

THE DESIGN AND APPLICATION OF INTERVAL TYPE-2 FUZZY HYBRID SYSTEM BASED ON TIME SERIES PREDICTION

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ABSTRACT. *In many fields, it is an important and difficult problem to improve the prediction accuracy, especially to improve the prediction accuracy of time series. Time series forecasting method is the most basic approach for system prediction. At present, there are both advantages and disadvantages for time series forecasting method, and there are no methods being the best prediction method. The theoretical and experimental results show that, hybrid models can effectively improve their forecasting performances and forecasting accuracy. In this paper, fuzzy time series based interval type-2 hybrid systems models are designed by combining autoregressive integrated moving average (ARIMA) models, subtractive clustering, neural networks, and fuzzy logic systems. Then the Back-Propagation (BP) algorithms are used to tune the parameters of the proposed systems. The simulation results show that, it is feasible and effective to apply the designed models to fuzzy time series prediction based on the data of Australian thermal coal spot trading price.*

Keywords: ARIMA, Fuzzy logic system, Neural network, Back-Propagation algorithms, Time series prediction

1. Introduction. In the field of time series prediction, the most powerful approach to solve prediction problem is not only to be able to predict accurately, but also to discover the dynamic behavior of time series and reveal the law behind the dynamic phenomenon. The autoregressive integrated moving average (ARIMA) model is a well-known time series prediction method put forward by Box and Jenkins in the early 1970s [1]. Once ARIMA model is identified, the past value and the present value from the time series can predict future values.

Fuzzy neural network system (FNNS) is developed based on neural network and fuzzy logic system, and the fusion of the two has made up for deficiency of neural network in fuzzy data processing and deficiency of fuzzy logic in learning, which is system integrating language calculations, logical reasoning, distributed processing and nonlinear dynamics into one. The current researches on fuzzy neural network mainly focus on applications in the learning algorithms in fuzzy neural network, determination of FNNS structure, the extraction and refinement of fuzzy rules, auto-adaptive control of fuzzy neural network and predictive control, etc.

Traditionally, autoregressive moving average model is linear model which is applied most widely in time series prediction [2]. The application of fuzzy logic system in time series prediction has also made some research achievements [3-5], and the application of fuzzy neural network system in time series prediction research also shows that the fuzzy neural network can replace the traditional ARIMA structure [6-13]. At present, there are both advantages and disadvantages for time series forecasting method, and there are no methods being the best prediction method. The theoretical and experimental results show that, hybrid models can effectively improve their forecasting performances and forecasting accuracy. This paper designs interval type-2 fuzzy hybrid systems (IT2FHSs) of the time

series prediction based on ARIMA model, subtractive clustering, neural network and fuzzy logic system. The linear model and fuzzy neural network appear in the hybrid form, which can reflect its superiority in the predicting process and make the accuracy of hybrid system apply to the time series prediction better than that of using these models separately for time series prediction.

2. Designing IT2FHSs Based on Time Series Prediction. In this paper, IT2FHSs are designed by combining ARIMA with subtractive clustering, neural network and fuzzy logic system based on time series prediction.

2.1. Determination of the input layer nodes by ARIMA model. The input layer nodes are determined by ARIMA model. Then we establish the following models through model identification, model order determination, model parameter estimation and model prediction. The model is as follows:

$$\begin{cases} x_t = \varphi_0 + \varphi_1 x_{t-1} + \dots + \varphi_p x_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \dots - \theta_q \varepsilon_{t-q} \\ E(\varepsilon_t) = 0, \text{Var}(\varepsilon_t) = \sigma_\varepsilon^2 \\ E(\varepsilon_t \varepsilon_s) = 0, s \neq t \\ E(x_s \varepsilon_t) = 0, \forall s < t \end{cases}$$

The above model has established p input data, a linear function relationship between output data, in which $\varphi_1, \varphi_2, \dots, \varphi_p$ are the autoregressive coefficients, $\theta_1, \theta_2, \dots, \theta_q$ are moving coefficients, E is the expected value, Var is variance, and $\varepsilon_t, \varepsilon_{t-1}, \dots, \varepsilon_{t-p}$ are the data residual sequence. Therefore, the number of the input layer node of hybrid model is p .

