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ROBUST STABILIZATION AND CONTROLLER DESIGN FOR A CLASS OF NETWORKED CONTROL SYSTEMS WITH NONUNIFORM SAMPLING AND NONLINEARITY

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Abstract. *This paper investigates the robust stabilization problem for a class of networked control systems with time-varying delays, time-varying sampling intervals and nonlinearity. The nonlinear perturbation of the system is time-varying and satisfies quadratic constraints. The network-induced delays and sampling intervals are time-varying and bounded. By using an input delay approach, the network induced delays, sampling intervals and nonlinear perturbation are presented in one framework. Using Lyapunov functional approach and introducing relax variables technology, the stability condition of the systems is proposed, which describes the maximum allowable nonlinear bound, the maximum allowable sampling intervals and the maximum allowable network delay bound. By using the cone complementary liberalization (CCL) algorithm and linear matrix inequalities (LMIs), the feedback gain of the robust state feedback controller is obtained. An example demonstrates the effectiveness of the proposed methods.*
Keywords: Networked control systems, Nonlinearity, Time-varying delays, Time-varying sampling

1. **Introduction.** Networked control system (NCS) integrates communication network with control system to attain low cost, simple installation, easy maintenance and high flexibility. However, NCSs' challenges such as time-varying network-induced delays, time-varying sampling intervals or nonlinear disturbances in transmission channel bring influences on the stability and the performance of the NCS. Recently, some works have discussed the control problem of NCSs with delays and uniform sampling intervals [1,2]. In a networked control system, all signals are sampled by a sampler, and the sampling rate for each signal may be varying from sample to sample according to actual situations. The kind of sampling intervals is called as time-varying sampling intervals or nonuniform sampling intervals. Over the past decades, NCSs with nonuniform sampling have been studied extensively by many researchers using sampled-data control theory [3,4]. A series of works on stability and control problems for the sampled-data control systems with variable sampling has been investigated [5,6]. However, these results are hard to be applied to NCS. Presently, [7] and [8] respectively studied the control problem of NCSs with nonuniform sampling. Nevertheless, these results are not appropriate for networked control systems with nonlinear disturbance.

To the best of the authors' knowledge, most of the existing literature does not consider the case that time-varying delays, nonlinearity and nonuniform sampling simultaneously exist in the NCSs, and their effects on the NCSs' stability. Consequently, it is necessary to conduct an analysis on the network delays, sampling intervals and nonlinearity, and understand how much effects these three factors make on the overall systems. This paper will take an alternative look at robust stabilization of the NCS subject to time-varying delays, time-varying sampling intervals and nonlinear constraints. By using the input delay approach, the effects of network-induced delays and nonlinear disturbance and sampling intervals are included in the NCS model. By using the Lyapunov functional approach, cone complementary liberalization algorithm and the technology of inducing relax matrix variables, a sufficient criterion ensuring the NCS to be stable is derived. Furthermore, the controller design conditions are obtained which are dependent on sampling interval bounds, network-induced delay bounds and nonlinear bound.

The rest of the paper is organized as follows. In Section 2, we first introduce the characteristics of the NCS with nonlinearity, then develop the model of the NCS to describe both the nonuniform sampling and the nonlinearity in a unified framework. Section 3 deals with the stability analysis and controller design of the NCS, respectively. The proposed approach is illustrated in Section 4 through a numerical example. Section 5 concludes the paper.

2. **Problem Formulation.**

We consider an NCS with nonlinearity. The plant is a continuous-time linear time-invariant system whose state-space representation is given by

$$\dot{x}(t) = Ax(t) + Bu(t) + h(t, x(t)) \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the system state, and $u(t) \in \mathbf{R}^p$ is the control input. A, B are some constant matrices of appropriate dimensions, $h(t, x(t)) : [0, \infty) \times \mathbf{R}^n \to \mathbf{R}^n$ represents nonlinear uncertainties of the plant to be controlled. Assume that $h(t, x(t))$ is a piecewise-continuous nonlinear function in both arguments t and x, and satisfies the following quadratic constraint condition for $\forall t \geq 0$

$$h^T(t, x(t))h(t, x(t)) \leq \alpha^2 x^T(t)H^T Hx(t) \tag{2}$$

where $\alpha > 0$ is the bounding parameter on the uncertain function $h(t, x(t))$ and H is a constant matrix. Note that for any given H, Inequality (2) defines a class of piecewise-continuous functions

$$H_\alpha = \left\{h : R^{n+1} \to R^n \middle| \, h^T h \leq \alpha^2 x^T H^T Hx \text{ in the domains of continuity}\right\} \tag{3}$$

The class H_α is comprised of functions that satisfy $h(t, 0) = 0$ in their domains of continuity, and $x = 0$ is an equilibrium of system (1).

