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DECENTRALIZED CONTROL OF SWITCHED T-S FUZZY INTERCONNECTED SYSTEMS WITH ACTUATOR SATURATION

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Abstract. *This paper deals with the decentralized control problem for a class of switched Takagi-Sugeno (T-S) fuzzy interconnected system with actuator saturation. By using parallel distributed compensation (PDC) approach, a state feedback decentralized fuzzy controller is developed. The decentralized switching law and the stabilization sufficient conditions are proposed and proved based on the multiple Lyapunov function theory, which are formulated as linear matrix inequalities (LMIs). It is proved that proposed control scheme can guarantee that whole closed-loop system is asymptotically stable. A numerical example is given to illustrate the effectiveness of the proposed control method.*
Keywords: Switched systems, Fuzzy T-S interconnected system, Actuator saturation, Decentralized control

1. **Introduction.** Takagi-Sugeno (T-S) fuzzy model is a well known universal approximator, most complex nonlinear systems can be satisfactorily represented as T-S systems, and fuzzy control of nonlinear interconnected systems based on T-S fuzzy model has attracted great attention in recent years [1]. Especially, the decentralized control as an effective control approach has been extensively studied for the nonlinear interconnected systems, and some useful results have been obtained [2-4]. The works in [3] proposed a state feedback decentralized control approach for the T-S interconnected systems; [4] investigated a decentralized H_∞ control design for nonlinear interconnected systems via T-S fuzzy models. However, the above mentioned results are only suitable for the non-switched nonlinear interconnected systems, instead of the switched nonlinear interconnected systems.

Switched systems belong to a special class of hybrid systems since they can provide a valid modeling and control approach for many physical systems. In general, a switched system composes a family of continuous-time or discrete-time subsystems and a logical rule that orchestrates the switching between them. Recently the control design and stability analysis for switched fuzzy T-S systems has attracted more and more attention, see for example [5-8] and references therein. The works in [5,6] developed the dynamic output feedback H_∞ control problems for a class of continuous-time and discrete-time T-S fuzzy systems by using switching fuzzy controller. [7] proposed a switched state feedback control design method for a class of switched fuzzy systems; the authors in [8] investigated the exponential stability and asynchronous stabilization problems for a family of switched nonlinear systems based on a piecewise Lyapunov-like functions and minimum dwell time method, and relaxed stability conditions are developed in [9]. In addition, the aforementioned results do not consider the effect of the actuator saturation, actuator saturation is unavoidable in practical systems, and it can reduce the performance of a control system and some times even leads to the unsteadiness of the control systems [10-12]. To the best of our knowledge, there are no results on the switched T-S fuzzy interconnected systems with actuator saturation, which motivates us for this study.

In this paper, we focus on the decentralized control problem for switched T-S fuzzy interconnected system with actuator saturation. Utilizing the PDC approach, a state feedback decentralized controller is developed. The decentralized switching law and sufficient conditions of ensuring the system stability are given by using multiple Lyapunov function method and LMIs. It is shown that the proposed decentralized control approach can guarantee the closed-loop system with actuator saturation is asymptotically stable.

2. **System Description.** Consider a fuzzy interconnected system composed of N interconnected subsystems M_l, $l = 1, 2, \ldots, N$. The lth switched subsystem M_l is described as follows:

Rule: IF $z_{l1}(t)$ is $M^l_{\sigma_l i1}$, $z_{l2}(t)$ is $M^l_{\sigma_l i2}$, …, $z_{lp}(t)$ is $M^l_{\sigma_l ip}$, then

$$\dot{x}_l(t) = A^l_{\sigma_l i} x_l(t) + B^l_{\sigma_l i}\mathrm{sat}\left(u_{\sigma_l i}(t)\right) + \sum_{p=1,\, p\neq l}^{N} R^l_{p\sigma_l i} x_p(t) \tag{1}$$

where $z_l(t) = [z_{l1}(t), z_{l2}(t), \ldots, z_{lp}(t)]$ are the premise variables, and $M^l_{\sigma_l i1}, \cdots, M^l_{\sigma_l ip}$ are the fuzzy sets. $\sigma_l \in M_l = \{1, 2, \ldots, m_l\}$ is a piecewise constant function, which is called switching signal, $A^l_{\sigma_l i}(t)$ and $B^l_{\sigma_l i}(t)$ are known constant matrices with appropriate dimensions, $R^l_{p\sigma_l i} x_p(t)$ is the nonlinear interconnection between the lth and pth subsystems. $x_l(t) \in R^n$ is the state variable, $\mathrm{sat}(u_{\sigma_l i}(t)) \in R^m$ is the control input with actuator saturation, which is defined as [11]