2.2. The structure of IT2FHSs. The first step is to determine the nodes of the networks, and the second step is to determine the rules by subtractive clustering. Then we combine fuzzy logic system with neural network to design IT2FHSs based on time series prediction. The architecture of IT2FHSs with six layers is shown in Figure 1.

Consider an IT2FHS having n inputs $x_1 \in X_1, \dots, x_n \in X_n$ and one output $y \in Y$. An IT2FHS is also described by fuzzy IF-THEN rules that represent input-output relations

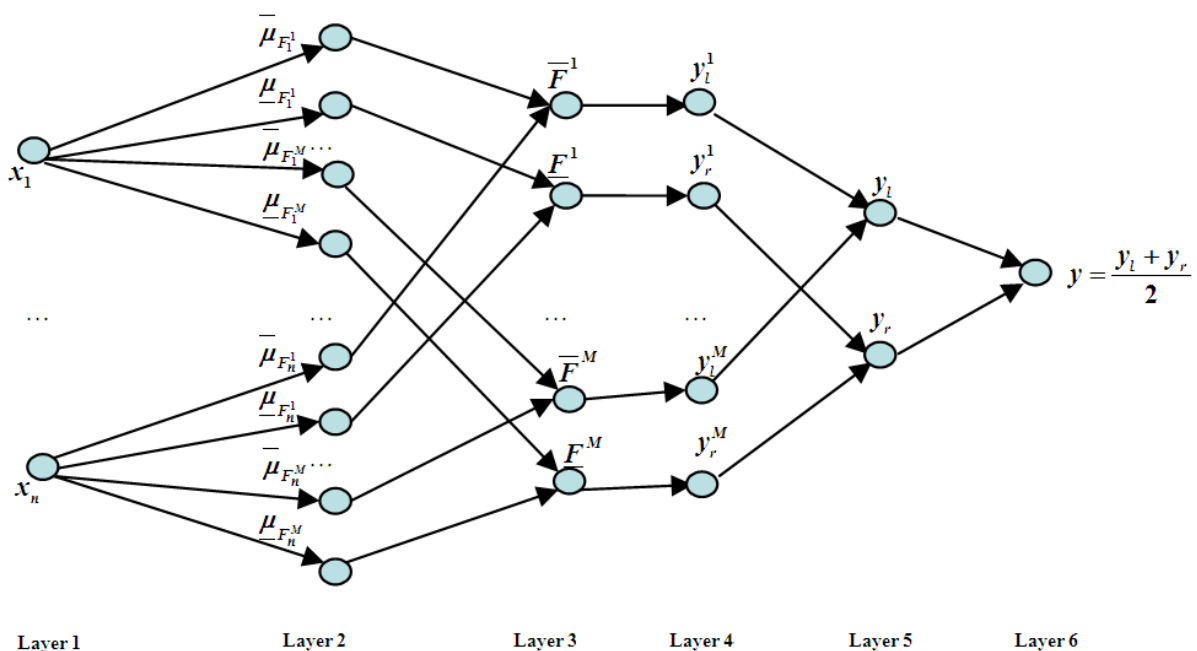


FIGURE 1. The structure of IT2FHSs

of a system. Assume the system adopts M rules and the k th rule can be expressed as follows:

R^k : IF x_1 is \tilde{F}_1^k and x_2 is \tilde{F}_2^k and \dots and x_n is \tilde{F}_n^k

Then Y^k is $C_0^k + C_1^k x_1 + C_2^k x_2 + \dots + C_n^k x_n \quad k = 1, \dots, M$

where $\tilde{F}_1^k, \tilde{F}_2^k, \dots, \tilde{F}_n^k$ are interval type-2 antecedent fuzzy sets. $C_0^k, C_1^k, \dots, C_n^k$ are consequent interval type-1 fuzzy sets, where $C_i^k = [c_i^k - s_i^k, c_i^k + s_i^k]$, c_i^k denotes the center (mean) of C_i^k , and s_i^k denotes the spread of C_i^k ($i = 0, \dots, n$).

