

NOVEL GRADIENT-ADAPTIVE ALGORITHM WITH IMPROVED CONVERGENCE PROPERTY FOR VEHICLE NOISE ATTENUATION

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ABSTRACT. *The filtered-x LMS algorithm has difficulty in reducing vehicle acceleration noise because choosing its optimum value is a challenge with rapidly changing noise conditions. The Mathews algorithm was proposed to address this problem. However, its convergence speed is slow, i.e., it has an inferior convergence property. This paper presents a new method to improve the convergence property of the Mathews algorithm and deal with vehicle acceleration noise. We confirm the algorithm's convergence property experimentally.*

Keywords: Noise control, Adaptive control, fx-LMS algorithm, Mathews algorithm

1. **Introduction.** Active noise control (ANC) is a common method used commercially to deal with acoustic noise. And the filtered-x LMS (fx-LMS) algorithm is widely used for even acceleration noise control. However, these methods have difficulty in reducing vehicle acceleration noise actually. This is because the value of the convergence coefficient of the fx-LMS is fixed and unchangeable, but the optimum convergence coefficient of the time-varying noise signal changes with the variation of the signal.

To deal with time-varying signals, Mathews and Xie [1] provide a stochastic gradient adaptive filtering algorithm to increase the convergence rate. Their algorithm updates the time-varying convergence coefficient proportionally to the negative gradient of the squared error with respect to the convergence coefficient. However, because vehicle acceleration noise varies rapidly, the convergence coefficient may not attain the optimum value quickly enough. Thus, a faster algorithm for time-varying signals is required.

In this paper, we propose a new algorithm based on the Mathews algorithm. This algorithm improves the convergence property, allowing more rapid attainment of the optimum value for time-varying changes.

2. Algorithm.

2.1. **The filtered x-LMS algorithm.** The filtered x-LMS algorithm can be written as:

$$y(n) = H^T(n)x(n) \quad (1)$$

$$z(n) = G^T y(n) \quad (2)$$

$$e(n) = d(n) - z(n) \quad (3)$$

$$r(n) = \hat{G}^T x(n) \quad (4)$$

$$H(n+1) = H(n) + \mu e(n)r(n) \quad (5)$$

In the above equations, $d(n)$ is the desired signal of the adaptive filter, $x(n)$ is the input vector to the adaptive filter, $e(n)$ is the error signal, and $H(n)$ is the vector of adaptive filter coefficients.

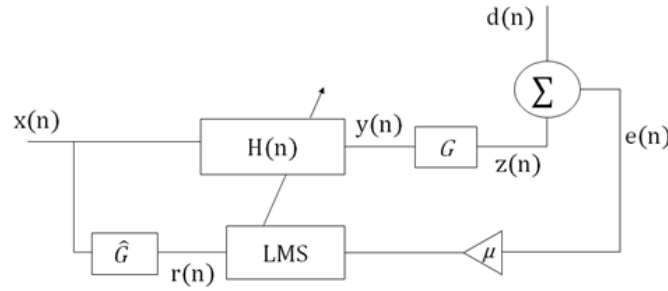


FIGURE 1. Block diagram of the fx-LMS

2.2. **Mathews algorithm.** Mathews algorithm can be written as:

$$e(n) = d(n) - z(n) \quad (6)$$

$$\mu(n) = \mu(n-1) - \frac{\rho}{2} \frac{\partial}{\partial \mu(n-1)} e^2(n) = \mu(n-1) - \rho e(n)e(n-1)X^T(n-1)X(n) \quad (7)$$

$$0 < \mu(n) < \frac{2}{3\text{tr}\{R\}} \quad (8)$$

and

$$H(n+1) = H(n) - \frac{\mu(n)}{2} \frac{\partial e^2(n)}{\partial H(n)} = H(n) + \mu(n)e(n)r(n) \quad (9)$$

where ρ is a small positive constant. R is the autocorrelation matrix of the input vector $X(n)$ and $\text{tr}\{\cdot\}$ denotes the trace of the matrix (\cdot) .

The variation of the convergence coefficient is determined by the error signal and the input vector.