$$\mathrm{sat}(u_{\sigma_l i}) = \left[\mathrm{sat}\left(u^1_{\sigma_l i}\right), \mathrm{sat}\left(u^2_{\sigma_l i}\right), \ldots, \mathrm{sat}\left(u^m_{\sigma_l i}\right)\right]^T$$

where

$$\mathrm{sat}\left(u^j_{\sigma_l i}\right) = \mathrm{sgn}\left(u^j_{\sigma_l i}\right)\min\left\{1, \left|u^j_{\sigma_l i}\right|\right\}, \quad j \in \{1, 2, \ldots, m\}$$

The overall switched fuzzy systems are inferred as follows

$$\dot{x}_l(t) = \sum_{i=1}^{r_{\sigma_l}} \mu^l_{\sigma_l i}(z(t)) \left\{ A^l_{\sigma_l i} x_l(t) + B^l_{\sigma_l i}\mathrm{sat}(u_{\sigma_l i}(t)) + \sum_{p=1,\, p\neq l}^{N} R^l_{p\sigma_l i} x_p(t) \right\} \tag{2}$$

where $\mu^l_{\sigma_l i}(z(t)) = h^l_{\sigma_l i}(z(t)) \Big/ \sum_{i=1}^{r_{\sigma_l}} h^l_{\sigma_l i}(z(t))$, $h^l_{\sigma_l i}(z(t)) = \prod_{j=1}^{p} M^l_{\sigma_l ij}(z_j(t))$, r_{σ_l} is the number of fuzzy rules.

For the matrices $H^p_l \in R^{n\times n}$, denote the qth row of H^p_l as H^{pq}_l and define

$$\ell\left(H^p_l\right) := \{x_l(t) \in R^n : |H^{pq}_l| \leq 1,\ q = 1, 2, \ldots, m\} \tag{3}$$

Let 1 or 0 be the diagonal elements of ν, which are $m \times m$ diagonal matrices. Suppose that each element of ν is labeled as E_l, $l = 1, 2, \ldots, 2^m$. Denote $E^-_l = I - E_l$. Note that E^-_l is also an element of ν if $E_l \in \nu$.

The following lemma, which captures certain properties of dynamical system with actuator saturation, will be used in this paper.

Lemma 2.1. [11] *Let $F \in R^{m\times n}$ and $H \in R^{m\times n}$ be given. If $x_l(t) \in \ell(F)$, then $\mathrm{sat}(Fx_l(t))$ can be rewritten as:*

$$\mathrm{sat}(Fx_l(t)) = \sum_{k=1}^{2^m} \eta_k(t)\left(E^k_l F + E^{k-}_l H\right) x_l(t) \tag{4}$$

where $\eta_k(t)$ for $k = 1, 2, \ldots, 2^m$ are some scalars, $0 \leq \eta_k(t) \leq 1$ and $\sum_{k=1}^{2^m} \eta_k(t) = 1$.

The purpose of this study is to determine a state feedback decentralized controller with the decentralized switching law $\sigma(x(t)) = [\sigma_1(x_1(t))\ \sigma_2(x_2(t))\ \cdots\ \sigma_N(x_N(t))]^T$ such that the closed-loop fuzzy system is asymptotically stable.

3. **Main Results.** Based on PDC principle, we design the following fuzzy controllers as

$$u_{\sigma_l i}(t) = -\sum_{i=1}^{r_{\sigma_l}} \mu_{\sigma_l i}^l(z(t))\text{sat}\left(K_{\sigma_l i}^l x_l(t)\right) \tag{5}$$

So the closed-loop fuzzy switched system is represented as follows

$$\dot{x}_l(t) = \sum_{i=1}^{r_{\sigma_l}}\sum_{j=1}^{r_{\sigma_l}} \mu_{\sigma_l i}^l(z(t))\mu_{\sigma_l j}^l(z(t))\left\{A_{\sigma_l i}^l x_l(t) - B_{\sigma_l i}^l \text{sat}\left(K_{\sigma_l j}^l x_l(t)\right) + \sum_{p=1,\, p\neq l}^{N} R_{p\sigma_l i}^l x_p(t)\right\} \tag{6}$$

By using Lemma 2.1, system (6) can be represented as

$$\begin{aligned}\dot{x}_l(t) = &\sum_{k=1}^{2^m}\eta_k(t)\sum_{i=1}^{r_{\sigma_l}}\sum_{j=1}^{r_{\sigma_l}} \mu_{\sigma_l i}^l(z(t))\mu_{\sigma_l j}^l(z(t))\Bigg\{A_{\sigma_l i}^l x_l(t)\\ &- B_{\sigma_l i}^l\left(E_l^k K_{\sigma_l j}^l + E_l^{k-} H_{\sigma_l j}^l\right)x_l(t) + \sum_{p=1,\, p\neq l}^{N} R_{p\sigma_l i}^l x_p(t)\Bigg\}\end{aligned} \tag{7}$$

A set of sufficient conditions on the stability for system (7) is provided in the following theorem.