Define the zero-order hold control action

$$u_c(t) = u_d(s_k) = Kx(s_k), \; s_k \leq t < s_{k+1} \tag{4}$$

where K is the feedback gain to be determined. u_d is a discrete-time control signal and the time s_k is the sampling instant satisfying $0 = s_0 < s_1 < \cdots < s_k < \cdots$. The sampling interval $T_k = s_{k+1} - s_k$ may vary but it is bounded. In this paper, it is assumed that T_k is time-varying and its lower bound and upper bound are known:

$$0 < T_m \leq T_k \leq T_M \tag{5}$$

where T_m and T_M depend on networked types. Modeling of continuous-time systems with discrete-time control inputs was investigated by [5]. The digital control law may be represented as follows by using input delay approach:

$$u_c(t) = Kx(s_k) = Kx(t - (t - s_k)) = Kx(t - \tau(t)), \; s_k \leq t < s_{k+1} \tag{6}$$

where $\tau(t) = t - s_k$ is piecewise linear with the derivative $\dot{\tau}(t) = 1$ for $t \neq s_k$, and $0 \leq \tau(t) \leq T_M$, $\forall t \geq s_0$. In NCSs, the control signals from the sampler at s_k take δ_k

time units to reach the actuator, then $u(t) = u_c(t - \delta_k) = Kx(t - \delta_k - \tau(t - \delta_k))$. In the following, $d(t) = \tau(t - \delta_k) + \delta_k$, so, $s_k = t - d(t)$ for $s_k + \delta_k \le t \le s_{k+1} + \delta_{k+1}$. Note that the variable sampling interval and networked delay are integrated in a single delay $d(t)$. Network communication delay δ_k is naturally assumed as $\delta_m \le \delta_k \le \delta_M$. The piecewise-constant control law (6) can be represented as a continuous-time controller with a time-varying piecewise continuous delay:

$$u(t) = Kx(t - d(t)),\ s_k + \delta_k \le t < s_{k+1} + \delta_{k+1} \tag{7}$$

where $d(t) = t - s_k$, is piecewise linear with the derivative $\dot{d}(t) = 1$ for $t \ne s_k + \delta_k$, and

$$\delta_m \le d(t) \le T_M + \delta_M \tag{8}$$

The case considered in this paper is that network communication delay is smaller than sampling interval, and thus we have $\delta_m \le \delta_M \le T_m \le T_M$. Let $\delta_M/T_m \le \rho \le 1$, then we can write (8) as

$$\delta_m \le d(t) \le T_M + \frac{\delta_M + \rho T_m}{2} = \overline{\sigma} \tag{9}$$

We call $\overline{\sigma}$ the maximum allowable equivalent delay bound (MAEDB). Combining the controller (7) into the NCS (1), we can obtain the closed-loop networked control system:

$$\dot{x}(t) = Ax(t) + BKx(t - d(t)) + h(t, x(t)),\ s_k + \delta_k \le t < s_{k+1} + \delta_{k+1} \tag{10}$$

3. **Main Results.**

Theorem 3.1. *For given scalars δ_m, δ_M, T_m, T_M $(0 \le \delta_m \le \delta_M \le T_m \le T_M)$, $\alpha > 0$, $0 \le \gamma < 1$, $\varepsilon > 0$, $\delta_M/T_m \le \rho \le 1$, and a matrix K, the closed-loop NCS (10) is asymptotically stable if there exist matrices $P = P^T > 0$, $Q_i = {Q_i}^T > 0$ $(i = 1, 2, 3)$, $Z_j = {Z_j}^T > 0$ $(j = 1, 2, 3)$, and N, T, M, R, E, S of appropriate deimensions, such that*

$$\begin{bmatrix} \Gamma & G_H^T \\ * & -\frac{1}{\varepsilon * \alpha^2} I \end{bmatrix} < 0 \tag{11}$$

