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JOINT TIME SYNCHRONIZATION AND LOCALIZATION FOR SENSOR NETWORK WITH UNKNOWN PROPAGATION SPEED

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Abstract. *By considering the propagation speed as unavailable, linear estimators are proposed for the joint time synchronization and localization problem. The proposed linear estimators including the least square (LS) and weighting least square (WLS) estimators represent the parameter estimates as the algebraic solutions which avoid the problem of local optimum of the numerical calculation. Then we derive the Cramer-Rao lower bound (CRLB) of the joint estimation problem. The simulations show that the positioning accuracy of the designed linear estimators can be close to the CRLB performance of the estimation problem. Compared with the LS estimator, the accuracy performance of the WLS estimator is better due to the efficient weights.*
Keywords: Wireless sensor networks, Time synchronization, Localization, Least square, Weighting least square

1. **Introduction.** By consisting of lightweight, low-cost, low-power and multi-functional sensors, wireless sensor networks (WSNs) has driven a myriad of monitoring, tracking, and control applications. Often, the data collected by a sensor node must be tied with the sensor position in order to be meaningful. In WSNs, in general, only a few sensor nodes, called as anchor nodes in the sequel, know their positions a priori, while the remaining source nodes with unknown positions are required to be localized [1]. To locate the source node, the relative distance should be measured or estimated between the anchor nodes and source nodes. The most important kinds utilize the received signal strength (RSS), angle of arrival (AOA) [2], and signal propagation time, respectively. Signal propagation time based algorithms estimate the object location using the time that it takes the signal to travel from the transmitter to the target and from there to the receivers. They achieve very accurate estimation of object location if combined with high-precision timing measurement techniques, such as ultrawideband (UWB) signaling, which allows centimeter and even submillimeter accuracy. The algorithms based on signal propagation time can be further classified into time of arrival (TOA) [3] and time difference of arrival (TDOA) [4]. TOA algorithms employ the information of the absolute signal travel time from the transmitter to the target and thence to the receivers.

To accurately estimate the source locations, there are a lot of algorithms for the source location estimation by using TOA-based ranging technique in the past years. Maximum likelihood (ML) estimator is asymptotically optimal, but the ML estimator is nonconvex and its performance highly relies on the initial solution provided for the iterative solver. To overcome the shortcoming of ML estimator, the linear estimator [5] or convex optimization method is proposed by converting the cost function of ML estimator into linear or convex problem model.

In TOA-based model the source locations are estimated by using the propagation time which is the time difference from the transmitter to receiver. However, each sensor node has independent crystal and specific clock circuit. Due to the initialization or clock deflection, the direct observation time of sensor nodes would be inaccurate without time synchronization. TOA-based localization method is highly dependent on time synchronization in the asynchronous networks. Recently, joint time synchronization and localization have been considered in the literature. By considering the practical clocks with internal delays and clock skews, the estimation of source location is proposed in [6] by a weighted least squares (WLS) estimator when a two-way ranging (TWR) protocol is employed for asynchronous networks. To alleviate the computational complexity, two iterative approaches including expectation maximization (EM) and least squares are proposed to estimate the clock parameters and the location of the unknown node [7]. For jointly estimating clock parameters and the locations of multiple sensor nodes, two computationally efficient algebraic solutions are developed by using the timing measurements between source nodes and anchor nodes [8]. The performance of linear estimator is not enough robust especially when the noises are high. In [9], a semidefinite programming (SDP) algorithm is proposed for cooperative joint sensor synchronization and localization. Moreover, the complexity of the proposed SDP is especially high for a large number of variables.

On the other hand, the standard choice for the underwater WSNs communication is to utilize acoustic waves but the speed of sound is a function of temperature, pressure, salinity and depth in the oceans, which implies that the signal propagation speed is also subject to uncertainties. While in underground WSNs and in-solid scenarios where seismic/vibrational sensor data are processed, the propagation speed is unknown and depends strongly on the propagation medium. In this paper linear estimators are proposed for the joint time synchronization and localization problem when considering the propagation speed as unavailable. The linear estimators including the LS and WLS estimators are designed by transforming the nonlinear equations to linear equations and obtain the closed-form solution to the parameters estimates. This paper contains a number of symbols. Following the convention, we represent the matrices as bold case letters. If we denote the matrices as $(*)$, $(*)^{-1}$ represents matrix inverse. $\|*\|$ denotes ℓ_2 norm.