Theorem 3.1. *For the fuzzy switched system (7), if there exist non-positive (non-negative) $\beta_{\sigma_l\lambda_l} \in R$ ($l = 1, 2, \ldots, N$, $\sigma_l, \lambda_l = 1, 2, \ldots, M_l$, $\sigma_l \neq \lambda_l$), positive definite matrices $P_{\sigma_l}^l$ with appropriate dimensions and $\delta_{\sigma_l i} > 0$, $\varepsilon_{\sigma_l i} > 0$, $\xi > 0$ ($i = 1, 2, \ldots, r_{\sigma_l}$), satisfying the following conditions*

$$\Gamma_{\sigma_l ij} + \sum_{\lambda_l=1,\, \lambda_l\neq\sigma_l}^{M_l} \beta_{\sigma_l\lambda_l}\left(P_{\lambda_l}^l - P_{\sigma_l}^l\right) < 0 \tag{8}$$

with

$$\begin{aligned}\Gamma_{\sigma_l ij} = &A_{\sigma_l i}^{l\ T} P_{\sigma_l}^l + P_{\sigma_l}^l A_{\sigma_l i}^l - \delta_{\sigma_l j}P_{\sigma_l}^l B_{\sigma_l j}^l E_l^k E_l^{kT} B_{\sigma_l i}^{l\ T} P_{\sigma_l}^l - \delta_{\sigma_l j}P_{\sigma_l}^l B_{\sigma_l i}^l E_l^k E_l^{kT} B_{\sigma_l j}^{l\ T} P_{\sigma_l}^l\\ &- \varepsilon_{\sigma_l j}P_{\sigma_l}^l B_{\sigma_l j}^l E_l^{k-} E_l^{k-T} B_{\sigma_l i}^{l\ T} P_{\sigma_l}^l - \varepsilon_{\sigma_l j}P_{\sigma_l}^l B_{\sigma_l i}^l E_l^{k-} E_l^{k-T} B_{\sigma_l j}^{l\ T} P_{\sigma_l}^l\\ &+ \frac{1}{\xi}(N-1)P_{\sigma_l}^l P_{\sigma_l}^l + \sum_{p=1,\, p\neq l}^{N} \xi\left(R_{l\sigma_l i}^p\right)^T R_{l\sigma_l i}^p\end{aligned}$$

then the controller (4), with the switching law $\sigma = \sigma(x(t))$ can guarantee the closed-loop switched system (7) is asymptotical stability. Moreover, the control gain matrices are $K_{\sigma_l i}^l = \delta_{\sigma_l i}\left(B_{\sigma_l i}^l E_l^k\right)^T P_{\sigma_l}^l$ and $H_{\sigma_l i}^l = \varepsilon_{\sigma_l i}\left(B_{\sigma_l i}^l E_l^{k-}\right)^T P_{\sigma_l}^l$.

Proof: Without loss of generality, we assume $\beta_{\sigma_l\lambda_l} \geq 0$. Obviously, for every $x_l(t) \in R^n\backslash\{0\}$, there exists a $\sigma_l \in M_l$ such that $x_l^T(t)(P_{\lambda_l} - P_{\sigma_l})x_l(t) \geq 0$, $\forall\lambda_l \in \underline{M_l}$, then from the matrix Inequality (8), we have

$$x_l^T(t)\left[\Gamma_{\sigma_l ij} + \sum_{\lambda_l=1,\, \lambda_l\neq\sigma_l}^{M_l} \beta_{\sigma_l\lambda_l}\left(P_{\lambda_l}^l - P_{\sigma_l}^l\right)\right]x_l(t) < 0 \tag{9}$$

