{\Gamma_1}\widetilde{\xi}_\omega^2 + \widetilde{x}_2^2 + \widetilde{x}_3^2 + \frac{1}{\Gamma_3}\widetilde{\xi}_v^2\right) \geq 0 \tag{6}$$

Then, the time derivative of V is given as

$$\begin{aligned} \dot{V} &= \widetilde{x}_1\dot{\widetilde{x}}_1 + \frac{1}{\Gamma_1}\widetilde{\xi}_\omega\dot{\widetilde{\xi}}_\omega + \widetilde{x}_2\dot{\widetilde{x}}_2 + \widetilde{x}_3\dot{\widetilde{x}}_3 + \frac{1}{\Gamma_3}\widetilde{\xi}_v\dot{\widetilde{\xi}}_v \\ &= \widetilde{x}_1\widetilde{u}_1 + \widetilde{\xi}_\omega\left(\widetilde{x}_1 + \frac{1}{\Gamma_1}\dot{\widetilde{\xi}}_\omega\right) + \widetilde{x}_2\left(\widetilde{u}_2 - \widetilde{x}_3\widetilde{\xi}_\omega\right) + \widetilde{x}_3\widetilde{x}_2\left(\omega_d - \widetilde{u}_1 + \widetilde{\xi}_\omega\right) + \widetilde{\xi}_v\left(\widetilde{x}_2 + \frac{1}{\Gamma_3}\dot{\widetilde{\xi}}_v\right) \\ &= -k_1\widetilde{x}_1^2 + \widetilde{x}_2(-k_1\widetilde{x}_1\widetilde{x}_3 - k_2\widetilde{x}_2 - \omega_d\widetilde{x}_3) + \widetilde{x}_3\widetilde{x}_2\left(\omega_d + k_1\widetilde{x}_1\right) \end{aligned}$$

$$= -k_1\widetilde{x}_1^2 - k_2\widetilde{x}_2^2 \leq 0 \tag{7}$$

Therefore, the error chained form (3) is asymptotically stable.

Then, we can obtain the control input v_{cr} and ω_r for the mobile kinematic system (1) as

$$\begin{cases} v_{cr} = v_d - \widetilde{u}_2 - \widetilde{\xi}_v + (x\sin\phi - y\cos\phi)\left(\omega_d - \widetilde{u}_1 - \widetilde{\xi}_\omega\right) \\ \omega_r = \omega_d - \widetilde{u}_1 - \widetilde{\xi}_\omega \end{cases} \tag{8}$$

where, $v_d = \sqrt{\dot{x}_{cd}^2 + \dot{y}_{cd}^2}$, $\phi_d = \arctan\left(\dot{y}_{cd}/\dot{x}_{cd}\right)$, and $\omega_d = \left(\ddot{y}_{cd}\dot{x}_{cd} - \ddot{x}_{cd}\dot{y}_{cd}\right)/v_d^2$, where, x_{cd} and y_{cd} are the desired trajectories of robot in x and y directions respectively.

3.2. **Computed torque controller.** The reference linear and angular velocities had been designed by the adaptive feedback linearization controller shown in Subsection 3.1. In this subsection, a computed torque controller is proposed. This controller takes account of the specific vehicle dynamics to convert a steering system command into the torque for the actual robot. The control method is given as:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \boldsymbol{M} \begin{bmatrix} \dot{v}_{cr} + k_{p1}(v_{cr} - v_c) \\ \dot{\omega}_r + k_{p2}(\omega_r - \omega) \end{bmatrix} \tag{9}$$

where k_{p1} and k_{p2} are control parameters of the feedback linear and angular velocities.

The stability analysis of the control law (9) is given as follows. First, let the tracking error of v_c and ω be set in Equation (10):

$$e_v(t) = v_{cr}(t) - v_c(t),\ e_\omega(t) = \omega_r(t) - \omega(t) \tag{10}$$

Substituting the control law (9) and Equation (10) into dynamic Equation (2) yields the following equation:

$$\dot{e}_v(t) + k_{p1}e_v(t) = 0,\ \dot{e}_\omega(t) + k_{p2}e_\omega(t) = 0 \tag{11}$$

Here, the values of k_{p1} and k_{p2} are always set as a positive constant. Then, we obtain the following equations:

$$\lim_{t\to\infty} e_v(t) = \lim_{t\to\infty}[v_{cr}(t) - v_c(t)] = 0,\ \lim_{t\to\infty} e_\omega(t) = \lim_{t\to\infty}[\omega_r(t) - \omega(t)] = 0 \tag{12}$$

when using the control method of Equation (9), the stability of the linear and angular velocities tracking can be ensured by appropriate selection of k_{p1} and k_{p2}.

4. **Simulations.** This section describes simulations conducted with the proposed controller to verify the effectiveness of the proposed algorithm. The physical parameters of the robot used in the simulations are $2b = 0.3$m, $2a = 0.4$m, $M = 15$kg, $I = 0.28\text{kg}\cdot\text{m}^2$, $J_0 = 0.0125\text{kg}\cdot\text{m}^2$, $J_\omega = 0.0144\text{kg}\cdot\text{m}^2$, $r = 0.122$m, and $k = 3$. Since the arc line is difficult to track compared to linear line, in this paper, to thoroughly illustrate the effectiveness of the proposed control method, the robot is assumed to follow a sinusoid path which is described by

$$y_{cd} = 2 - \sin(x_{cd}) \tag{13}$$

The desired trajectory is given as follows:

$$x_{cd} = \pi - t/4,\ y_{cd} = 2\sin(\pi - t/4) \tag{14}$$

Our objective is to steer the robot, subject to unknown disturbances, to verify the suitability of the proposed control method to unknown disturbances, and simulations using the method with and without the adaptive law were both conducted. The unknown disturbances we used in simulations were constant set as $\xi_v = \xi_\omega = 0.2$. To show the effectiveness of the adaptive controller in relation to unknown disturbances, simulations are conducted with and without unknown disturbances. The parameters of the proposed control method are adjusted by manually selecting, the best tracking results under the assumption that unknown disturbances are zero. The selected control parameters are given as $k_{p1} = 6.2$, $k_{p2} = 6.9$, $k_1 = 168$, $k_2 = 164$, $\Gamma_1 = 193$, and $\Gamma_3 = 180$ and $\widetilde{\xi}_v(0) = \widetilde{\xi}_\omega(0) = 0$.