Layer 1: Input layer:

$$X = (x_1, x_2, \dots, x_n)^T$$

Layer 2: Interval type-2 membership functions layer:

$$\tilde{\mu}_{\tilde{F}_i^k} = \left[\underline{\mu}_{\tilde{F}_i^k}, \overline{\mu}_{\tilde{F}_i^k} \right] \quad i = 1, 2, \dots, n \quad k = 1, 2, \dots, M$$

where $\underline{\mu}_{\tilde{F}_i^k} = \exp \left[-\frac{1}{2} \left(\frac{x_i - \underline{m}_{\tilde{F}_i^k}}{\underline{\sigma}_{\tilde{F}_i^k}} \right)^2 \right]$, $\overline{\mu}_{\tilde{F}_i^k} = \exp \left[-\frac{1}{2} \left(\frac{x_i - \overline{m}_{\tilde{F}_i^k}}{\overline{\sigma}_{\tilde{F}_i^k}} \right)^2 \right]$.

Layer 3: Firing strength layer:

$$F^k = \left[\underline{F}^k, \overline{F}^k \right] \quad k = 1, 2, \dots, M$$

where $\underline{F}^k = \prod_{i=1}^n \underline{\mu}_{\tilde{F}_i^k}(x_i)$, $\overline{F}^k = \prod_{i=1}^n \overline{\mu}_{\tilde{F}_i^k}(x_i)$.

Layer 4: Consequent of rule layer:

$$Y^k = [y_l^k, y_r^k] \quad k = 1, 2, \dots, M$$

where $y_l^k = \sum_{i=1}^n c_i^k x_i + c_0^k - \sum_{i=1}^n |x_i| s_i^k - s_0^k$, $y_r^k = \sum_{i=1}^n c_i^k x_i + c_0^k + \sum_{i=1}^n |x_i| s_i^k + s_0^k$.

Layer 5: Type-reduction layer: y_l, y_r are computed by the Karnik-Mendel (KM) algorithm [11] as follows:

$$y_l = \frac{\sum_{k=1}^M F^k y_l^k}{\sum_{k=1}^M F^k} = \frac{\sum_{k=1}^L \overline{F}^k y_l^k + \sum_{k=L+1}^M \underline{F}^k y_l^k}{\sum_{k=1}^L \overline{F}^k + \sum_{k=L+1}^M \underline{F}^k}$$

$$y_r = \frac{\sum_{k=1}^M F^k y_r^k}{\sum_{k=1}^M F^k} = \frac{\sum_{k=1}^R \underline{F}^k y_r^k + \sum_{k=R+1}^M \overline{F}^k y_r^k}{\sum_{k=1}^R \underline{F}^k + \sum_{k=R+1}^M \overline{F}^k}$$

Layer 6: Output layer:

$$f_s(X) = \frac{y_l + y_r}{2}$$

2.3. Optimization of system parameters. The system parameters are optimized by the BP algorithm in this paper. In BP algorithm [4], the parameters are not fixed in advance and parameters before and after rules are obtained through the turning. Given the input and output training data pair $(X : Y)$, design the system shown in Figure 1. For the j th sample, define the error function $E^j = \frac{1}{2} * [f_s(X^j) - Y^j]^2$, $j = 1, \dots, K - n$, use the BP algorithm to minimize the errors, and then obtain the recirculation of turning parameters $m_{F_i^l}, \sigma_{F_i^l}, c_0^l, c_1^l, \dots, c_n^l, s_0^k, s_1^k, \dots, s_n^k$.

