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HIGH QUALITY BLIND IMAGE DEBLURRING USING HYPER-LAPLACIAN PRIOR

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Abstract. *Blind image deblurring is a longstanding challenge in image processing and computer vision. In this paper, we propose a new blind image deblurring approach using hyper-Laplacian prior. Firstly, maximum a posterior (MAP) based method is applied to generating the model of estimating blur kernel. Then, we use an alternating minimization scheme to solve the kernel estimation model. Furthermore, the generalization of soft-thresholding (GST) operator is adopted in order to deal with the non-convex problem caused by hyper-Laplacian prior. Finally, when the blur kernel has been estimated, a non-blind deblurring method is chosen to recover the latent image. Experimental results demonstrate that the images restored by the proposed method are superior to the state-of-the-art blind deblurring algorithms in quality.*
Keywords: Blind image deblurring, Hyper-Laplacian, Maximum a posterior (MAP), Generalization of soft-thresholding (GST)

1. **Introduction.** Blurry images are often caused by some reasons, for example the relative motion between the camera and the scene [1], out of focus [2] and low light conditions [3]. The blur sometimes can be used for artistic purpose, such as emphasizing the dynamic nature of a scene. However, it often acts as an undesired effect that we want to reduce or even to remove, for example, in many photos taken by handheld camera, traffic monitoring, medical film observation, and many scientific fields. So there is a great practical significance in dealing with the blurry image. In our work, we mainly focus on blind image deblurring. The process of image degradation is usually modeled as $y = x * k + n$, where y is the blurred image, x is the latent image, k is the blur kernel, which is also known as the point spread function (PSF), $*$ is the convolution operator and n often denotes the additive white Gaussian noise. The target of blind deblurring is to obtain x and k with the given y, a well-known ill-posed inverse problem. As the process of dealing with the degraded model above is a deconvolution operation, sometimes image deblurring is also called image deconvolution.

Image deblurring has recently received a lot of attention. According to the fact that whether the blur kernel is known or not, it is often divided into two categories: non-blind and blind deblurring. In non-blind deblurring, the blur kernel is assumed already known or computed in some way. So the task is only to recover the latent image. Krishnan and Fergus [4] proposed a fast non-blind image deconvolution method using hyper-Laplacian prior $\left(p(x) \propto e^{-k|x|^{\alpha}}\right)$, typically with $0.5 \leq \alpha \leq 0.8$. By using hyper-Laplacian prior the heavy-tailed distribution of gradients in a natural image can be better modeled than using

Laplacian and Gaussian priors. However, it will generate a confused non-convex problem due to hyper-Laplacian with $\alpha < 1$. Krishnan and Fergus analyzed two specific fraction values, i.e., $\alpha = \frac{1}{2}$ and $\alpha = \frac{2}{3}$ by finding the roots of a cubic and quartic polynomial, respectively. However, such method needs to compute different polynomials according to the different α. Besides, selecting the best real root seems very troublesome. Zuo et al. [5] proposed a generalization of soft-thresholding (GST) operator for non-convex l_α-norm minimization ($0 \leq \alpha < 1$) problem. The GST operator can easily solve the non-convex sparse problems with arbitrary α values. While in most circumstances, the problem we meet is a blind deconvolution owing to the unknown PSF. So compared with the non-blind deconvolution, the task of the blind deconvolution becomes more challenging.

In this paper, we describe a blind deblurring approach based on hyper-Laplacian prior. Compared with the previous work, this prior is mainly used for recovering the latent image directly in non-blind deblurring. In our work, we use it to estimate the blur kernel. Besides, in order to solve the non-convex problem caused by hyper-Laplacian prior, we utilize the GST operator. At last, when the blur kernel has been estimated, we employ a non-blind method to recover the final image. Experimental results restored by our method prove better quality than that of Fergus et al. [6], Cho and Lee [7] and Pan et al. [8].

The rest of the paper is organized as follows. Section 2 mainly analyzes the blind deblurring model by using MAP based approach. Section 3 describes the process of solving blind deblurring model via an alternative minimization scheme. Section 4 demonstrates the experimental results compared with state-of-the-art blind deblurring algorithms. Finally, the conclusions are given in Section 5.