Let $\Omega_{\sigma_l}^l = \left\{x_l(t) \in R_n | x_l^T(t)(P_{\lambda_l} - P_{\sigma_l})x_l(t) \geq 0,\ \forall x_l(t) \neq 0\right\}$, then for any $l \in \underline{N}$, we have $\bigcup_{\sigma_l}\Omega_{\sigma_l}^l = R^n\backslash\{0\}$. Construct the sets $\tilde{\Omega}_1^l = \Omega_1^l, \tilde{\Omega}_2^l = \Omega_2^l, \cdots, \tilde{\Omega}_{M_l}^l = \Omega_{M_l}^l - \bigcup_{i=1}^{M_l-1}\tilde{\Omega}_i^l$, it is easy to see that $\bigcup_{i=1}^{M_l}\tilde{\Omega}_i^l = R^n - \{0\}$, and $\tilde{\Omega}_i^l \cap \tilde{\Omega}_j^l = \phi$, $i \neq j$. Therefore, we design the switching law as

$$\sigma_l(x_l(t)) = \gamma \text{ when } x_l(t) \in \tilde{\Omega}_i^l \tag{10}$$

The Lyapunov function is chosen as follows:

$$V(t) = x_l^T(t)P_{\sigma_l}^l x_l(t) \tag{11}$$

The derivative of $V(t)$ can be written as

$$\begin{aligned}
\dot{V}(t) &= \dot{x}_l^T(t)P_{\sigma_l}^l x_l(t) + x_l^T(t)P_{\sigma_l}^l \dot{x}_l(t) \\
&= \sum_{k=1}^{2^m} \eta_k(t) \sum_{i=1}^{r_{\sigma_l}} \sum_{j=1}^{r_{\sigma_l}} \mu_{\sigma_l i}^l(z(t))\mu_{\sigma_l j}^l(z(t)) \Bigg\{ x_l^T(t) \left[{A_{\sigma_l i}^l}^T P_{\sigma_l}^l + P_{\sigma_l}^l A_{\sigma_l i}^l - \left(E_l^k K_{\sigma_l j}^l\right)^T \right. \\
&\quad \left. \times {B_{\sigma_l i}^l}^T P_{\sigma_l}^l - P_{\sigma_l}^l B_{\sigma_l i}^l E_l^k K_{\sigma_l j}^l - \left(E_l^{k-} H_{\sigma_l j}^l\right)^T {B_{\sigma_l i}^l}^T P_{\sigma_l}^l - P_{\sigma_l}^l B_{\sigma_l i}^l E_l^{k-} H_{\sigma_l j}^l \right] x_l(t) \\
&\quad + \sum_{p=1,\, p\neq l}^{N} x_p^T(t) \left(R_{p\sigma_l i}^l\right)^T P_{\sigma_l}^l x_l(t) + x_l^T(t) P_{\sigma_l}^l \sum_{p=1,\, p\neq l}^{N} R_{p\sigma_l i}^l x_p(t) \Bigg\} \\
&= \sum_{k=1}^{2^m} \eta_k(t) \sum_{i=1}^{r_{\sigma_l}} \sum_{j=1}^{r_{\sigma_l}} \mu_{\sigma_l i}^l(z(t))\mu_{\sigma_l j}^l(z(t)) \Bigg\{ x_l^T(t) \Big[{A_{\sigma_l i}^l}^T P_{\sigma_l}^l + P_{\sigma_l}^l A_{\sigma_l i}^l \\
&\quad - \delta_{\sigma_l j} P_{\sigma_l}^l B_{\sigma_l j}^l E_l^k {E_l^k}^T \times {B_{\sigma_l i}^l}^T P_{\sigma_l}^l - \delta_{\sigma_l j} P_{\sigma_l}^l B_{\sigma_l i}^l E_l^k {E_l^k}^T {B_{\sigma_l j}^l}^T P_{\sigma_l}^l \\
&\quad - \varepsilon_{\sigma_l j} P_{\sigma_l}^l B_{\sigma_l j}^l E_l^{k-} {E_l^{k-}}^T {B_{\sigma_l i}^l}^T P_{\sigma_l}^l - \varepsilon_{\sigma_l j} P_{\sigma_l}^l B_{\sigma_l i}^l E_l^{k-} \times {E_l^{k-}}^T {B_{\sigma_l j}^l}^T P_{\sigma_l}^l \Big] x_l(t) \\
&\quad + \sum_{p=1,\, p\neq l}^{N} x_p^T(t) \left(R_{p\sigma_l i}^l\right)^T P_{\sigma_l}^l x_l(t) + x_l^T(t) P_{\sigma_l}^l \sum_{p=1,\, p\neq l}^{N} R_{p\sigma_l i}^l x_p(t) \Bigg\}
\end{aligned} \tag{12}$$