2. **Problem Specification.**

In a 2-dimensional plane, the positions of M anchor nodes are known and denoted as $\mathbf{x}_i = [x_i \quad y_i]^T$, $i = 1, 2, \ldots, N$. These anchor nodes are used to determine the position of a source node, which is denoted as $\mathbf{x} = [x \quad y]^T$. To locate the source nodes, a TOA-based scheme of two-way message exchanges is introduced and shown as Figure 1. A message is sent from anchor node i at observation time T_i and received by the source node at observation time R_i which is not accurate due to the imperfect crystal clock of the source node. According to the clock model in [9], the observation time is linearized with the real time and can be modeled as

$$\begin{cases} T_i = \omega_i t_i + \theta_i \\ R_i = \omega_x t_i' + \theta_x \end{cases} \tag{1}$$

where $i = 1, 2, \ldots, M$, and t_i and t_i' are the real time at anchor node i and the source node, respectively. ω_i and ω_x are the clock skew of anchor node i and the source node. θ_i and θ_x are the clock offset of anchor node i and the source node. So the propagation distance d_i is written as

$$d_i = c\left(\frac{R_i}{\omega_x} - \frac{\theta_x}{\omega_x} + \frac{\theta_i}{\omega_i} - \frac{T_i}{\omega_i}\right) + cn_i \tag{2}$$

where the clock skew ω_i and offset θ_i of the anchor nodes are considered as known, and c is propagation speed and assumed to be unknown due to the uncertainty environment. n_i is the additive time measurement noise with zero mean and variance δ_i^2. Subsequently,

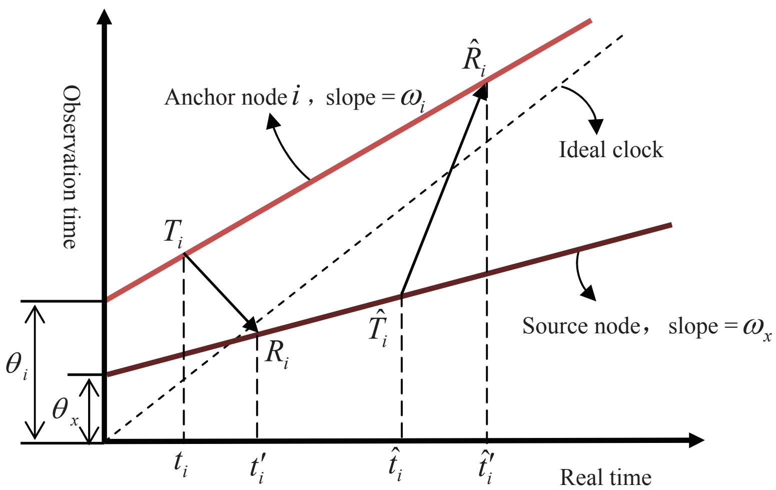

Figure 1. Two-way message exchange between anchor node and source node

source node replies anchor node i with another message sent at $\hat{T}_i$ and is received by anchor node i at $\hat{R}_i$. The similar clock observation can be obtained with

$$\begin{cases} \hat{T}_i = \omega_x \hat{t}_i + \theta_x \\ \hat{R}_i = \omega_i \hat{t}'_i + \theta_i \end{cases} \tag{3}$$

So the propagation distance is equal to $c\left(\hat{t}'_i - \hat{t}_i\right)$ and also given by

$$d_i = c\left(\frac{\hat{R}_i}{\omega_i} - \frac{\theta_i}{\omega_i} + \frac{\theta_x}{\omega_x} - \frac{\hat{T}_i}{\omega_x}\right) + c\hat{n}_i \tag{4}$$

where $\hat{n}_i$ is the measurement noise with zero mean and variance $\hat{\delta}_i^2$.