Figure 4 shows the path tracking ability of the robot by the proposed method. Clearly, by using the proposed controller, the robot can successfully track a sinusoid path without the unknown disturbances. While considering unknown disturbances, although tracking errors become a little larger, the robot still can successfully track the desired path and trajectory.

Figure 5 shows the path tracking ability of the robot by the proposed method without the adaptive law. In this condition, the robot can still successfully track the desired path while having not taken the unknown disturbances into consideration. However, while the unknown disturbances are considered, the robot cannot track the path, and the trajectory tracking errors of the x and y are too large. Comparing the results shown in Figure 4 and Figure 5 further verifies the suitability and superiority of the proposed control method to the unknown disturbances.

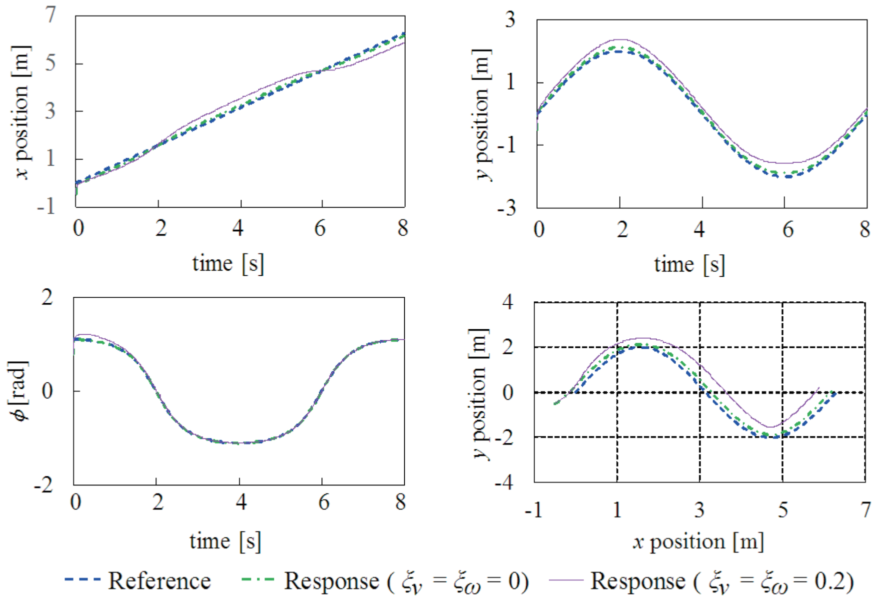

FIGURE 4. Simulation results using the proposed control method

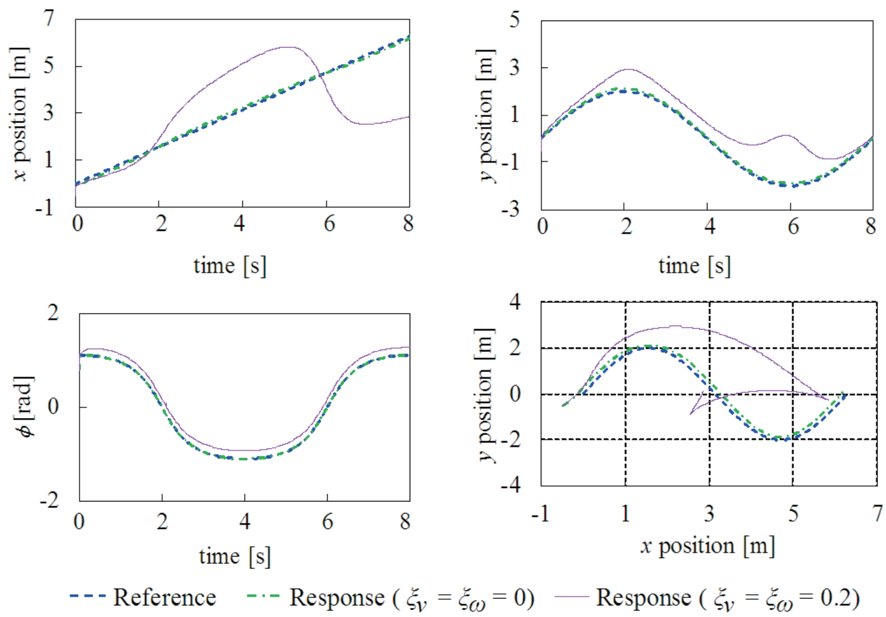

FIGURE 5. Simulation results without the adaptive law

5. **Conclusion.** In this paper, a double closed-loop controller is proposed for the path tracking of a mobile robot considering the unknown disturbances. Firstly, an adaptive feedback linearization is designed based on the kinematics to deal with the unknown disturbances and non-holonomic problem, and then a computed torque controller based on the dynamics is designed to generate the actual torques for the robot. Finally, simulations are conducted and the results demonstrate the effectiveness of the proposed controller. In future, the effectiveness of the proposed method will be validated by experiments.

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