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A MULTI-STAGE SCHEDULING METHOD FOR MEDICAL RESOURCE ORDER AND DISTRIBUTION BASED ON INFLUENZA DIFFUSION MODEL

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Abstract. *This paper presents a multi-stage scheduling model for medical resource order and distribution that can be used in control of influenza diffusion. The research problem is formulated as a mixed 0-1 integer programming model, which systematically considers the decision-making at different layers of local hospitals, distribution centers, suppliers, and the transportation between them. The main contribution of this study is the interaction between the course of influenza diffusion and the dynamic medical resource allocation. A numerical example is presented to illustrate the model. The test results show the good performance of this model.*
Keywords: Medical resource, Influenza diffusion, Mixed integer programming, Scheduling

1. **Introduction.** Influenza is an acute viral infection that spreads easily from person to person, which occurs globally with an annual attack rate estimated at 5%-10% in adults and 20%-30% in children. The illness can result in hospitalization and death mainly among high-risk groups [1]. It is now widely recognized that influenza is a serious public health problem for our society, which can cause severe social and economic losses. Therefore, how to forecast the number of the infected people by influenza diffusion dynamics and how to assign the medical resource attract more and more attentions in recent years. The main contribution of this paper is to construct the interaction relationship between them.

There are two main streams of literature related to our research: the diffusion dynamics and the medical resource allocation problems. A great number of analytical works on epidemic diffusion have focused on the compartmental epidemic models, e.g., [2-5], the epidemic diffusions in these works are formulated as ordinary differential equations. In these models, the total population is divided into several classes and each class of people is closed into a compartment. Recently, Kim et al. described the transmission of avian influenza between birds and humans [6]. Liu and Zhang presented a Susceptible-Exposed-Infected-Recovered-Susceptible (SEIRS) epidemic model based on a scale-free network [7]. Samsuzzoha et al. proposed a diffusive epidemic model to describe the transmission of influenza. The ordinary differential equations were solved numerically by using the splitting method under different initial distributions of population density [8]. Further, Samsuzzoha et al. presented a vaccinated diffusive compartmental epidemic model to explore the impact of vaccination as well as diffusion on the transmission dynamics of influenza [9].

The second stream of the relevant research is medical resource allocation. Brandeau et al. studied resource allocation problems in an effort to control infectious diseases in multiple independent populations using cost-effectiveness analysis and methods [10]. Wei and Özdamar described an integrated location-distribution model for coordinating logistics support and evacuation operations in disaster response activities [11]. As to the variability and uncertainty characteristics of the demand, Wei and Kumar presented a meta-heuristic of ant colony optimization for solving the logistics problem arising in disaster relief activities [12]. Qin constructed a multi-objective stochastic programming model with time-varying demand for emergency logistics network based on epidemic diffusion rule [13]. For more results on this topic, we refer readers to [14-16].

Although researches on the compartmental epidemic models and medical resource allocation have been conducted by many scholars, there is a dearth of research on the combination between them. Therefore, the main contribution of this paper is to bridge these two streams of research by presenting a hybrid model that embeds a forecast of the influenza spread into dynamic logistics planning of medical resource allocation and distribution. A similar work can be found in [17]. They presented a time-space network model for studying the dynamic impact of medical resource allocation in controlling an epidemic spread. Although that paper addresses a related topic, it takes a different model approach of time-space network, which is built upon a linear-growth-factor estimator. Furthermore, as limited by the time-space network structure, it does not incorporate many practical and important features of a logistics system that are explicitly modeled in this manuscript, such as the unit variable cost in different layers, the order size, and the safe stock.

The rest of this paper is organized as follows. In Section 2, we propose a time-discrete version of SEIR model which is used to forecast the number of infected people and a dynamic logistics model to allocate medical resource. In Section 3, the solution procedure is introduced. In Section 4, numerical tests are given. Finally, Section 5 concludes the paper.