Let $\alpha_x = \frac{c}{\omega_x}$, $\beta_x = \frac{c\theta_x}{\omega_x}$, $\tau_i = \frac{\theta_i}{\omega_i} - \frac{T_i}{\omega_i}$ and $\hat{\tau}_i = \frac{\hat{R}_i}{\omega_i} - \frac{\theta_i}{\omega_i}$, (2) and (4) are modified as

$$\begin{cases} d_i = R_i\alpha_x - \beta_x + c\tau_i + cn_i \\ d_i = -\hat{T}_i\alpha_x + \beta_x + c\hat{\tau}_i + c\hat{n}_i \end{cases} \tag{5}$$

Apparently the parameters τ_i and $\hat{\tau}_i$ are known since the anchor nodes are assumed to be synchronized and the clock parameters of anchor nodes are assumed to be known. When the noises are Gaussian, the well known ML estimator of the proposed model is simply obtained by the following minimization problem

$$\begin{aligned} &\min_{\mathbf{x},\alpha_x,\beta_x,c} \sum_{i=1}^{M} \left\{ \frac{\left[d_i - \left(R_i\alpha_x - \beta_x + c\tau_i\right)\right]^2}{\delta_i^2} + \frac{\left[d_i - \left(-\hat{T}_i\alpha_x + \beta_x + c\hat{\tau}_i\right)\right]^2}{\hat{\delta}_i^2} \right\} \\ &\text{s.t.} \quad d_i = \|\mathbf{x} - \mathbf{x}_i\| \end{aligned} \tag{6}$$

The optimization problem in (5) is highly nonlinear and nonconvex and solved by the iterative numerical methods, which requires an initial point. If the initial point is not sufficiently close to the global minimum, the numerical methods may converge to a local minimum or a saddle point causing a large estimation error. So a closed-form solution is introduced to avoid the shortcoming of the ML estimator and ensure the global convergence in the following.

3. **Linear Estimator.** To obtain a closed-form solution for the proposed model, the cost function of the ML estimator is firstly formulated as a linear function. When the fist

expression is added into the second one in (5), we can obtain that

$$2d_i = \left(R_i - \hat{T}_i\right)\alpha_x + c\left(\tau_i + \hat{\tau}_i\right) + c\left(n_i + \hat{n}_i\right) \tag{7}$$

Since clock skew ω_x is very close to one, we can assume that $\frac{1}{\omega_x} = 1 + \gamma_x$ (γ_x is a variable close to zero). So (7) is rewritten as

$$2d_i = \lambda_i c + \rho_i c\gamma_x + c\left(n_i + \hat{n}_i\right) \tag{8}$$

where $\lambda_i = \left(R_i - \hat{T}_i + \tau_i + \hat{\tau}_i\right)$, $\rho_i = R_i - \hat{T}_i$. Squaring both sides of (8) and neglecting the high order terms, we represent (8) as

$$-8\mathbf{x}_i^T\mathbf{x} + 4\mathbf{x}^T\mathbf{x} - \lambda_i^2c^2 - 2\lambda_i\rho_i c^2\gamma_x = -4\mathbf{x}_i^T\mathbf{x}_i + \varepsilon_i \tag{9}$$

where $\varepsilon_i = \left(\lambda_i + \rho_i\gamma_x\right)c^2\left(n_i + \hat{n}_i\right)$, $i = 1, 2, \ldots, M$. Let $\mathbf{z} = \begin{bmatrix}\mathbf{x} & \mathbf{x}^T\mathbf{x} & c^2 & c^2\gamma_x\end{bmatrix}$, (9) can be rewritten as matrix form

$$\mathbf{Az} = \mathbf{b} + \varepsilon \tag{10}$$

where the row vector of $\mathbf{A}$ is equal to $[-8\mathbf{x}_i^T \quad 4 \quad -\lambda_i^2 \quad -2\lambda_i\rho_i]$. The element value of $\mathbf{b}$ and ε are equal to $[-4\mathbf{x}_i^T\mathbf{x}_i]$ and $[\varepsilon_i]$, respectively. $\mathbf{A} \in \mathbb{R}^{M\times 4}$, $\mathbf{b} \in \mathbb{R}^M$ and $\varepsilon \in \mathbb{R}^M$. So the weighting least square (WLS) solution to (10) is

$$\mathbf{z} = \left(\mathbf{A}^T\mathbf{\Sigma}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^T\Sigma^{-1}\mathbf{b} \tag{11}$$

where $\mathbf{\Sigma} = E\left(\varepsilon^T\varepsilon\right)$ which is given by

$$\mathbf{\Sigma} = \operatorname{diag}\left\{\left(\lambda_i + \rho_i\gamma_x\right)^2 c^4\left(\delta_i^2 + \hat{\delta}_i^2\right)\right\} \tag{12}$$