The whole process is as follows:

(1) Initialization of system parameters

$$\overline{m} = 200 + 0.2 * rands(M, n)$$

$$\overline{\sigma} = 538 + 0.2 * rands(M, n)$$

$$\begin{aligned} \underline{m} &= 199 + 0.2 * rands(M, n) \\ \underline{\sigma} &= 540 + 0.2 * rands(M, n) \\ c &= 0.2 + rand(M, n + 1) \\ s &= 0.3 + rand(M, n + 1) \end{aligned}$$

where “*” denotes product operation, and \bar{m} , $\bar{\sigma}$, \underline{m} , $\underline{\sigma}$ are spaced and center of rules antecedent. c , s are spaced and center of rules consequent.

(2) Circulation type to optimize system parameters

$$\begin{aligned} \Delta \bar{\sigma}_{F_i^k} &= \eta \frac{\partial E^j}{\partial \bar{\sigma}_{F_i^k}} = \eta * e^j * \frac{1}{2} * \exp \left(-\frac{1}{2} \left[\frac{x_i^j - \bar{m}_{F_i^k}}{\bar{\sigma}_{F_i^k}} \right]^2 \right) * \frac{(x_i^j - \bar{m}_{F_i^k})^2}{\bar{\sigma}_{F_i^k}^3} * (y_l^k + y_r^k) \\ \Delta \underline{\sigma}_{F_i^k} &= \eta \frac{\partial E^j}{\partial \underline{\sigma}_{F_i^k}} = \eta * e^j * \frac{1}{2} * \exp \left(-\frac{1}{2} \left[\frac{x_i^j - \underline{m}_{F_i^k}}{\underline{\sigma}_{F_i^k}} \right]^2 \right) * \frac{(x_i^j - \underline{m}_{F_i^k})^2}{\underline{\sigma}_{F_i^k}^3} * (y_l^k + y_r^k) \\ \Delta \bar{m}_{F_i^k} &= \eta \frac{\partial E^j}{\partial \bar{m}_{F_i^k}} = \eta * e^j * \frac{1}{2} * \exp \left(-\frac{1}{2} \left[\frac{x_i^j - \bar{m}_{F_i^k}}{\bar{\sigma}_{F_i^k}} \right]^2 \right) * \frac{x_i^j - \bar{m}_{F_i^k}}{\bar{\sigma}_{F_i^k}^2} * (y_l^k + y_r^k) \\ \Delta \underline{m}_{F_i^k} &= \eta \frac{\partial E^j}{\partial \underline{m}_{F_i^k}} = \eta * e^j * \frac{1}{2} * \exp \left(-\frac{1}{2} \left[\frac{x_i^j - \underline{m}_{F_i^k}}{\underline{\sigma}_{F_i^k}} \right]^2 \right) * \frac{x_i^j - \underline{m}_{F_i^k}}{\underline{\sigma}_{F_i^k}^2} * (y_l^k + y_r^k) \\ \Delta c_i^k &= \eta \frac{\partial E^j}{\partial c_i^k} = \eta * e^j * \frac{1}{2} * \frac{\bar{F}^k + \underline{F}^k}{\sum_{k=1}^R \underline{F}^k + \sum_{k=R+1}^M \bar{F}^k} * x_i^j + \eta * e^j * \frac{1}{2} * \frac{\bar{F}^k + \underline{F}^k}{\sum_{k=1}^L \bar{F}^k + \sum_{k=L+1}^M \underline{F}^k} * x_i^j \\ \Delta s_i^k &= \eta \frac{\partial E^j}{\partial s_i^k} = \eta * e^j * \frac{1}{2} * \frac{\bar{F}^k + \underline{F}^k}{\sum_{k=1}^R \underline{F}^k + \sum_{k=R+1}^M \bar{F}^k} * |x_i^j| - \eta * e^j * \frac{1}{2} * \frac{\bar{F}^k + \underline{F}^k}{\sum_{k=1}^L \bar{F}^k + \sum_{k=L+1}^M \underline{F}^k} * |x_i^j| \end{aligned}$$

$i = 0, \dots, n. \quad x_0^j = 1$

where “*” denotes product operation, and \bar{m} , $\bar{\sigma}$, \underline{m} , $\underline{\sigma}$ are spaced and center of rules antecedent. c , s are spaced and center of rules consequent, η denotes learning rate, $e^j = f_s(X^j) - Y^j$ is error value, and F^k is firing strength of rules. Δc_i^k , Δs_i^k are parameters associated with recursions of $c_0, c_1, \dots, c_n, s_0, s_1, \dots, s_n$ to update the design parameters of this system.