2. **Model Formulation.**

2.1. **Demand forecasting.** In this phase, we use a classical SEIR model to forecast the number of infected individuals. In such a model, the total population is divided into four classes, i.e., the susceptible individuals S, the exposed individuals E, the infected individuals I, and the recovered individuals R. As is shown in Figure 1, individuals enter into the compartment through $S(t)$ at rate λN and exit the compartment at rate $\lambda(S(t)+E(t)+I(t)+R(t))$. Herein, N is the number of whole population, and λ is the entry or exit rate of the population. Without considering the natural birth rate and death rate of population, we can use an Ordinary Differential Equation (ODE) to describe the epidemic spread process, which is illustrated as the following equations.

$$\begin{cases} S'(t) = \lambda N - \beta S(t)I(t) - \lambda S(t) \\ E'(t) = \beta S(t)I(t) - \gamma E(t) - \lambda E(t) \\ I'(t) = \gamma E(t) - \lambda I(t) - \delta I(t) \\ R'(t) = \delta I(t) - \lambda R(t) \\ N = S(t) + E(t) + I(t) + R(t) \end{cases} \tag{1}$$

Herein, all the parameters λ, β, γ and δ are positive constants. β is the transmission probability of the influenza. γ represents the rate where the exposed individuals become

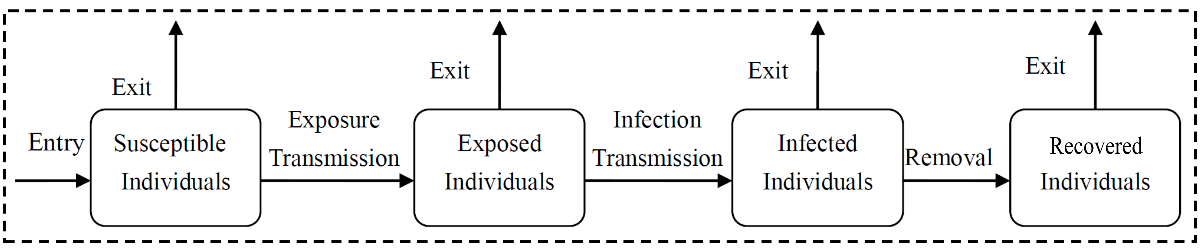

FIGURE 1. Schematic diagram of SEIR model

infective. δ is the recovered rate. ODE(1) represents the change rate of the susceptible, the exposed, the infected, and the recovered individuals. It is difficult to get the analytical solution for such ordinary differential equations. To facilitate the model formulation in the following section, ODE(1) can be time discretized into the following difference equations, for any $t = 0, 1, \ldots, T-1$:

$$\begin{cases} S(t+1) = S(t) + \lambda N - \beta S(t)I(t) - \lambda S(t) \\ E(t+1) = E(t) + \beta S(t)I(t) - \gamma E(t) - \lambda E(t) \\ I(t+1) = I(t) + \gamma E(t) - \lambda I(t) - \delta I(t) \\ R(t+1) = R(t) + \delta I(t) - \lambda R(t) \\ N = S(t+1) + E(t+1) + I(t+1) + R(t+1) \end{cases} \tag{2}$$

Given the initial values of $S(0)$, $E(0)$, $I(0)$, $R(0)$, Equation (2) can be used to forecast the number of infected individuals. Meanwhile, a linear function $D = F(I)$ is designed to describe the relationship between the infected individuals and the demand of medical resource, which will be introduced in subsection.

2.2. **The dynamic logistics model.** In this section, we prescribe a dynamic logistics model based on the forecasted demand. The logistics model consists of three parts: the layer of local hospitals, the layer of distribution centers and the layer of suppliers, as well as the decision variables, parameters, and relevant costs in each layer. Notations in the model formulation are given as follows.

The parameters:

$i/j/k$: the index of the suppliers/the distribution centers/the hospitals

$h_{1k}/g_{1j}/f_{1i}$: the fixed cost of hospital k/the fixed order cost in DC j/supplier i

$h_{2k}/g_{2j}/f_{2i}$: the unit variable cost at hospital k/the unit carrying cost at DC j/the unit variable cost at supplier i

$h_k(t)/g_j(t)/f_i(t)$: the total cost at hospital k/DC j/supplier i on day t

$v_j^s/v_j^0/V_j$: the safety stock/the order size/the storage capacity at DC j

$u^i(t)/w^j(t)$: the total transportation cost for supplier i/DC j on day t

D_i: the production constraints of supplier i

m: the average daily dose for a patient

X_k: the capacity of the hospital k

$d_i(t)$: the total orders that supplier i receives on day t

The decision variables:

$x_k(t)$: the number of beds assigned for epidemic patients in hospital k at day t