By the inequality $X^TY + Y^TX \leq \xi X^TX + \xi^{-1}Y^TY$ $(\xi > 0)$, (12) can be rewritten as

$$\begin{aligned}
\dot{V}(t) &\leq \sum_{k=1}^{2^m} \eta_k(t) \sum_{i=1}^{r_{\sigma_l}} \sum_{j=1}^{r_{\sigma_l}} \mu_{\sigma_l i}^l(z(t))\mu_{\sigma_l j}^l(z(t)) \Bigg\{ x_l^T(t) \Big[{A_{\sigma_l i}^l}^T P_{\sigma_l}^l + P_{\sigma_l}^l A_{\sigma_l i}^l \\
&\quad - \delta_{\sigma_l j} P_{\sigma_l}^l B_{\sigma_l j}^l E_l^k {E_l^k}^T \times {B_{\sigma_l i}^l}^T P_{\sigma_l}^l - \delta_{\sigma_l j} P_{\sigma_l}^l B_{\sigma_l i}^l E_l^k {E_l^k}^T {B_{\sigma_l j}^l}^T P_{\sigma_l}^l \\
&\quad - \varepsilon_{\sigma_l j} P_{\sigma_l}^l B_{\sigma_l j}^l E_l^{k-} {E_l^{k-}}^T {B_{\sigma_l i}^l}^T P_{\sigma_l}^l - \varepsilon_{\sigma_l j} P_{\sigma_l}^l B_{\sigma_l i}^l \times E_l^{k-} {E_l^{k-}}^T {B_{\sigma_l j}^l}^T P_{\sigma_l}^l \Big] x_l(t) \\
&\quad + \sum_{p=1,\, p\neq l}^{N} \left[\xi x_p^T(t) \left(R_{p\sigma_l i}^l\right)^T R_{p\sigma_l i}^l x_p(t) + \frac{1}{\xi} x_l^T(t) P_{\sigma_l}^l P_{\sigma_l}^l x_l(t) \right] \Bigg\} \\
&= \sum_{k=1}^{2^m} \eta_k(t) \sum_{i=1}^{r_{\sigma_l}} \sum_{j=1}^{r_{\sigma_l}} \mu_{\sigma_l i}^l(z(t))\mu_{\sigma_l j}^l(z(t)) x_l^T(t) \Bigg\{ {A_{\sigma_l i}^l}^T P_{\sigma_l}^l + P_{\sigma_l}^l A_{\sigma_l i}^l \\
&\quad - \delta_{\sigma_l j} P_{\sigma_l}^l B_{\sigma_l j}^l E_l^k {E_l^k}^T \times {B_{\sigma_l i}^l}^T P_{\sigma_l}^l - \delta_{\sigma_l j} P_{\sigma_l}^l B_{\sigma_l i}^l E_l^k {E_l^k}^T {B_{\sigma_l j}^l}^T P_{\sigma_l}^l \\
&\quad - \varepsilon_{\sigma_l j} P_{\sigma_l}^l B_{\sigma_l j}^l E_l^{k-} {E_l^{k-}}^T {B_{\sigma_l i}^l}^T P_{\sigma_l}^l - \varepsilon_{\sigma_l j} P_{\sigma_l}^l B_{\sigma_l i}^l \times E_l^{k-} {E_l^{k-}}^T {B_{\sigma_l j}^l}^T P_{\sigma_l}^l \\
&\quad + \frac{1}{\xi}(N-1) P_{\sigma_l}^l P_{\sigma_l}^l + \sum_{p=1,\, p\neq l}^{N} \xi \left(R_{l\sigma_l i}^p\right)^T R_{l\sigma_l i}^p \Bigg\} x_l(t)
\end{aligned} \tag{13}$$

By (8) and the decentralized switching law, we have $\dot{V}(t) < 0$. Therefore, the switched closed-loop fuzzy system with actuator saturation is asymptotically stable.

Note that matrix inequalities $\Gamma_l = \Gamma_{\sigma_l ij} + \sum_{\lambda_l=1,\, \lambda_l\neq\sigma_l}^{M_l} \beta_{\sigma_l\lambda_l}\left(P_{\lambda_l}^l - P_{\sigma_l}^l\right) < 0$ are not linear matrix inequalities (LMIs). Therefore, we should transform $\Gamma_l < 0$ into LMIs to obtain positive definite matrices $P_{\sigma_l}^l$, control gain matrices $K_{\sigma_l i}^l$ and $H_{\sigma_l i}^l$.