(3) The parameter after optimization

$$\begin{aligned} new \bar{\sigma}_{F_i^k} &= old \bar{\sigma}_{F_i^k} + \Delta \bar{\sigma}_{F_i^k} + r * \Delta \bar{\sigma}_{F_i^k}' \\ new \underline{\sigma}_{F_i^k} &= old \underline{\sigma}_{F_i^k} + \Delta \underline{\sigma}_{F_i^k} + r * \Delta \underline{\sigma}_{F_i^k}' \\ new \bar{m}_{F_i^k} &= old \bar{m}_{F_i^k} + \Delta \bar{m}_{F_i^k} + r * \Delta \bar{m}_{F_i^k}' \\ new \underline{m}_{F_i^k} &= old \underline{m}_{F_i^k} + \Delta \underline{m}_{F_i^k} + r * \Delta \underline{m}_{F_i^k}' \\ new c_0^k &= old c_0^k + \Delta c_1^k + r * \Delta c_1^k' \\ &\dots \\ new c_n^k &= old c_n^k + \Delta c_n^k + r * \Delta c_n^k' \\ new s_0^k &= old s_0^k + \Delta s_1^k + r * \Delta s_1^k' \\ &\dots \\ new s_n^k &= old s_n^k + \Delta s_n^k + r * \Delta s_n^k' \end{aligned}$$

where $\Delta \bar{m}_{F_i^{k'}}', \Delta \bar{\sigma}_{F_i^{k'}}', \Delta \underline{m}_{F_i^{k'}}', \Delta \underline{\sigma}_{F_i^{k'}}', \Delta c_0^{k'}, \dots, \Delta c_n^{k'}, \Delta s_0^{k'}, \dots, \Delta s_n^{k}'$ denote parametric variations, $\Delta \bar{m}_{F_i^k}, \Delta \bar{\sigma}_{F_i^k}, \Delta \underline{m}_{F_i^k}, \Delta \underline{\sigma}_{F_i^k}, \Delta c_0^k, \dots, \Delta c_n^k, \Delta s_0^k, \dots, \Delta s_n^k$ denote parametric variations in last epoch, and r is momentum coefficient.

3. Forecasting the Price of Australian Thermal Coal Spot Trading. Select 269 data in total of Australia spot trading price from January 5, 2006 to January 20, 2011, add the uniformly distributed noise between $[-0.2, 0.2]$ to indicate that there is error in measurement, and use MATLAB software to draw these 269 time series data with noise.

This paper established model ARIMA for the time series to predict through five processes, namely, model identification, model order determination, model parameter estimation, establishment of linear relationship between input and output data and model prediction. The IT2FHS is programmed by MATLAB to obtain the tracking results of predicted output data and actual output data, which are shown in Figure 2.