$z_j^i(t)$: the shipment from supplier i to DC j on day t

$y_k^j(t)$: the shipment from DC j to hospital k on day t

$v_j(t)$: the inventory level of DC j on day t

$\alpha_k(t)$: a 0-1 variable that indicates whether the hospital k is opened

$\varepsilon_j^i(t)$: a 0-1 variable that indicates whether DC j receives orders from supplier i

$\omega_i(t)$: a 0-1 variable that indicates whether supplier i is opened

Based on these notations, the scheduling model can be formulated as follows:

$$MinZ = \sum_{t=1}^{T} \left(\sum_{k=1}^{K} h_k(t) + \sum_{j=1}^{J} \left(g_j(t) + w^j(t) \right) + \sum_{i=1}^{I} \left(f_i(t) + u^i(t) \right) \right) \tag{3}$$

Subject to:

$$\sum_{k=1}^{K} x_k(t) = I(t), \quad \forall t \tag{4}$$

$$x_k(t) \le \alpha_k(t) X_k, \quad \forall k, t \tag{5}$$

$$v_j^s \le v_j(t) \le V_j, \quad \forall j, t \tag{6}$$

$$v_j(t) = \sum_{i=1}^{I} \varepsilon_j^i(t) v_j^0 - \sum_{k=1}^{K} y_k^j(t), \quad \forall j, t \tag{7}$$

$$mx_k(t) = \sum_{j=1}^{J} y_k^j(t), \quad \forall k, t \tag{8}$$

$$d_i(t) \le D_i, \quad \forall i, t \tag{9}$$

$$v_j^i(t) \le M\varepsilon_j^i(t), \quad \forall i, j, t \tag{10}$$

$$\alpha_k(t), \varepsilon_j^i(t), \omega_i(t) \in \{0, 1\}, \quad \forall i, j, k, t \tag{11}$$

$$x_k(t), y_k^j(t), z_j^i(t), v_j(t) \in Int, \quad \forall i, j, k, t \tag{12}$$

The objective function (3) denotes the minimization of total logistics cost, including the operation cost of hospitals, the inventory holding cost, the order cost of DCs and the transportation cost between DCs and hospitals, the operation cost of suppliers and the transportation cost between suppliers and DCs. (4) ensures that all the patients are assigned to hospitals. (5) denotes the capacity of hospitals. (6) denotes the inventory level of DCs. (7) indicates the flow conservation in DCs. (8) guarantees the demand of the patients. (9) denotes the production capacity of suppliers. (10) indicates the constraint whether supplier runs its operation. (11) and (12) are variable constraints.

3. **The Solution Procedure.** The model is formulated as a mixed 0-1 linear integer programming problem. Such a problem can be solved by some available optimization software, e.g., MATLAB. The procedure is described as follows:

Step 1: Set the initial values for the parameters.

Step 2: Use Equation (2) to forecast the number of infected individuals.

Step 3: Let $t = 1, \ldots, T$ denote a particular day in the cycle.

Step 4: Call function intlinprog() to solve the logistics model and obtain the scheduling results on day t.

Step 5: $t > T$? If not, repeat Step 4.

Step 6: Record the data of optimal schedules and the total operation costs.

4. **Numerical Tests.** To test how well the proposed model can be applied in the real world, we perform several numerical tests based on historical operating data of a certain kind of medical resource from a hospital in Nanjing, China, with reasonable simplifications. The tests were performed on a PC equipped with an Intel(R) Core(TM) 2.13 GHz CPU and 2.00 GB of RAM in the environment of Microsoft Windows 7.