Pre- and post-multiplying both side of (8) by matrix $Q^l_{\sigma_l} = {P^l_{\sigma_l}}^{-1}$ and using Schur's complement, then we have

$$\begin{bmatrix} \Pi_{\sigma_l ij} & Q^l_{\sigma_l} & \cdots & Q^l_{\sigma_l} & Q^l_{\sigma_l}\left(R^l_{1\sigma_l i}\right)^T & \cdots & Q^l_{\sigma_l}\left(R^l_{N\sigma_l i}\right)^T \\ * & -\beta^{-1}_{\sigma_l 1}Q^l_1 & \cdots & 0 & 0 & \cdots & 0 \\ * & * & \ddots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & -\beta^{-1}_{\sigma_l M_l}Q^l_{M_l} & 0 & \cdots & 0 \\ * & * & * & * & -\frac{1}{\xi}I & \cdots & 0 \\ * & * & * & * & * & \ddots & \vdots \\ * & * & * & * & * & * & -\frac{1}{\xi}I \end{bmatrix} < 0 \tag{14}$$

with

$$\begin{aligned} \Pi_{\sigma_l ij} &= Q^l_{\sigma_l}\left(A^l_{\sigma_l i}\right)^T + A^l_{\sigma_l i}Q^l_{\sigma_l} - \delta_{\sigma_l j}B^l_{\sigma_l j}E^k_l\left(B^l_{\sigma_l i}E^k_l\right)^T - \delta_{\sigma_l j}B^l_{\sigma_l i}E^k_l\left(B^l_{\sigma_l j}E^k_l\right)^T \\ &\quad -\varepsilon_{\sigma_l j}B^l_{\sigma_l j}E^{k-}_l\left(B^l_{\sigma_l i}E^{k-}_l\right)^T - \varepsilon_{\sigma_l j}B^l_{\sigma_l i}E^{k-}_l\left(B^l_{\sigma_l j}E^{k-}_l\right)^T + \frac{1}{\xi}(N-1)I \\ &\quad - \sum_{\lambda_l=1,\, \lambda_l\neq\sigma_l}^{M_l}\beta_{\sigma_l\lambda_l}Q^l_{\sigma_l} \\ Q^l_{\lambda_l} &= {P^l_{\lambda_l}}^{-1},\quad \lambda_l = 1, 2, \ldots, M_l. \end{aligned}$$

The matrices $Q^l_{\sigma_l}$ (thus $P^l_{\sigma_l} = {Q^l_{\sigma_l}}^{-1}$) can be obtained by solving the LMIs in (14). By substituting $P^l_{\sigma_l}$ into $K^l_{\sigma_l i} = \delta_{\sigma_l i}\left(B^l_{\sigma_l i}E^k_l\right)^T P^l_{\sigma_l}$ and $H^l_{\sigma_l i} = \varepsilon_{\sigma_l i}\left(B^l_{\sigma_l i}E^{k-}_l\right)^T P^l_{\sigma_l}$, we can easily solve $K^l_{\sigma_l i}$ and $H^l_{\sigma_l i}$.

4. **Simulation Study.** Consider the following switched T-S fuzzy interconnected system with actuator saturation:

$$\dot{x}_1(t) = \begin{bmatrix} \dot{x}^1_1(t) \\ \dot{x}^1_2(t) \end{bmatrix} = \sum_{i=1}^{2}\mu^l_{\sigma_l i}(z(t))\left\{A^1_{\sigma_l i}x_1(t) + B^1_{\sigma_l i}\mathrm{sat}(u_{\sigma_l i}(t)) + R^1_{2\sigma_l i}x_2(t)\right\}$$

$$\dot{x}_2(t) = \begin{bmatrix} \dot{x}^2_1(t) \\ \dot{x}^2_2(t) \end{bmatrix} = \sum_{i=1}^{2}\mu^l_{\sigma_l i}(z(t))\left\{A^2_{\sigma_l i}x_2(t) + B^2_{\sigma_l i}\mathrm{sat}(u_{\sigma_l i}(t)) + R^2_{1\sigma_l i}x_1(t)\right\}$$