where

$$\Gamma = \begin{bmatrix} \Gamma_1 & N & T & M & R & E & S & {A_c}^T Z_1 & {A_c}^T Z_2 & {A_c}^T Z_3 \\ * & -\frac{1}{\gamma\bar{\sigma}} Z_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\frac{1}{\bar{\sigma} - \gamma\underline{\sigma}} Z_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\frac{1}{\sigma} Z_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\frac{1}{\sigma} Z_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\frac{1}{\sigma} Z_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{\sigma} Z_3 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\frac{1}{\bar{\sigma}} Z_1 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\frac{1}{\bar{\sigma}} Z_2 & 0 \\ * & * & * & * & * & * & * & * & * & -\frac{1}{\bar{\sigma}} Z_3 \end{bmatrix} \tag{12}$$

$$\begin{aligned} \Gamma_1 &= \Gamma_{11} + \Gamma_{12} + {\Gamma_{12}}^T \\ \Gamma_{12} &= [\ N + S \quad M - E + R - T \quad E \quad -M - R - S \quad T - N \quad 0\ ] \end{aligned} \tag{13}$$

$$\Gamma_{11} = \begin{bmatrix} PA + A^T P + Q_1 + Q_2 + Q_3 & PBK & 0 & 0 & 0 & P \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & -Q_1 & 0 & 0 & 0 \\ * & * & * & -Q_2 & 0 & 0 \\ * & * & * & * & -Q_3(1-\gamma) & 0 \\ * & * & * & * & * & -\varepsilon I \end{bmatrix}$$

$$A_c = [\ A \quad BK \quad 0 \quad 0 \quad 0 \quad I\ ],\ G_H = [\ H \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\ ]$$

$$\sigma = T_M + \frac{\delta_M + \rho T_m}{2} - \delta_m,\ \bar{\sigma} = T_M + \frac{\delta_M + \rho T_m}{2},\ \underline{\sigma} = \delta_m \tag{14}$$

Proof: Consider the following piecewise Lyapunov functional:

$$\begin{aligned} V(t) &= x^T(t)Px(t) + \int_{t-\tau_m}^{t} x^T(s)Q_1x(s)ds + \int_{t-\left(T_M+\frac{\tau_M+\rho T_m}{2}\right)}^{t} x^T(s)Q_2x(s)ds \\ &\quad + \int_{t-\alpha d(t)}^{t} x^T(s)Q_3x(s)ds + \int_{-\left(T_M+\frac{\tau_M+\rho T_m}{2}\right)}^{0} \int_{t+\beta}^{t} \dot{x}^T(s)Z_1\dot{x}(s)dsd\beta \\ &\quad + \int_{-\left(T_M+\frac{\tau_M+\rho T_m}{2}\right)}^{-\tau_m} \int_{t+\beta}^{t} \dot{x}^T(s)Z_2\dot{x}(s)dsd\beta + \int_{-\left(T_M+\frac{\tau_M+\rho T_m}{2}\right)}^{0} \int_{t+\beta}^{t} \dot{x}^T(s)Z_3\dot{x}(s)dsd\beta \end{aligned} \tag{15}$$

$P = P^T > 0$, $Q_m = {Q_m}^T > 0$ $(m = 1, 2, 3)$, $Z_j = {Z_j}^T > 0$ $(j = 1, 2, 3)$.

Obviously, $V(t)$ is discontinuous. Note that $V(t)$ is continuous and positive except the instants $s_k + \delta_k$. Therefore, we consider the following two cases.

1). For $s_k + \delta_k < t < s_{k+1} + \delta_{k+1}$, calculating the derivative of $V(t)$ with respect to t along the solutions of the system (10), and denoting $h_1 = \tau_m$, $h_2 = T_M + \frac{\tau_M+\rho T_m}{2}$, it yields that