2.3. **Proposed algorithm.** Our proposed algorithm is based on the Mathews algorithm. The algorithm updates the time-varying ρ proportionally to the negative of the gradient of the squared error with respect to ρ . This algorithm can be written as:

$$e(n) = d(n) - z(n) \quad (10)$$

$$\mu(n) = \mu(n-1) - \frac{\rho}{2} \frac{\partial}{\partial \mu(n-1)} e^2(n) = \mu(n-1) - \rho(n)e(n)e(n-1)X^T(n-1)X(n) \quad (11)$$

$$0 < \mu(n) < \frac{2}{3\text{tr}\{R\}} \quad (12)$$

$$\rho(n) = \rho(n-1) - \frac{\alpha}{2} \frac{\partial}{\partial \rho(n-1)} e^2(n) = \rho(n-1) - \alpha e(n)e(n-1)X^T(n-1)X(n) \quad (13)$$

and

$$H(n+1) = H(n) - \frac{\mu(n)}{2} \frac{\partial e^2(n)}{\partial H(n)} = H(n) + \mu(n)e(n)r(n) \quad (14)$$

where α is a small positive constant which controls the adaptive behavior of $\rho(n)$.

3. **Simulation.** We apply fx-LMS, the Mathews algorithm, and our proposed algorithm to attenuating vehicle acceleration noise. The adaptive filter $H(z)$ has 256 coefficients, $d(n)$ is the acceleration noise of a motorcycle (Figure 2), and $x(n)$ is a sine wave whose frequency is changed with the frequency variation of $d(n)$.