where

$$A^1_{11} = \begin{bmatrix} 6.2 & -2.1 \\ 2.3 & -2.8 \end{bmatrix},\ A^1_{12} = \begin{bmatrix} 6.5 & -2.8 \\ 2.7 & -3 \end{bmatrix},\ A^1_{21} = \begin{bmatrix} -2.9 & 7.8 \\ -5.1 & 0.5 \end{bmatrix},$$
$$A^1_{22} = \begin{bmatrix} 2.2 & -7.6 \\ 3.5 & -4.6 \end{bmatrix},\ A^2_{11} = \begin{bmatrix} 1.1 & 0.8 \\ 0.1 & 0.5 \end{bmatrix},\ A^2_{12} = \begin{bmatrix} 0.6 & 0.5 \\ 1.1 & 0.2 \end{bmatrix},\ A^2_{21} = \begin{bmatrix} 0.1 & 1.6 \\ -1.3 & 0.6 \end{bmatrix},$$
$$A^2_{22} = \begin{bmatrix} 2.2 & -7.6 \\ 3.5 & -4.6 \end{bmatrix},\ B^1_{11} = B^1_{12} = \begin{bmatrix} -1 \\ 0.05 \end{bmatrix},\ B^1_{21} = B^1_{22} = \begin{bmatrix} 0.21 \\ -0.13 \end{bmatrix},$$
$$B^2_{11} = B^2_{12} = \begin{bmatrix} 01 \\ -2.35 \end{bmatrix},\ B^2_{21} = B^2_{22} = \begin{bmatrix} -5.5 \\ -3.3 \end{bmatrix},\ R^1_{211} = \begin{bmatrix} 0 & 0 \\ 0.02 & 0 \end{bmatrix},$$
$$R^1_{212} = \begin{bmatrix} 0 & 0 \\ 0.01 & 0 \end{bmatrix},\ R^1_{221} = \begin{bmatrix} 0 & 0 \\ 0.02 & 0 \end{bmatrix},\ R^1_{222} = \begin{bmatrix} 0 & 0 \\ 0.01 & 0.01 \end{bmatrix},$$
$$R^2_{111} = \begin{bmatrix} 1 & 0 \\ 0.01 & 0 \end{bmatrix},\ R^2_{112} = \begin{bmatrix} 0 & 2 \\ 0.02 & 0 \end{bmatrix},\ R^2_{121} = \begin{bmatrix} 1 & 0 \\ 0.01 & 0 \end{bmatrix},\ R^2_{122} = \begin{bmatrix} 0 & 2 \\ 0.02 & 0 \end{bmatrix},$$
$$E^k_1 = E^k_2 = 1,\ \ E^{k-}_1 = E^{k-}_2 = 0.$$

Then the corresponding fuzzy membership functions are listed as follows:

$$\mu_{11}^{1}\left(x_1^1(t)\right)=\mu_{21}^{1}\left(x_1^1(t)\right)=1-1/\left(1+e^{-15x_1^1(t)}\right),$$
$$\mu_{12}^{1}\left(x_1^1(t)\right)=\mu_{22}^{1}\left(x_1^1(t)\right)=1/\left(1+e^{-15x_1^1(t)}\right),$$
$$\mu_{11}^{2}\left(x_1^2(t)\right)=\mu_{21}^{2}\left(x_1^2(t)\right)=1-1/\left(1+e^{-4x_1^2(t)}\right),$$
$$\mu_{12}^{2}\left(x_1^2(t)\right)=\mu_{22}^{2}\left(x_1^2(t)\right)=1/\left(1+e^{-4x_1^2(t)}\right).$$

The design parameters are chosen as

$$\varepsilon_{11}=\varepsilon_{12}=6,\ \varepsilon_{21}=\varepsilon_{22}=7,\ \xi=3,\ \beta_{12}=\beta_{21}=1,\ \delta_{11}=\delta_{12}=10,\ \delta_{21}=\delta_{22}=15.$$

By solving (14), we can obtain the positive definite matrices $P_{\sigma_l}^l$ and $K_{\sigma_l i}^l$ as follows:

$$P_1^1=\begin{bmatrix}2.1256 & -0.5255\\ -0.5255 & 2.2892\end{bmatrix},\quad P_2^1=\begin{bmatrix}1.7917 & -0.7477\\ -0.7477 & 2.7710\end{bmatrix},$$
$$P_1^2=\begin{bmatrix}0.0653 & -0.0061\\ -0.0061 & 0.0171\end{bmatrix},\quad P_2^2=\begin{bmatrix}0.0194 & -0.0134\\ -0.0134 & 0.0235\end{bmatrix}.$$
$$K_{11}^1=K_{12}^1=\begin{bmatrix}-0.5187 & 6.8996\end{bmatrix},\quad K_{21}^1=K_{22}^1=\begin{bmatrix}7.1018 & -7.7569\end{bmatrix},$$
$$K_{11}^2=K_{12}^2=\begin{bmatrix}7.7956 & -0.4635\end{bmatrix},\quad K_{21}^2=K_{22}^2=\begin{bmatrix}-0.9338 & -0.0543\end{bmatrix}.$$