4.1. **Case data.** In the SEIR model, the initial values of the parameters are set as follows: $S(0) = 9955$, $E(0) = 40$, $I(0) = 5$, $R(0) = 0$, $N = 10000$, $\beta = 4 \times 10^{-5}$, $\gamma = 0.6$, $\delta = 0.3$, $\lambda = 1 \times 10^{-3}$. In the dynamic logistics model, we have 6 local hospitals (H), 3 distribution centers (DC), 3 suppliers (S), $T = 42$ and $m = 1$. The unit transportation cost from supplier (S1/S2/S3) to distribution centers (DC1, DC2, DC3) is 2, 4, 6/ 4, 3, 2/ 3, 5, 2 (dollar). Similarly, the unit transportation cost from DC1/ DC2/ DC3 to 6 local hospitals (H1, H2, H3, H4, H5, H6) is 6, 2, 6, 7, 4, 2/ 4, 9, 5, 3, 8, 5/ 5, 2, 1, 9, 7, 4 (dollar). The operation cost parameters of the 6 local hospitals are given as follows: the capacities of 6 local hospitals are 50, 55, 50, 55, 50, 60 (person), the fixed costs are 35, 34, 33, 32, 32, 30 (dollar), and the unit variable costs are 2.5, 2.6, 2.7, 2.8, 2.9, 3.0 (dollar). Likewise, the operation cost parameters of DCs are introduced: the capacities of 3 DCs are 120, 150, 120 (dose); the ordering costs are 6, 6.5, 7 (dollar); the unit carrying costs are 1.4, 1.3, 1.2 (dollar); the order sizes are 40, 30, 40 (dose); the safety stocks are 10, 10, 10 (dose). Finally, the operation cost parameters of the suppliers are expressed: the capacities of 3 suppliers are 80, 90, 100 (dose); the fixed costs are 120, 110, 115 (dollar); the unit variable costs are 6, 7, 6.5 (dollar).

4.2. **Test results.** Figure 2 shows the inventory level over time in each of three DCs during the entire 120 days. It is observed that the order levels are high in the second and third stage, but low in the first and fourth stage, which well shows that the order size varies with the infected individuals. Moreover, they all begin with a certain stock and then are maintained at the low level in most of the days except for the necessary reorder point. The inventory-carrying performance can minimize the inventory ordering and carrying cost while meeting the hospitals' need. Namely, firstly, the DCs are deposited a certain level of initial inventory, for ensuring supply to the hospitals during the lead time of their first order. Secondly, they are operated to minimize the total logistics cost, therefore the inventory level in the DCs should be as low as possible while coordinated with the medicine order planning and the medicine distribution scheduling. The trajectory in Figure 2 illustrates an almost just-in-time mechanism in the medicine supply chain of our model.