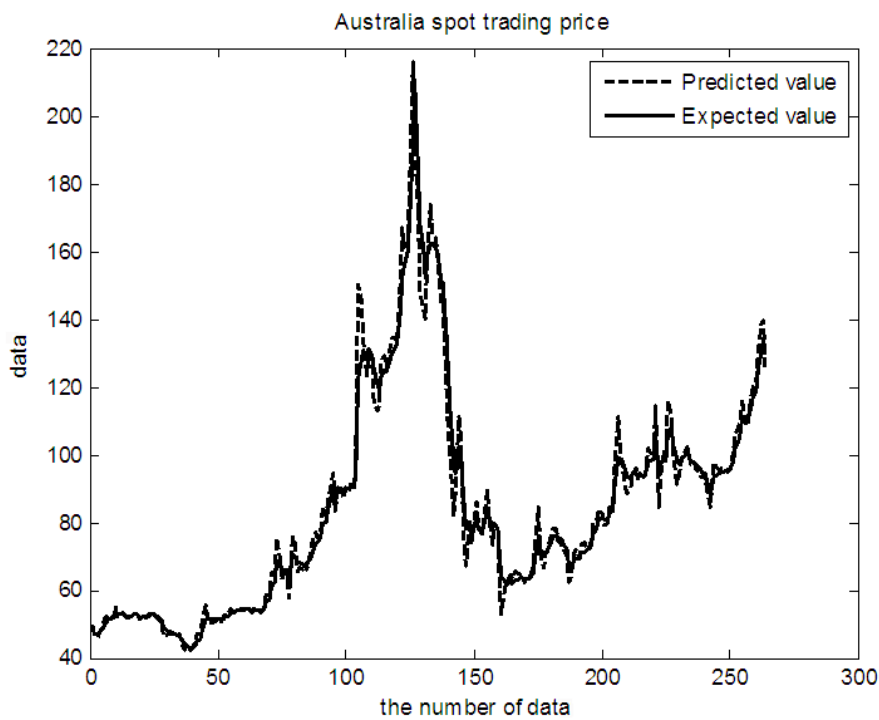


FIGURE 2. The tracking diagram of time series data

The root mean square error of the system: $RMSE = 5.3359$. Because the error is very small, the prediction effect is good. The classical model of time series is established as follows:

$$\begin{cases} x_t = 0.5176x_{t-1} + 0.3889x_{t-2} + 0.3575x_{t-3} + 0.2220x_{t-4} + 0.2337x_{t-5} + \varepsilon_t \\ E(x_s \varepsilon_t) = 0, \forall s < t \end{cases}$$

The model established above has established 5 input data, a linear function relationship between output data. Therefore, determine the input layer node of hybrid model is 5.

Australian thermal coal spot trading price has 269 data in total, which constitute 264 sets of input-output data pairs in total, use the former 134 data pairs to design the hybrid system, and use the latter 130 data pairs to test the system. This data set is divided into two subsets: one subset $(X^{(1)} : Y^{(1)}), (X^{(2)} : Y^{(2)}), \dots, (X^{(134)} : Y^{(134)})$ is the training subset, and the other subset $(X^{(135)} : Y^{(135)}), (X^{(136)} : Y^{(136)}), \dots, (X^{(264)} : Y^{(264)})$ is the testing subset. 134 rules can be obtained from 134 input-output training pairs, in which redundant rules will appear, screen with subtractive clustering is used to define the density function and the data radius of each data point in the space, according to the data point

density around each data point, calculate the possibility of this point as the cluster center, and the selected data point of cluster center is with the highest density function values, thus determining the number of cluster centers to be 30. Finally, 30 rules and five inputs are obtained. The k th rule can be expressed as follows:

R^k : IF x_1 is \tilde{F}_1^k and x_2 is \tilde{F}_2^k and x_3 is \tilde{F}_3^k and x_4 is \tilde{F}_4^k and x_5 is \tilde{F}_5^k

Then y is $Y^k = C_0^k + C_1^k x_1 + C_2^k x_2 + C_3^k x_3 + C_4^k x_4 + C_5^k x_5 \quad k = 1, \dots, 30$

where $\tilde{F}_1^k, \tilde{F}_2^k, \dots, \tilde{F}_5^k$ are interval type-2 antecedent fuzzy sets. $C_0^k, C_1^k, \dots, C_5^k$ are consequent interval type-1 fuzzy sets, where $C_i^k = [c_i^k - s_i^k, c_i^k + s_i^k]$, c_i^k denotes the center (mean) of C_i^k , and s_i^k denotes the spread of C_i^k ($i = 0, \dots, 5$).