$$\begin{aligned} \dot{V}(t) &\leq 2x^T(t)P(Ax(t) + BKx(t-d(t)) + h(t,x(t))) + \sum_{i=1}^{3} x^T(t)Q_ix(t) \\ &\quad - x^T(t-h_1)Q_1x(t-h_1) - x^T(t-h_2)Q_2x(t-h_2) \\ &\quad - (1-\gamma)x^T(t-\gamma d(t))Q_3x(t-\gamma d(t)) + (Ax(t) + BKx(t-d(t)) \\ &\quad + h(t,x(t)))^T(h_2Z_1 + (h_2-h_1)Z_2 + h_2Z_3)(Ax(t) + BKx(t-d(t)) + h(t,x(t))) \\ &\quad - \int_{t-\gamma d(t)}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-d(t)}^{t-\gamma d(t)} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-h_2}^{t-d(t)} \dot{x}^T(s)Z_1\dot{x}(s)ds \\ &\quad - \int_{t-h_2}^{t-d(t)} \dot{x}^T(s)Z_2\dot{x}(s)ds - \int_{t-d(t)}^{t-h_1} \dot{x}^T(s)Z_2\dot{x}(s)ds - \int_{t-h_2}^{t} \dot{x}^T(s)Z_3\dot{x}(s)ds \\ &\quad + 2\zeta^T(t)N\left[x(t) - x(t-\gamma d(t)) - \int_{t-\gamma d(t)}^{t} \dot{x}(s)ds\right] + 2\zeta^T(t)T\left[x(t-\gamma d(t))\right. \\ &\quad \left. - x(t-d(t)) - \int_{t-d(t)}^{t-\gamma d(t)} \dot{x}(s)ds\right] + 2\zeta^T(t)M\left[x(t-d(t)) - x(t-h_2)\right. \\ &\quad \left. - \int_{t-h_2}^{t-d(t)} \dot{x}(s)ds\right] + 2\zeta^T(t)R\left[x(t-d(t)) - x(t-h_2) - \int_{t-h_2}^{t-d(t)} \dot{x}(s)ds\right] \\ &\quad + 2\zeta^T(t)E\left[x(t-h_1) - x(t-d(t)) - \int_{t-d(t)}^{t-h_1} \dot{x}(s)ds\right] \\ &\quad + 2\zeta^T(t)S\left[x(t) - x(t-h_2) - \int_{t-h_2}^{t} \dot{x}(s)ds\right] \\ &\leq \zeta^T(t)\left(\tilde{\Gamma}_1 + \gamma h_2NZ_1^{-1}N^T + (1-\gamma)h_2TZ_1^{-1}T^T + h_{12}M{Z_1}^{-1}M^T + h_{12}R{Z_2}^{-1}R^T\right. \\ &\quad \left. + h_{12}EZ_2^{-1}E^T + h_{12}S{Z_3}^{-1}S^T + \tilde{A}_c^TU\tilde{A}_c\right)\zeta(t) + \varepsilon h^T(t,x(t))h(t,x(t)) \\ &\quad - \int_{t-\gamma d(t)}^{t} \mathcal{H}_1Z_1^{-1}{\mathcal{H}_1}^Tds - \int_{t-d(t)}^{t-\gamma d(t)} \mathcal{H}_2Z_1^{-1}{\mathcal{H}_2}^Tds - \int_{t-h_2}^{t-d(t)} \mathcal{H}_3{Z_1}^{-1}{\mathcal{H}_3}^Tds \end{aligned}$$

$$-\int_{t-h_2}^{t-d(t)}\mathcal{H}_4 {Z_2}^{-1}{\mathcal{H}_4}^T ds-\int_{t-d(t)}^{t-h_1}\mathcal{H}_5 Z_2^{-1}{\mathcal{H}_5}^T ds-\int_{t-h_2}^{t}\mathcal{H}_6 Z_3^{-1}{\mathcal{H}_6}^T ds$$

where

$$\mathcal{H}_1=\zeta^T(t)N+\dot{x}^T(s)Z_1,\ \mathcal{H}_2=\zeta^T(t)T+\dot{x}^T(s)Z_1,\ \mathcal{H}_3=\zeta^T(t)M+\dot{x}^T(s)Z_1$$
$$\mathcal{H}_4=\zeta^T(t)R+\dot{x}^T(s)Z_2,\ \mathcal{H}_5=\zeta^T(t)E+\dot{x}^T(s)Z_2,\ \mathcal{H}_6=\zeta^T(t)S+\dot{x}^T(s)Z_3$$

$$\zeta(t)=\begin{bmatrix} x^T(t) & x^T(t-d(t)) & x^T(t-\tau_m) & x^T\left(t-\left(T_M+\frac{\tau_M+\rho T_m}{2}\right)\right) & x^T(t-\gamma d(t)) & h(t,x(t))\end{bmatrix}^T$$