The estimation error in $\mathbf{z}$ is denoted as $\Delta\mathbf{z}$, which can be represented as

$$\Delta\mathbf{z} = \left(\mathbf{A}^T\mathbf{\Sigma}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{\Sigma}^{-1}\varepsilon \tag{13}$$

where ε is the noise vector, so $\Delta\mathbf{z}$ is unknown. However, it is straightforward to show that $\Delta\mathbf{z}$ has zero mean, when the noise ε is Gaussian with zero mean.

According to the definition of $\mathbf{z}$, we obtain the source position estimate $\mathbf{x} = \mathbf{z}(1:2)$, the propagation speed estimate $c = \sqrt{\mathbf{z}(4)}$ and the clock estimate ω_x which is obtained with

$$\omega_x = \frac{z(4)}{z(4) + z(5)} \tag{14}$$

To further estimate the clock offset θ_x, the first expression is subtracted from the second one in (5), then we can obtain that

$$2\theta_x = R_i + \hat{T}_i + \left(\tau_i - \hat{\tau}_i\right)\omega_x + \omega_x\left(n_i - \hat{n}_i\right) \tag{15}$$

By neglecting the noise term, the estimated clock offset θ_x can be written as

$$\theta_x = \sum_{i=1}^{M} 0.5\left(R_i + \hat{T}_i + \left(\tau_i - \hat{\tau}_i\right)\omega_x\right) \tag{16}$$

4. **CRLB Performance.** When the propagation speed is unavailable to the estimator, the clock parameters and the propagation speed are considered as unknown and should also be estimated along with the source locations. To derive the CRLB of the estimation problem, a new vector is defined and denoted as $\phi = [\mathbf{x} \quad \omega_x \quad \theta_x \quad \mathbf{c}]^T \in \mathbb{R}^5$. Then the probability density function of all measurements is written as

$$p\left(\mathbf{R};\phi\right) = \prod_{i=1}^{M}\left\{\frac{1}{2\pi\delta_i\hat{\delta}_i}\exp\left\{-\frac{\left(\frac{d_i}{c} - \frac{R_i}{\omega_x} + \frac{\theta_x}{\omega_x} - \tau_i\right)^2}{2\delta_i^2} - \frac{\left(\frac{d_i}{c} + \frac{\hat{T}_i}{\omega_x} - \frac{\theta_x}{\omega_x} - \hat{\tau}_i\right)^2}{2\hat{\delta}_i^2}\right\}\right\} \tag{17}$$

where $\mathbf{R}$ denotes the measurement vector. So when the clock parameters, propagation speed and source locations are unknown, the logarithm of the probability density function is represented as

$$\ln p(\mathbf{R};\phi) = k - \left(\mathbf{r}(\phi)^T \mathbf{\Sigma}^{-1}\mathbf{r}(\phi) + \hat{\mathbf{r}}(\phi)^T \hat{\mathbf{\Sigma}}^{-1}\hat{\mathbf{r}}(\phi)\right) \tag{18}$$

where $\mathbf{\Sigma} = \text{diag}\{\delta_i^2\}$, $\hat{\mathbf{\Sigma}} = \text{diag}\left\{\hat{\delta}_i^2\right\}$, $\mathbf{r}(\phi) = [r_1 \quad r_2 \quad \dots \quad r_M]$, $\hat{\mathbf{r}}(\phi) = [\hat{r}_1 \quad \hat{r}_2 \quad \dots \quad \hat{r}_M]$.