2. **Blind Deblurring Model Based on MAP.** In this section, we adopt MAP based method to analyze the above degraded process of the image and then the blind deblurring model generates. The MAP is a mode of the posterior distribution. It can be used to obtain an estimation of unobserved quantities on the basis of empirical data. In our work, the target of blind deblurring is to obtain x and k with the given y. So we can adopt MAP based approach to analyze this ill-posed inverse problem. The MAP maximizes a joint posterior probability distribution with respect to the blur kernel k and the latent image x,

$$(x^*, k^*) = \underset{x,k}{\operatorname{argmax}}\, P(x, k|y). \tag{1}$$

According to the Bayes theorem, this posterior distribution is proportional to the product of a likelihood, a prior on the latent image x and a prior on the blur kernel k,

$$P(x, k|y) \propto P(y|x, k)P(x)P(k). \tag{2}$$

Next we apply negative logarithmic operation on Equation (2) and adopt the corresponding likelihood and priors. Then work out Equation (1) can be transformed into solving the following Equation (3)

$$(x^*, k^*) = \underset{x,k}{\operatorname{argmin}} \left\{ \|x * k - y\|_2^2 + \lambda \|\nabla x\|_\alpha^\alpha + \gamma \|k\|_2^2 \right\}, \tag{3}$$

where $\nabla = [\nabla_h, \nabla_v]$, ∇_h and ∇_v are the horizontal and vertical gradient operators, respectively; λ and γ are the regularization parameters, $0.5 \leq \alpha \leq 0.8$. The right side of Equation (3) consists of three terms. The first term is the data fitting term derived from the degraded model of the image. The second term is the sparse hyper-Laplacian distribution prior of x in the gradient domain. The third term is the Tikhonov regularization on the blur kernel k.

3. Deblurring Algorithm.

3.1. PSF estimation. Equation (3) can be solved by alternatively computing

$$x^* = \underset{x}{\text{argmin}} \left\{ \|x * k - y\|_2^2 + \lambda \|\nabla x\|_\alpha^\alpha \right\} \tag{4}$$

and

$$k^* = \underset{k}{\text{argmin}} \left\{ \|x * k - y\|_2^2 + \gamma \|k\|_2^2 \right\}. \tag{5}$$

x **subproblem**: Equation (4) is a non-convex problem due to hyper-Laplacian prior term. Here, a half-quadratic splitting method is used to solve this problem. With the introduction of an auxiliary variable g corresponding to ∇x, Equation (4) can be rewritten as

$$(x^*, g^*) = \underset{x,g}{\text{argmin}} \left\{ \|x * k - y\|_2^2 + \beta \|\nabla x - g\|_2^2 + \lambda \|g\|_\alpha^\alpha \right\}, \tag{6}$$

when β is close to ∞, the solution of Equation (6) approaches that of Equation (4). With this formulation, Equation (6) can be efficiently solved through alternatively minimizing x and g independently by fixing another variable.

The value of g is initialized to be zero. In each iteration, with the given g, x can be obtained by solving the following subproblem

$$x^* = \underset{x}{\text{argmin}} \left\{ \|x * k - y\|_2^2 + \beta \|\nabla x - g\|_2^2 \right\}. \tag{7}$$

The solution of x can be obtained by

$$x^* = \mathcal{F}^{-1} \left(\frac{\overline{\mathcal{F}(k)}\mathcal{F}(y) + \beta \left(\overline{\mathcal{F}(\nabla h)}\mathcal{F}(g_h) + \overline{\mathcal{F}(\nabla v)}\mathcal{F}(g_v) \right)}{\overline{\mathcal{F}(k)}\mathcal{F}(k) + \beta \left(\overline{\mathcal{F}(\nabla h)}\mathcal{F}(\nabla h) + \overline{\mathcal{F}(\nabla v)}\mathcal{F}(\nabla v) \right)} \right), \tag{8}$$

where $\mathcal{F}(\cdot)$ and $\mathcal{F}(\cdot)^{-1}$ denotes the fast fourier transform (FFT) and the inverse FFT, respectively. The $\overline{\mathcal{F}(\cdot)}$ is complex conjugate operator. Given a fixed x, g can be obtained by solving Equation (9)

$$g^* = \underset{g}{\text{argmin}} \left\{ \beta \|\nabla x - g\|_2^2 + \lambda \|g\|_\alpha^\alpha \right\}. \tag{9}$$

Lemma 3.1. *The GST operator*

Let z be a single variable, if the optimal solution of

$$\min_z \left\{ \frac{1}{2}(z - f)^2 + \mu |z|^q \right\} \tag{10}$$

is z^*, then z^* is defined as

$$z^* = \begin{cases} 0, & \textit{otherwise}, \\ sgn(f) S_q^{GST}(|f|; \mu), & \textit{if } |f| > \tau_q^{GST}(\mu), \end{cases} \tag{11}$$

where the thresholding value $\tau_q^{GST}(\mu)$ is

$$\tau_q^{GST}(\mu) = (2\mu(1-q))^{\frac{1}{2-q}} + \mu q (2\mu(1-q))^{\frac{q-1}{2-q}}. \tag{12}$$

More details about $S_q^{GST}(|f|; \mu)$ can refer to [5].