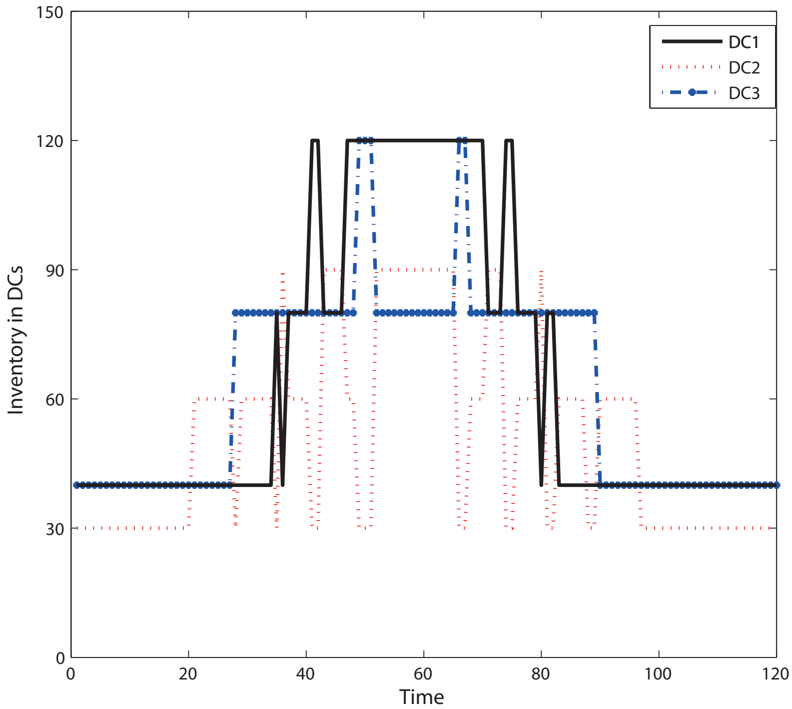

FIGURE 2. Inventory level in the DCs

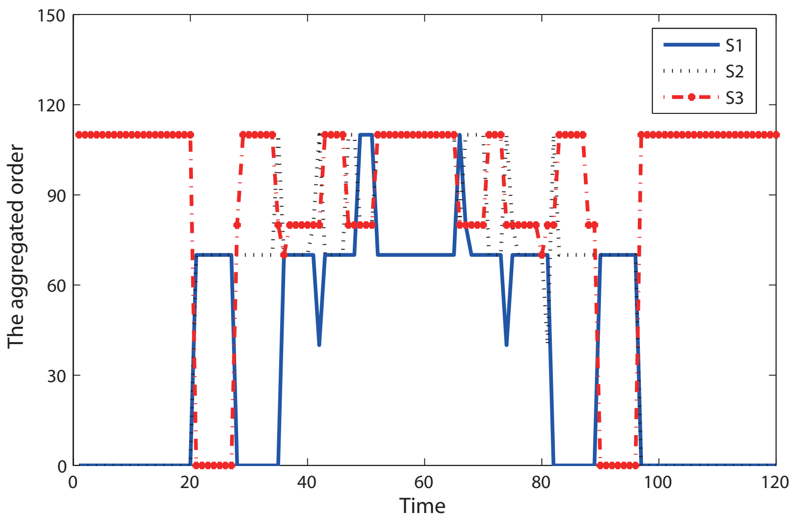

FIGURE 3. Production level in the suppliers

Figure 3 shows the production levels of the three suppliers during the entire 120 days. It can be seen that the production patterns of all the suppliers are similar where the output levels are high in the second and third cycle, but low in the first and fourth cycle. This is well expected because in the first cycle when the epidemic just bursts, the infected population size is small and the need for medicine is relatively low. In the second and third cycle when the epidemic diffusion is at its maximum scale, the number of infected people reaches the peak and the demand for the medicine is at the highest level, thus so would be the production level. In the fourth cycle, when the epidemic diffusion is brought under control and the infected population diminishes, the medicine demand decreases and the production slows down.

Figure 4 shows the number of patients assigned to each hospital on the 66th day. On this particular day, Hospitals 2, 3, 4 and 8 are operated at capacity, Hospital 1 is used to treat the infected individuals on that day but is not filled up, while Hospital 5 is not in operation. Considering the cost structure of the hospitals, such assignment avoids unnecessary fixed cost of operations and takes full advantages of opened hospital resource. It makes a common sense that it is not desired to have more than one hospital operated under capacity in such a cost structure.

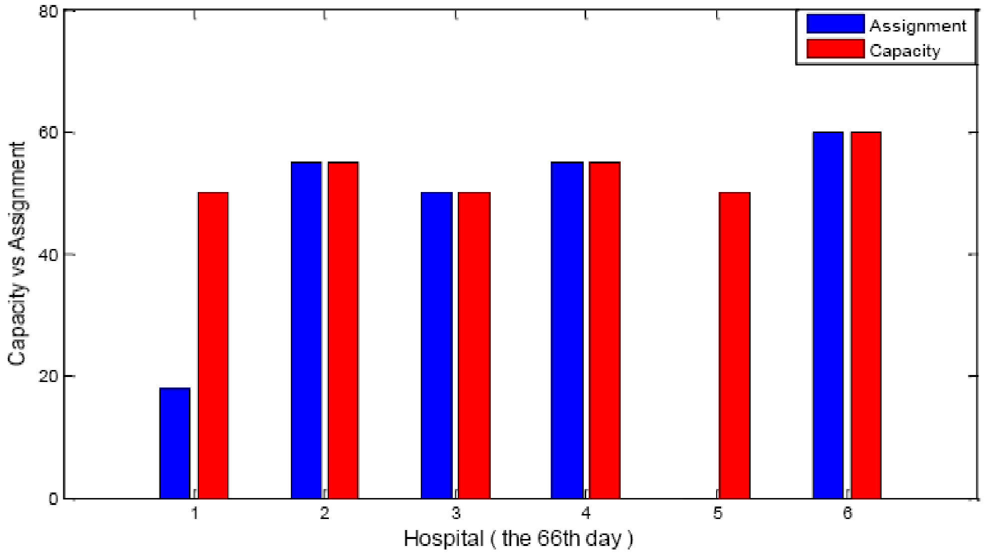

Figure 4. Hospital assignment on 66th day

4.3. **Sensitivity analyses.** To understand the influence of the related parameters on the proposed model, two parameters are discussed here. We test three situations for each of them: $\beta_1 = 2.9 \times 10^{-5}$, $\beta_2 = 3 \times 10^{-5}$, $\beta_3 = 3.1 \times 10^{-5}$, $\delta_1 = 0.29$, $\delta_2 = 0.30$ and $\delta_3 = 0.31$. Figure 5 shows that the total cost grows with the increase of the parameter β. The reason is that there will be more people who get exposed to the disease and get infected with the parameter β increasing. Hence, the demand for medical resource will grow, which leads to a rise of the total cost. Similarly, Figure 6 shows that the total cost will decrease with the increase of the parameter δ. That means there will be more infected population who get recovered, which reduces the demand of medical resource as well as the total cost.