$$\begin{cases} r_i = \frac{d_i}{c} - \frac{R_i}{\omega_x} + \frac{\theta_x}{\omega_x} - \tau_i \\ \hat{r}_i = \frac{d_i}{c} + \frac{\hat{T}_i}{\omega_x} - \frac{\theta_x}{\omega_x} - \hat{\tau}_i \end{cases} \tag{19}$$

where $i = 1, 2, \dots, M$. The CRLB of the unknown parameters are the diagonal elements of the inverse of the Fisher information matrix (FIM). The FIM of the joint estimation problem is obtained as

$$\mathbf{F} = -\frac{\partial^2 \ln p(\mathbf{R};\phi)}{\partial\phi^T \partial\phi} \tag{20}$$

Therefore, (20) can also be represented as

$$\mathbf{F} = \left(\frac{\partial \mathbf{r}(\phi)}{\partial\phi}\right)^T \mathbf{\Sigma}^{-1}\frac{\partial \mathbf{r}(\phi)}{\partial\phi} + \left(\frac{\partial \hat{\mathbf{r}}(\phi)}{\partial\phi}\right)^T \hat{\mathbf{\Sigma}}^{-1}\frac{\partial \hat{\mathbf{r}}(\phi)}{\partial\phi} \tag{21}$$

where $\frac{\partial \mathbf{r}(\phi)}{\partial\phi} = \left[r_i^\phi\right]$, $\frac{\partial \hat{\mathbf{r}}(\phi)}{\partial\phi} = \left[\hat{r}_i^\phi\right]$, $r_i^\phi = \left[r_i^{\mathbf{x}} \quad r_i^{\theta_x} \quad r_i^{\omega_x} \quad r_i^c\right]$, $\hat{r}_i^\phi = \left[\hat{r}_i^{\mathbf{x}} \quad \hat{r}_i^{\theta_x} \quad \hat{r}_i^{\omega_x} \quad \hat{r}_i^c\right]$,

$$\begin{cases} r_i^{\mathbf{x}} = \hat{r}_i^{\mathbf{x}} = \frac{(\mathbf{x}-\mathbf{x}_i)^T}{cd_i} \\ r_i^{\theta x} = \frac{1}{\omega_x}, \quad \hat{r}_i^{\theta x} = -\frac{1}{\omega_x} \\ r_i^{\omega x} = \frac{R_i-\theta_x}{\omega_x^2}, \quad \hat{r}_i^{\omega x} = \frac{\theta_x - T_i}{\omega_x^2} \\ r_i^c = \hat{r}_i^c = -\frac{d_i}{c^2} \end{cases} \tag{22}$$

where $i = 1, 2, \dots, M$. So the CRLB of unknown parameters is calculated by

$$\text{CRLB}([\phi]_k) = \left[\mathbf{F}^{-1}(\phi)\right]_{k,k} \tag{23}$$

where $k = 1, 2, \dots, 5$.

5. **Evaluation.** To test the performance of proposed algorithm, the simulations are conducted with MATLAB software. The anchors are deployed in a square area of 100 m × 100 m. The source target is set at the point $(50, 50)$ in a prior. In the region eight anchor nodes are set at the points $(60, 10), (80, 90), (35, 80), (10, 10), (50, 5), (30, 70), (20, 20), (30, 50)$. The measurement noises between the source node and each anchor node are subject to zero mean white Gaussian processes with zero mean and identical variance δ^2. In order to evaluate the performance in different conditions, root mean square error (RMSE) is used to evaluate the accuracy performance of the estimated parameter. We verify the performance of the proposed method through Monte Carlo (MC) simulations and the number of samples used in the MC procedure is 1000. We firstly give the performance in terms of RMSE as the noises increase.