According to Lemma 3.1, the solution of Equation (9) is

$$g^* = \begin{cases} 0, & \text{otherwise}, \\ \text{sgn}(\nabla x) S_\alpha^{GST} \left(|\nabla x|; \frac{\lambda}{2\beta} \right), & \text{if } |\nabla x| > \tau_\alpha^{GST} \left(\frac{\lambda}{2\beta} \right). \end{cases} \tag{13}$$

k **subproblem**: With the given x, we estimate the blur kernel k in the gradient space by

$$k^* = \underset{k}{\operatorname{argmin}} \left\{ \|\nabla x * k - \nabla y\|_2^2 + \gamma \|k\|_2^2 \right\}, \tag{14}$$

where $k(i,j) \geq 0$, $\sum_i \sum_j k(i,j) = 1$. Obviously, Equation (14) is a least squares minimization problem and the solution of it can be fast obtained by [7].

After obtaining k, we set the negative elements to 0, and normalize it to ensure that the sum of its elements is 1. The proposed kernel estimation process is performed in a coarse-to-fine manner using an image pyramid. Algorithm 1 shows the main steps for estimating blur kernel on one pyramid level.

Algorithm 1 Kernel Estimation

Input: Blurred image y, parameters λ, γ, $\beta_{\max}$ and α
Initialize x and k from the previous coarser level.
for $i = 1 \rightarrow 20$ **do**
  $\beta \leftarrow 2\lambda$
  while $\beta < \beta_{\max}$ **do**
    Estimate g according to (13)
    Estimate x according to (8)
    $\beta \leftarrow 2\beta$
  end while
  Estimate k according to (14)
  $\lambda \leftarrow \max\left(\frac{\lambda}{1.1}, 1e^{-4}\right)$
end for
Output: Blur kernel k

3.2. **Latent image estimation.** After the PSF has been estimated, the next work is to employ a non-blind deconvolution method to recover the latent image. Firstly, we estimate the latent image x_1 via Equation (3) with setting $\gamma = 0$. Secondly, we estimate the latent image x_2 by using the L_0-regularized gradient prior from [9]. Thirdly, we calculate a difference map between two estimated images and remove artifacts with bilateral filtering. Finally, we subtract the filtered difference map from x_1 and obtain the final latent image.

4. **Experimental Results.** In this section, we present the results of the proposed algorithm and compare it with the state-of-the-art blind deconvolution approaches of Fergus et al. [6], Cho and Lee [7] and Pan et al. [8]. Firstly, some implementation details about experiments are introduced. We empirically set the following parameters $\lambda = 4e^{-3}$, $\gamma = 2$ and $\beta_{\max} = 1e^5$, respectively. The value of α is typically set within the range of $[0.5, 0.8]$ according to [4]. At the same time, we conduct the abundant experiments about the choice of α in our work. Finally, we set $\alpha = 0.8$. The algorithms are tested on the dataset from [10], which consists of 4 images and 8 different spatially invariant kernels to give a total of 32 blurred images. The advantage of our proposed method can model the heavy-tailed distribution of gradients well. More importantly, it can estimate the PSF more precisely by using the GST operator.

In Figure 1, we plot the cumulative histogram of error ratios across the dataset for the 4 algorithms. Sum of squared differences error (SSDE) is used to estimate the accuracy of the final results. Error ratios less than 3 are considered visually plausible. Evidently, success percent of [6,7] is less than [8] and ours within this range. Although the values of [8] and ours are both 84% when the ratio is below 3, what is more, success percent of ours outperforms higher than [8] when the error ratio is below 2. So we can draw a conclusion that the results of our algorithm are more precise.