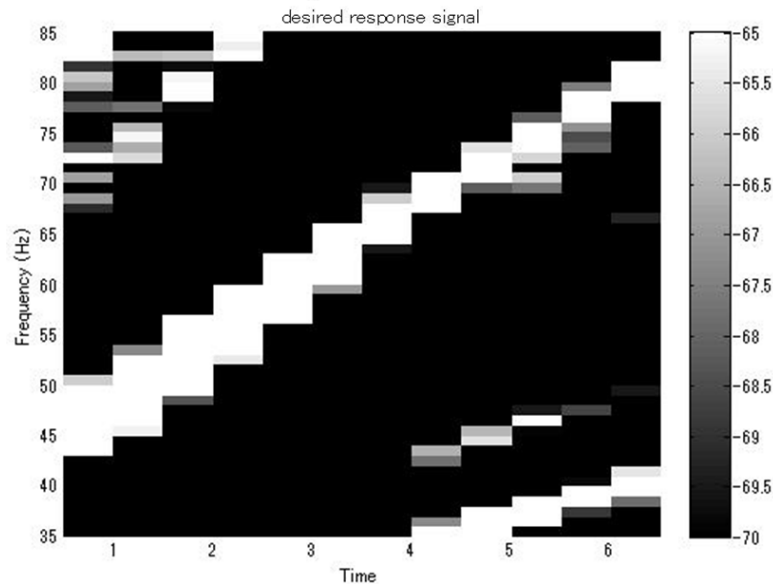


FIGURE 2. Spectrogram of the desired response signal

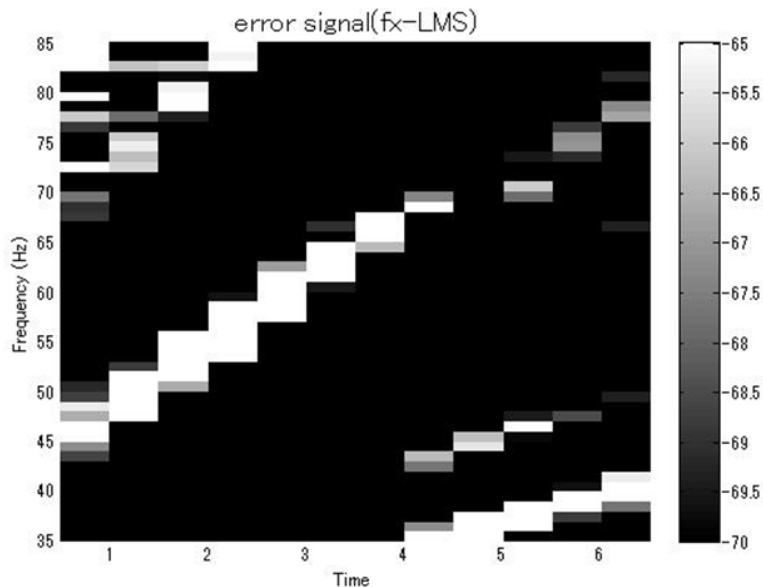


FIGURE 3. Spectrogram of the error signal from the fx-LMS algorithm

The optimum value can be found by recording the signal and then determining the optimum value through simulations. However, in practice, it is not feasible to record all cases of the signal. Hence, this approach is not typically applied to real scenarios.

Because it is difficult to determine the optimum value of $\mu(n)$ in practical applications, the initial value of $\mu(n)$ and $\rho(n)$ was set at the same positive integer for each algorithm. $\mu(n)$ is smaller than the optimum value and $\rho(n)$ is less than $\mu(n)$.

The results of the simulations are shown in Figures 3, 4, and 5. Figure 6 shows the difference between the averaged error signal and the averaged desired signal.

This indicates that the convergence property of our proposed algorithm is superior to that of the other two, and the Mathews algorithm is better than the fx-LMS algorithm.

The change of the convergence coefficient is presented in Figure 7. The convergence coefficient changes faster in our proposed algorithm than in the Mathews algorithm.

The convergence property of our proposed algorithm is superior to that of the Mathews algorithm. However, the convergence property becomes unstable for a large coefficient in

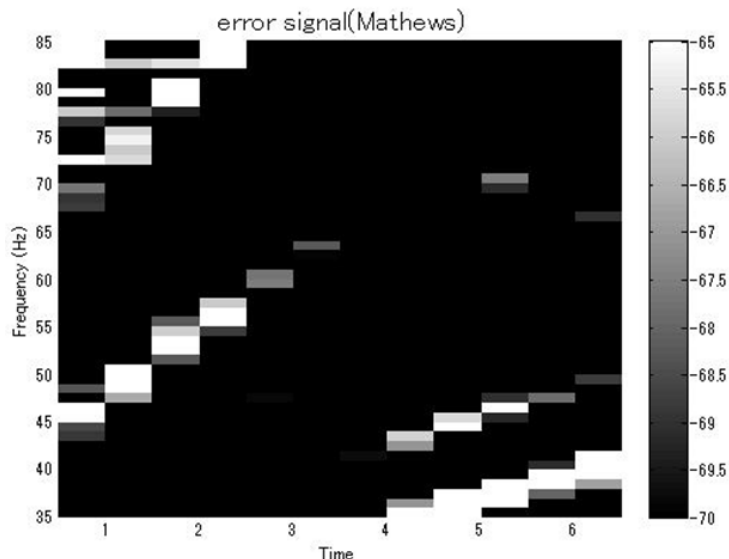


FIGURE 4. Spectrogram of the error signal from the Mathews algorithm

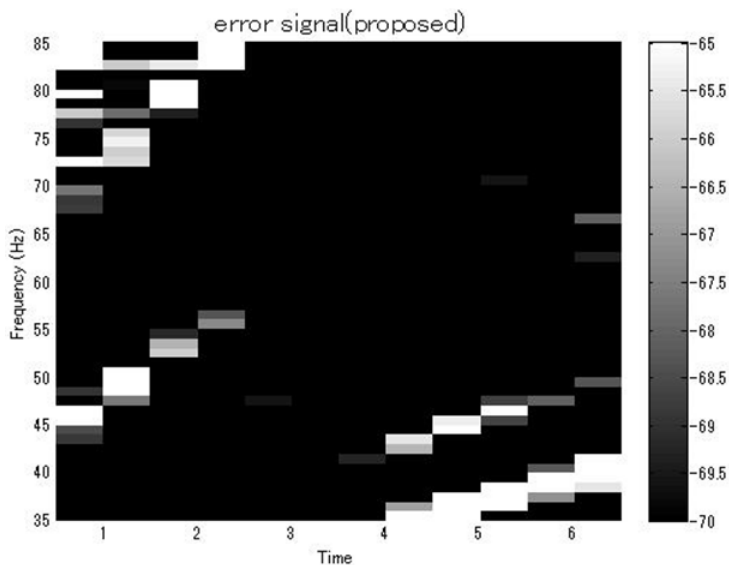


FIGURE 5. Spectrogram of the error signal from our proposed algorithm

both algorithms. To avoid this instability, we introduce a constraint on the convergence coefficient into our proposed algorithm.