5.1. **Estimated source location.** To test the impact of noises, the clock skew and propagation speed are set at 1.1 and 3×10^8 m/s in a prior, respectively. When the noise variance δ^2 is varied from 0.1^2 to 1^2, Figure 2 plots the performance comparison with the LS and WLS estimator. It can be seen from Figure 2 that the RMSE of the estimator source location is approximately linear with the noise. For instance, when the noise variance δ is set to 0.1, the RMSE of the LS estimator is about 0.022 m. However, the RMSE of the LS estimator is increased to 0.25 m, when the noise variance δ is set to

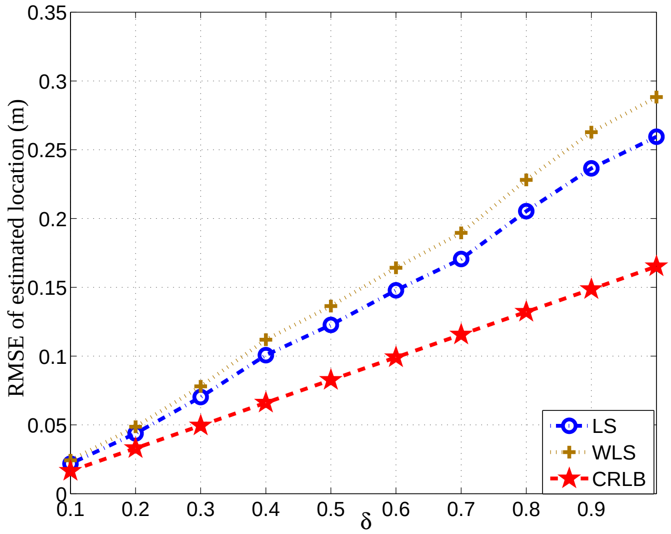

FIGURE 2. RMSE of estimated source location

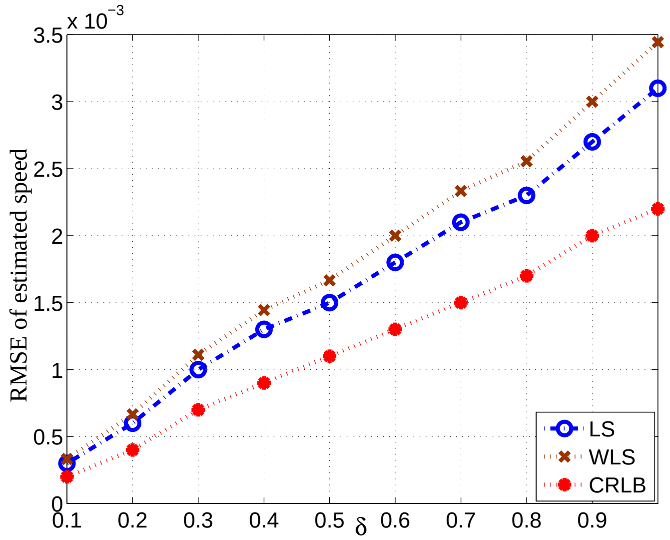

FIGURE 3. RMSE of estimated propagation speed

1. Due to the fact that the efficient weights are used in the WLS estimator, the RMSE of the WLS estimator is reduced compared with that of the LS estimator.

5.2. **Estimated propagation speed.** To test the RMSE performance of LS and WLS estimators when the propagation speed is unavailable, the noise variance δ is varied from 0.1^2 to 1^2. Figure 3 plots the performance comparison of estimated propagation speed as the noise increases. The order of LS and WLS estimators is the same as Figure 2. It is seen from Figure 3 that RMSE performance is approximately increased as the noise increases. When the noise covariance is set to 0.1^2, the RMSE of estimated propagation speed is about 2×10^{-4} m/s. When the noise covariance is increased to 1^2, the RMSE of estimated propagation speed is about 3.5×10^{-3} m/s. Compared with the LS estimator, the RMSE performance of WLS estimator is closer to the CRLB accuracy performance.

6. **Conclusion.** In this paper, the joint time synchronization and localization problem are considered by assuming the propagation speed as unavailable. To derive the source location and clock parameters, two linear estimators including the LS and WLS estimators are proposed by eliminating the nuisance parameters. The optimization problem

is converted into a linear least-squares estimation problem, so a closed-form solution to the joint estimation problem is obtained. The LS and WLS estimators do not require iteration or initialization compared with the solution to the numerical calculation. It is demonstrated that the performance of LS and WLS estimators is close to the CRLB performance of the proposed model. The proposed linear estimators have the advantages of low computation complexity and high positioning accuracy.

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