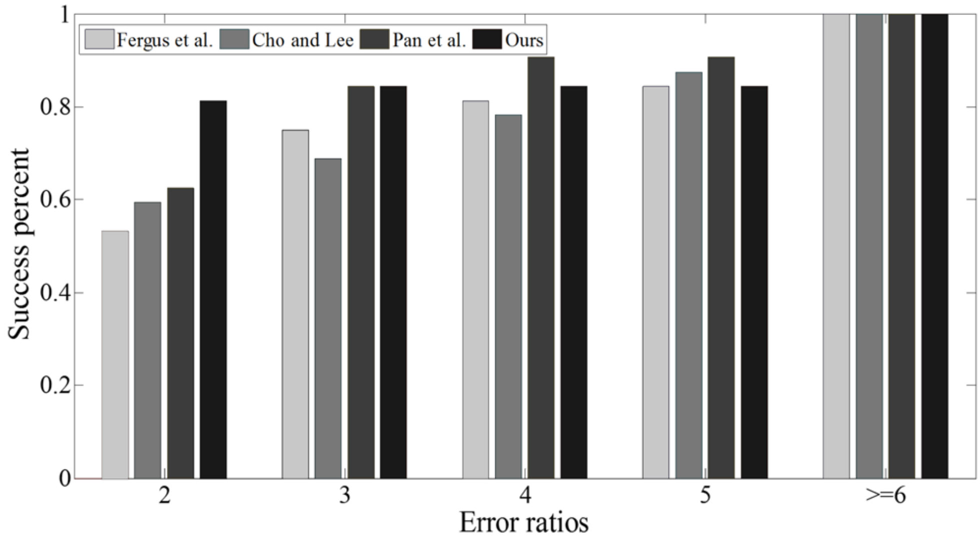

FIGURE 1. Cumulative histogram of the error ratios across the dataset

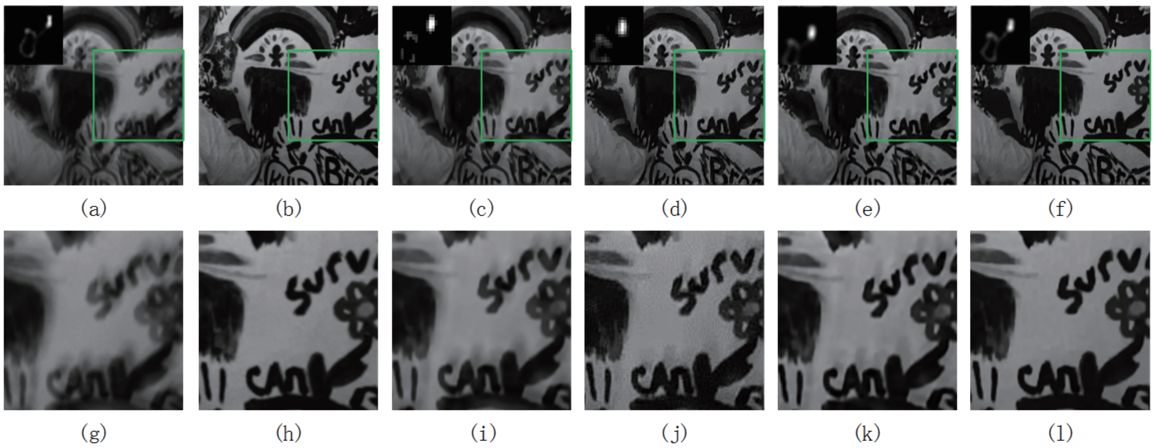

FIGURE 2. One visualization example. (a) Blurred image and ground truth kernel. (b) Original sharp image. (c)-(f) represent the results of [6-8], and ours, respectively. (g)-(l) are amplified from the regions in the black box corresponding to (a)-(f) for easier visual inspection.

Moreover, we adopt two quantitative image assessment methods (i.e., peak-signal-to-noise ratio (PSNR) and structural similarity index measure (SSIM)) to evaluate the quality of the restored images. PSNR and SSIM values in Table 1 (The numbers in the first column denote image with different PSFs, e.g., 3-2 denotes image 3 blurred by PSF 2) show that our approach obtained higher values than the other methods in most cases.

At last, an example from the above experiment is chosen and presented in Figure 2. In this example, it presents the kernels and the final results deblurred by [6-8], and ours. Besides, a representative region is extracted with green boxes and compared with the ground truth image. Result restored by the proposed method is very clear nearly with none artifacts and similar to the original image best.