Based on Figure 7, the maximum convergence coefficient is set to $5 * 10^{-6}$. If the calculated coefficient is greater than $5 * 10^{-6}$, it is lowered to equal $5 * 10^{-6}$. With this constraint, the proposed algorithm is stable.

4. Conclusions. This paper provides a new stochastic gradient adaptive filtering algorithm focused on the convergence coefficient to reduce vehicle acceleration noise. The difference between our proposed algorithm and Mathews algorithm is that, under the same conditions, the convergence property of our algorithm is superior to that of the Mathews algorithm; it attains the optimum value more quickly with time-varying change. This enables our proposed algorithm to deal more effectively with vehicle acceleration noise. However, the stability is affected under the high speed convergence property and a constraint is required.

In future work, we expect to reduce or eliminate the limitation on the convergence coefficient, and achieve algorithm stability.

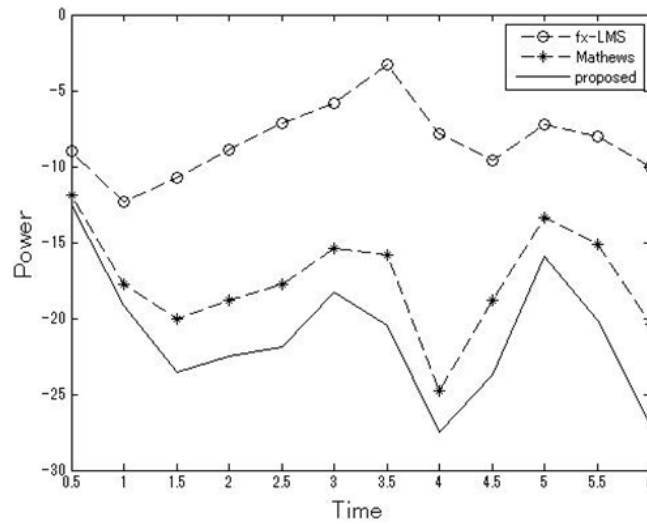


FIGURE 6. The average value of the power

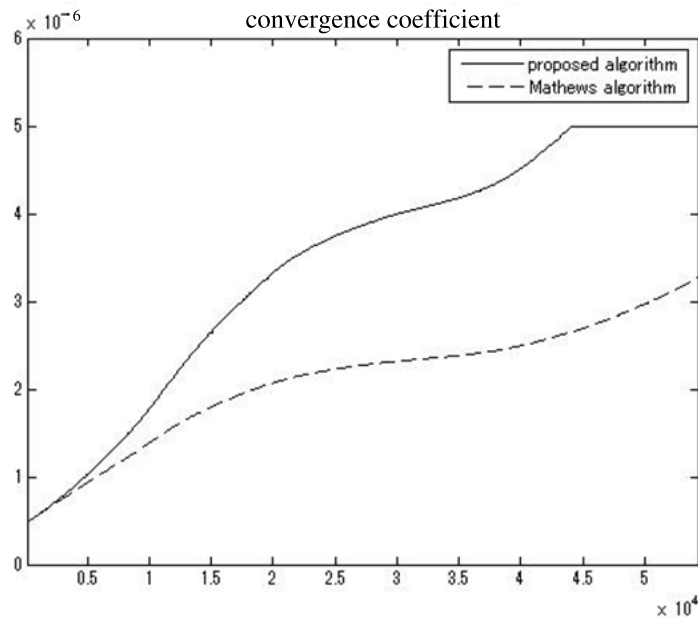


FIGURE 7. The change of the convergence coefficient

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