5. **Conclusion.** In this paper, we study a multi-stage scheduling model for medical resource allocation in the control of influenza diffusion. The dynamic logistics model in response to the demand forecasting is constructed as a 0-1 mixed linear integer programming problem, which mainly considers decision-making at the local hospitals, the distribution centers and the medicine suppliers. The innovation of our research against the existing works is the interaction between the course of influenza diffusion and the dynamic

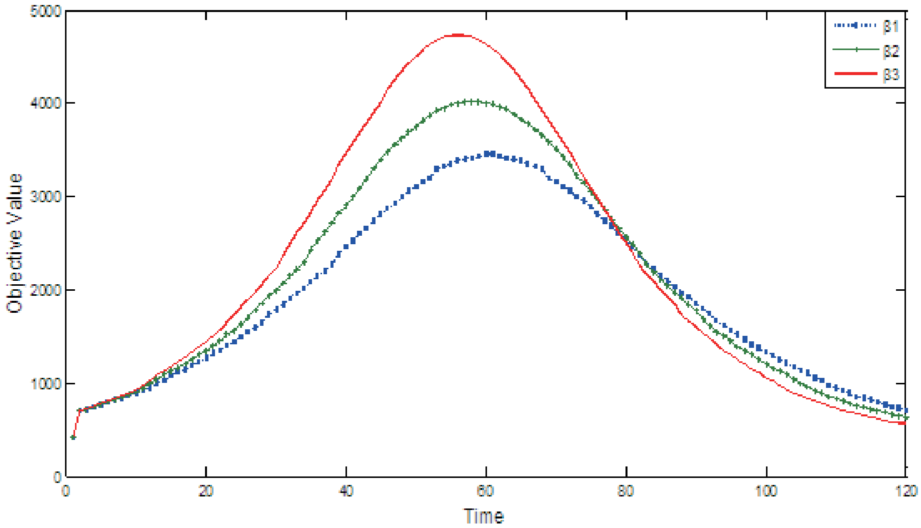

Figure 5. Sensitivity analysis on parameter β

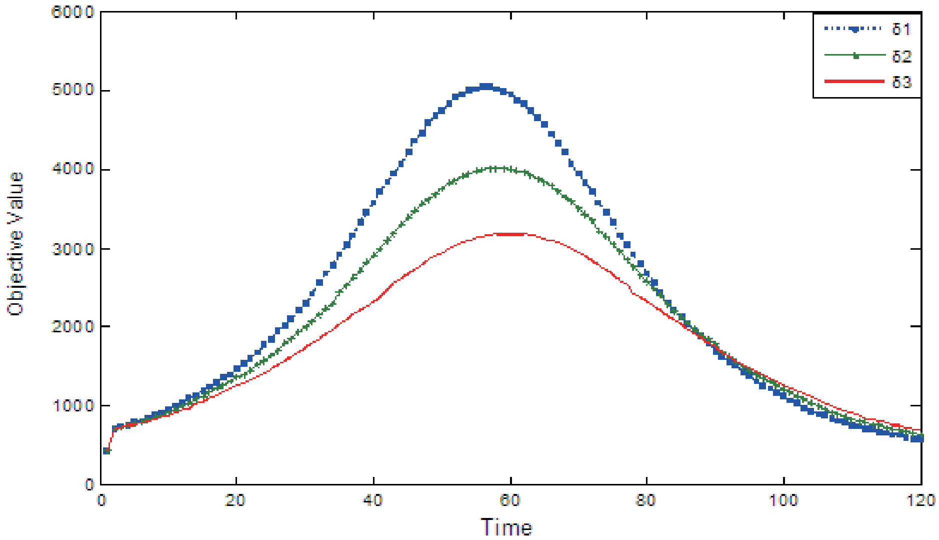

Figure 6. Sensitivity analysis on parameter δ

medical resource allocation. Future studies may address other realistic concerns such as the lead time of suppliers, the different order strategies of DCs, the random assignments of the infected people and the different exit rates of the population groups.

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