5. **Conclusion.** To sum up, we propose a blind image deblurring method with hyper-Laplacian prior. Based on MAP probabilistic framework, we put forward the model of blind deblurring. In order to solve this model, an alternating minimization scheme is used. Besides, the GST operator is introduced to deal with the non-convex optimal problem caused by the hyper-Laplacian prior. A non-blind deblurring method is adopted to recover the latent image when the PSF has been estimated. The experimental results prove that

Table 1. PSNR (dB) and SSIM values of different non-blind deconvolution methods. The numbers in bold denote the best assessment value.

images	PSNR (dB)					SSIM				
	Blurry	Fergus	Cho	Pan	Ours	Blurry	Fergus	Cho	Pan	Ours
1-1	22.57	25.32	27.12	25.35	**30.12**	0.73	0.85	0.86	0.85	**0.92**
1-2	21.59	24.58	25.87	25.06	**26.35**	0.68	0.83	0.83	0.85	**0.88**
1-3	24.97	26.93	28.67	28.24	**29.72**	0.81	0.88	0.89	**0.91**	**0.91**
1-4	17.10	22.61	16.81	19.95	**24.75**	0.46	0.73	0.38	0.56	**0.82**
1-5	24.25	**31.49**	29.03	28.94	26.52	0.79	**0.94**	0.91	0.93	0.90
1-6	21.96	22.37	24.24	**31.12**	26.94	0.71	0.78	0.79	**0.93**	0.90
1-7	19.07	22.17	23.80	20.51	**26.96**	0.56	0.78	0.77	0.58	**0.90**
1-8	18.74	20.75	19.54	22.08	**26.53**	0.52	0.67	0.53	0.69	**0.88**
2-1	21.14	22.75	24.50	28.14	**29.29**	0.62	0.76	0.79	0.90	**0.92**
2-2	20.11	**25.61**	23.48	22.62	22.89	0.55	**0.84**	0.80	0.77	0.79
2-3	22.91	25.77	28.80	26.45	**29.35**	0.71	0.84	0.90	0.88	**0.92**
2-4	17.13	**24.01**	23.53	17.13	16.98	0.38	**0.77**	0.73	0.35	0.38
2-5	22.12	25.33	**26.23**	22.83	24.55	0.67	0.86	**0.88**	0.79	0.86
2-6	20.86	20.70	**24.83**	19.72	24.21	0.60	0.67	0.79	0.60	**0.85**
2-7	18.21	23.23	20.29	17.60	**23.71**	0.43	0.79	0.57	0.45	**0.82**
2-8	18.01	24.59	24.40	17.72	**27.20**	0.39	0.80	0.79	0.40	**0.90**
3-1	20.95	20.06	25.10	21.12	**26.10**	0.74	0.74	0.87	0.79	**0.89**
3-2	19.88	24.39	25.86	23.44	**27.24**	0.68	0.88	0.88	0.87	**0.93**
3-3	23.68	27.23	28.57	25.56	**28.58**	0.84	**0.93**	**0.93**	0.91	0.92
3-4	15.69	22.97	**24.02**	19.35	16.67	0.40	**0.83**	0.81	0.70	0.36
3-5	22.06	**27.34**	25.98	22.77	26.47	0.78	**0.93**	0.87	0.84	0.89
3-6	20.53	19.17	**23.53**	23.49	22.59	0.70	0.70	0.80	**0.86**	0.81
3-7	16.93	21.32	24.36	17.50	**27.13**	0.46	0.81	0.86	0.54	**0.94**
3-8	16.99	20.65	22.09	21.08	**22.47**	0.47	0.75	0.80	0.76	**0.84**
4-1	21.88	22.28	26.36	25.45	**28.79**	0.77	0.78	0.86	0.86	**0.92**
4-2	20.50	23.41	25.50	19.81	**25.94**	0.70	0.83	0.83	0.68	**0.85**
4-3	24.72	**30.23**	28.83	28.38	29.06	0.85	**0.94**	0.91	0.93	0.93
4-4	16.54	16.76	**20.95**	15.75	16.33	0.49	0.18	**0.63**	0.38	0.43
4-5	23.52	24.64	24.79	21.26	**26.94**	0.82	0.86	0.88	0.80	**0.93**
4-6	21.66	22.02	**24.96**	22.92	24.13	0.75	0.79	0.81	0.79	**0.86**
4-7	17.88	18.67	**26.78**	19.32	25.23	0.54	0.53	0.89	0.59	**0.90**
4-8	18.08	18.43	22.25	20.02	**25.17**	0.56	0.45	0.72	0.62	**0.81**

the restored images are of higher quality than some state-of-the-art algorithms. However, the proposed approach is limited to dealing with spatially invariant blur kernel, so we will be committed to solving the blurred images with spatially variant kernel problem in the future.

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