By the Schur complements, combine (16) to obtain $\dot{V}(t)<0$ for all $s_k+\delta_k<t<s_{k+1}+\delta_{k+1}$.

$$\tilde{\Gamma}=\begin{bmatrix}
\tilde{\Gamma}_1 & N & T & M & R & E & S & \tilde{A}_c{}^T Z_1 & \tilde{A}_c{}^T Z_2 & \tilde{A}_c{}^T Z_3 & G_H^T\\
* & -\frac{1}{\gamma\bar{\sigma}}Z_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & -\frac{1}{\bar{\sigma}-\gamma\underline{\sigma}}Z_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & -\frac{1}{\sigma}Z_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & -\frac{1}{\sigma}Z_2 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & -\frac{1}{\sigma}Z_2 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & -\frac{1}{\sigma}Z_3 & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & * & -\frac{1}{\bar{\sigma}}Z_1 & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & -\frac{1}{\bar{\sigma}}Z_2 & 0 & 0\\
* & * & * & * & * & * & * & * & * & -\frac{1}{\bar{\sigma}}Z_3 & 0\\
* & * & * & * & * & * & * & * & * & * & -\frac{1}{\varepsilon*\alpha^2}I
\end{bmatrix}<0 \quad (16)$$

where

$$\tilde{\Gamma}_1=\tilde{\Gamma}_{11}+\Gamma_{12}+{\Gamma_{12}}^T$$

$$\tilde{\Gamma}_{11}=\begin{bmatrix}
PA+A^TP+Q_1+Q_2+Q_3 & PBK & 0 & 0 & 0 & P\\
* & 0 & 0 & 0 & 0 & 0\\
* & * & -Q_1 & 0 & 0 & 0\\
* & * & * & -Q_2 & 0 & 0\\
* & * & * & * & -Q_3(1-\alpha) & 0\\
* & * & * & * & * & -\varepsilon I
\end{bmatrix}$$

$$\tilde{A}_c=\begin{bmatrix} A & BK & 0 & 0 & 0 & I\end{bmatrix}$$

2). Note that the value of the state $x(t)$ before and after the instants $s_k+\delta_k$ remains unchanged. For $d(t)$, when $t=s_k+\delta_k$, $d(t)=\delta_k$; $t=(s_k+\delta_k)^-$, $d(t)=s_k+\delta_k-s_{k-1}$, so, the value of the fourth term $\int_{t-\alpha d(t)}^{t}x^T(s)Q_3x(s)ds$ in Lyapunov functional (15) does not increase at the instants $s_k+\delta_k$. It shows that Lyapunov functional (15) does not increase at the instants $s_k+\delta_k$. Thus, for $t\in[\ s_k+\delta_k,\ \ s_{k+1}+\delta_{k+1}\ )$, we have $V(t)-V(s_k+\delta_k)\le 0$. Since $\lim_{k\to\infty}t_k=\infty$, we have $\bigcup_{k=0}^{\infty}[\ s_k+\delta_k\ \ s_{k+1}+\delta_{k+1}\ )=[\ t_0,\ \ \infty\ )$. It follows that $V(t)-V(t_0)\le 0$. It shows that $V(t)$ is decreased. This completes the proof.

Theorem 3.2. *For given scalars δ_m, δ_M, T_m, T_M ($0\le\delta_m\le\delta_M\le T_m\le T_M$), $\alpha>0$, $0\le\gamma<1$, $\varepsilon>0$, $\delta_M/T_m\le\rho\le 1$, the closed-loop NCS (10) is asymptotically stable if there exist matrices $X=X^T>0$, $Q_i={Q_i}^T>0$ ($i=1,2,3$), $\tilde{Z}_j=\tilde{Z}_j^T>0$, $Z_{jL}={Z_{jL}}^T>0$ ($j=1,2,3$), $R_n={R_n}^T>0$, $R_{nL}={R_{nL}}^T>0$ ($n=1,2,3$), and $\tilde{N}$, $\tilde{T}$, $\tilde{M}$, $\tilde{R}$, $\tilde{E}$, $\tilde{S}$ of appropriate deimensions, such that*

$$\begin{bmatrix}\Xi & \tilde{H}\\ * & -\frac{1}{\varepsilon*\alpha^2}I\end{bmatrix}<0 \quad (17)$$