We choose singleton fuzzification, product implication and combine ARIMA model, subtractive clustering, neural network and fuzzy logic system to design the hybrid system model. The learning rate $\eta = 0.01$ and the momentum coefficient $r = 0.05$. The given initial parameters are as follows:

$$\begin{aligned} \bar{m} &= 200 + 0.2 * \text{rands}(32, 5) \\ \underline{m} &= 199 + 0.2 * \text{rands}(32, 5) \\ \bar{\sigma} &= 538 + 0.2 * \text{rands}(32, 5) \\ \underline{\sigma} &= 540 + 0.2 * \text{rands}(32, 5) \\ c &= 0.2 + \text{rand}(32, 6) \\ s &= 0.3 + \text{rand}(32, 6) \end{aligned}$$

The first 134 input-output data are used for training, i.e., the designs of the hybrid system. After 7000 times iteration, the system total error tends to be stable. After adjusting the parameters of the system, when the system is stable, the tracking results of the training data are shown in Figure 3. The hybrid system achieves optimization after training the parameters. The remaining 130 input-output data pairs are used for testing the designs. The tracking results are as shown in Figure 3.

Furthermore, since a T1FHS has the same structure as an IT2FHS, to compare the predicting results of the IT2FHS with its T1 counterpart. In the simulation, the rule number of T1FHS is chosen the same as the above IT2FHS, and their membership functions are

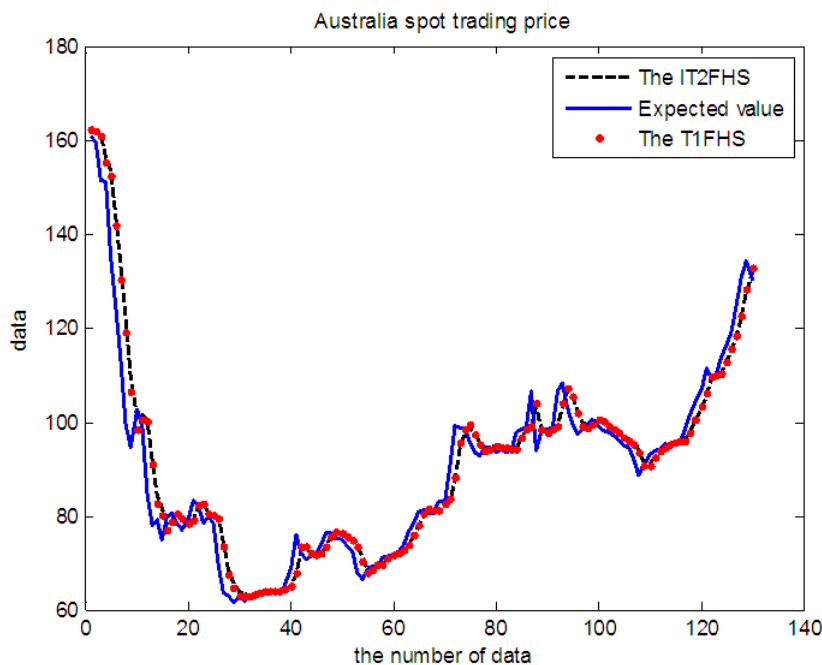


FIGURE 3. The tracking diagram of the testing data

TABLE 1. Root mean-squared error comparison

hybrid system	Australia spot trading price
Type-1 hybrid system	5.5442
Type-2 hybrid system	5.0912

all Gauss membership functions. The predicting results are shown in Figure 3. RMSE of different models are shown in Table 1.

From Figure 3 and Table 1, it can be seen that the IT2FHS has better performance than its T1 counterpart while the system contains some uncertainties.

4. Conclusions. In this paper, an IT2FHS was designed by combining ARIMA, subtractive clustering, neural network and fuzzy logic system and the proposed system was used to solve time series prediction problem of Australian thermal coal spot trading price. We use ARIMA to determine the input layer nodes, the subtractive clustering to extract rules, so as to determine the structure of the fuzzy hybrid system, and the BP algorithm is used to adjust the parameters of the designed IT2FHS. Simulation study shows that a good predicting result has been achieved. In addition, by comparing the simulation results of the IT2FHS with the T1FHS, it has been confirmed that the IT2FHS can achieve better forecasting result than its T1 counterpart while the proposed system contains some uncertainties. Our future work is to study the problem on the optimal structure of the IT2FHS by using genetic algorithms and other iterative learning algorithms.

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