$$\begin{bmatrix} -R_{1L} & X_L \\ * & -Z_{1L} \end{bmatrix} \leq 0, \begin{bmatrix} -R_{2L} & X_L \\ * & -Z_{2L} \end{bmatrix} \leq 0, \begin{bmatrix} -R_{3L} & X_L \\ * & -Z_{3L} \end{bmatrix} \leq 0 \tag{18}$$

$$R_{1L}R_1 = I,\ R_{2L}R_2 = I,\ R_{3L}R_3 = I,\ XX_L = I,\ Z_{1L}\tilde{Z}_1 = I,\ Z_{2L}\tilde{Z}_2 = I,\ Z_{3L}\tilde{Z}_3 = I \tag{19}$$

where

$$\Xi = \begin{bmatrix} \Xi_1 & \tilde{N} & \tilde{T} & \tilde{M} & \tilde{R} & \tilde{E} & \tilde{S} & A_L{}^T & A_L{}^T & A_L{}^T \\ * & -\frac{1}{\gamma\bar{\sigma}}\tilde{Z}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\frac{1}{\bar{\sigma}-\gamma\underline{\sigma}}\tilde{Z}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\frac{1}{\sigma}\tilde{Z}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\frac{1}{\sigma}\tilde{Z}_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\frac{1}{\sigma}\tilde{Z}_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{\sigma}\tilde{Z}_3 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\frac{1}{\bar{\sigma}}R_1 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\frac{1}{\bar{\sigma}}R_2 & 0 \\ * & * & * & * & * & * & * & * & * & -\frac{1}{\bar{\sigma}}R_3 \end{bmatrix} \tag{20}$$

$$\Xi_1 = \Xi_{11} + \Xi_{12} + \Xi_{12}{}^T$$

$$\Xi_{11} = \begin{bmatrix} AX + XA^T + \tilde{Q}_1 + \tilde{Q}_2 + \tilde{Q}_3 & BY & 0 & 0 & 0 & I \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & -\tilde{Q}_1 & 0 & 0 & 0 \\ * & * & * & -\tilde{Q}_2 & 0 & 0 \\ * & * & * & * & -\tilde{Q}_3(1-\alpha) & 0 \\ * & * & * & * & * & -\varepsilon I \end{bmatrix}$$

$$\Xi_{12} = \begin{bmatrix} \tilde{N} + \tilde{S} & \tilde{M} + \tilde{R} - \tilde{E} - \tilde{T} & \tilde{E} & -\tilde{M} - \tilde{R} - \tilde{S} & \tilde{T} - \tilde{N} \end{bmatrix} \tag{21}$$

$$A_L = \begin{bmatrix} AX^T & BY & 0 & 0 & 0 & I \end{bmatrix}^T,\ \tilde{H} = \begin{bmatrix} XH^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sigma = T_M + \frac{\delta_M + \rho T_m}{2} - \delta_m,\ \ \bar{\sigma} = T_M + \frac{\delta_M + \rho T_m}{2},\ \ \underline{\sigma} = \delta_m \tag{22}$$

$\underline{\sigma}$, σ, and $\bar{\sigma}$ are given in (22). In this case, the state-feedback gain is given by $K = YX^{-1}$.

Now using the modified cone complementary linearisation algorithm [9], the conditions in Theorem 3.2 are solvable. Although it is still not possible to always find the global optimal solution, the proposed nonlinear minimization problem is easier to solve than the original nonconvex feasibility problem.

4. **Example.** In the following, an example will be given to illustrate the effectiveness and applicability of the proposed approaches. Consider the following NCS with nonlinearity:

$$\dot{x}(t) = \begin{bmatrix} 1 & 1 \\ 0 & 0.99 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u(t) + h(t, x(t)) \tag{23}$$

with $H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Case 1. We first do not consider the effect of variable sampling; for this case, it means that $0 < d(t) < \bar{\sigma}$. By using the method provided in Theorem 3.2, we can find the upper bound of $d(t)$ is 0.30, while [10] is 0.2838, and [11] is 0.2509. It shows that we can provide much larger upper bound of delay. In the meanwhile, we can obtain nonlinear bound $\alpha_{\max} = 0.1703$. Compared with the result $\alpha_{\max} = 0.0013$ in [11] and $\alpha_{\max} = 0.1636$ in [10], our result in this paper allows a larger nonlinear bound. From this example, it can be seen that the method proposed in this paper is more effective than that in [10] and [11].

Case 2. We consider the settings with nonuniform sampling. Choosing the bounds of

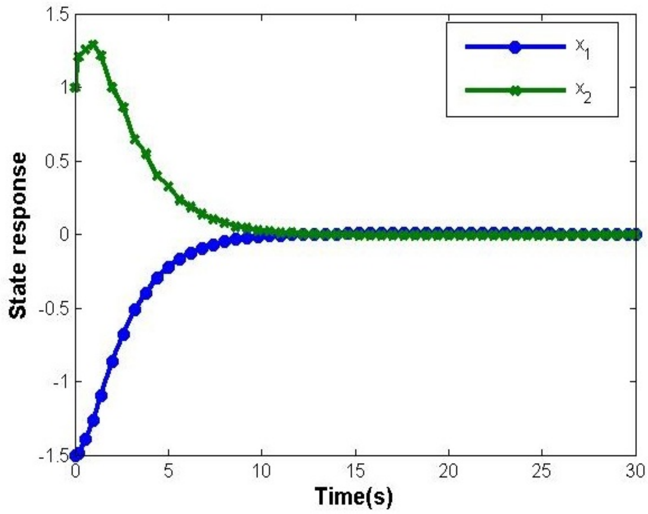

Figure 1. State response of the NCS with two transmission deadbands

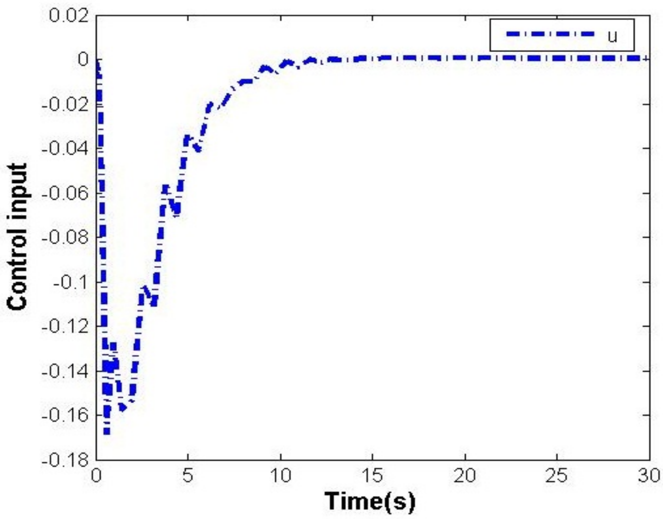

Figure 2. Control input of the NCS with two transmission deadbands

the network-induced delays are $\delta_m = 0.02$s and $\delta_M = 0.05$s, the bounds of sampling intervals are $T_m = 0.10$s and $T_M = 0.3$s. By Theorem 3.2, one can obtain the following solution: $X = 10^4 * \begin{bmatrix} 1.0702 & -0.9496 \\ -0.9496 & 1.5962 \end{bmatrix}$, $Y = 10^3 * \begin{bmatrix} 0.6932 & -2.6004 \end{bmatrix}$. Hence, the state feedback controller gain $K = \begin{bmatrix} -0.1690 & -0.2634 \end{bmatrix}$. Figure 1 and Figure 2 show the state response and control input of the closed-loop systems, respectively, where the initial state of the NCS is $x_0 = [1 \ \ -1.5]^T$. It can be seen that the closed-loop NCS is asymptotically stable with above obtained control gain K in Theorem 3.2. These have shown the effectiveness of controller design method proposed in this paper.

5. **Conclusions.** This paper studies the stability and stabilization problem for networked control systems subject to time-varying delays, time-varying sampling intervals, and nonlinear disturbances. By using the input delay approach, the network delays, sampling intervals and nonlinear disturbances are combined as a unified framework of the NCS model. By choosing a Lyapunov functional with discontinuity, a less conservative stability analysis condition of such NCS is derived. Moreover, by using the CCL algorithm, we

give robust controller design conditions which are associated with the bounds of network-induced delays, the bounds of sampling intervals and the bound of nonlinearity. Finally, an example is used to show the advantages of the proposed methods. Our future work will mainly focus on co-design strategy for nonlinear networked control systems based on variable sampling method. We will explore the relations of the sampling intervals and the systems performance.

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