Let

$$\Omega_1^1=\left\{x_1\in R^2\middle|x_1^T\left(P_2^1-P_1^1\right)x_1\geq 0,\ x\neq 0\right\},$$
$$\Omega_2^1=\left\{x_1\in R^2\middle|x_1^T\left(P_1^1-P_2^1\right)x_1\geq 0,\ x\neq 0\right\},$$
$$\Omega_1^2=\left\{x_2\in R^2\middle|x_2^T\left(P_2^2-P_1^2\right)x_2\geq 0,\ x\neq 0\right\},$$
$$\Omega_2^2=\left\{x_2\in R^2\middle|x_2^T\left(P_1^2-P_2^2\right)x_2\geq 0,\ x\neq 0\right\}.$$

Then we have $\Omega_1^1\cup\Omega_2^1=R^2\backslash\{0\}$, $\Omega_1^2\cup\Omega_2^2=R^2\backslash\{0\}$. The switching laws are constructed as

$$\sigma_1(x_1(t))=\begin{cases}1, & x_1(t)\in\Omega_1^1\\ 2, & x_1(t)\in\Omega_2^1\backslash\Omega_1^1\end{cases},\quad \sigma_2(x_2(t))=\begin{cases}1, & x_2(t)\in\Omega_1^2\\ 2, & x_2(t)\in\Omega_2^2\backslash\Omega_1^2\end{cases}$$

The initial condition is chosen as $\begin{bmatrix}1.1 & -4 & 1.5 & -0.2\end{bmatrix}^T$. Then, the simulation results are shown in Figures 1-4, where Figure 1 and Figure 2 show the trajectories of $x_{\sigma_l}^l$ (σ_l,

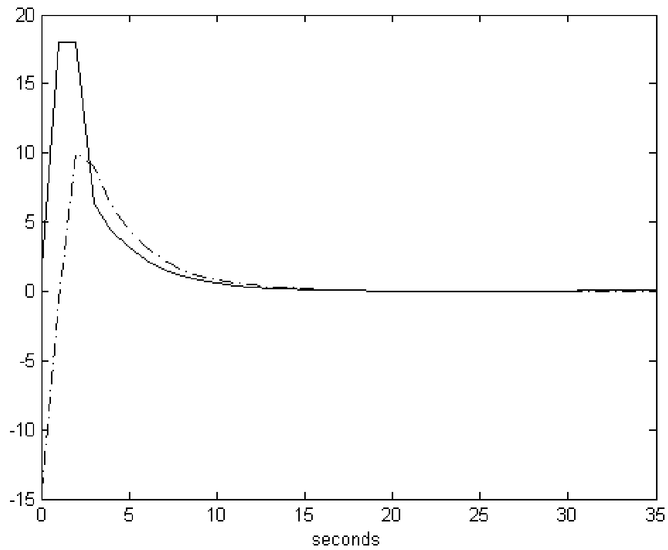

FIGURE 1. x_1^1 (solid line) and x_2^1 (dotted line)

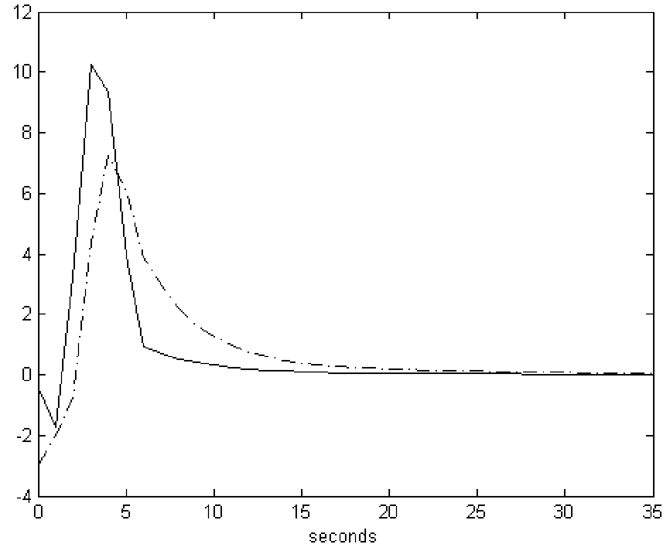

Figure 2. x_1^2 (solid line) and x_2^2 (dotted line)

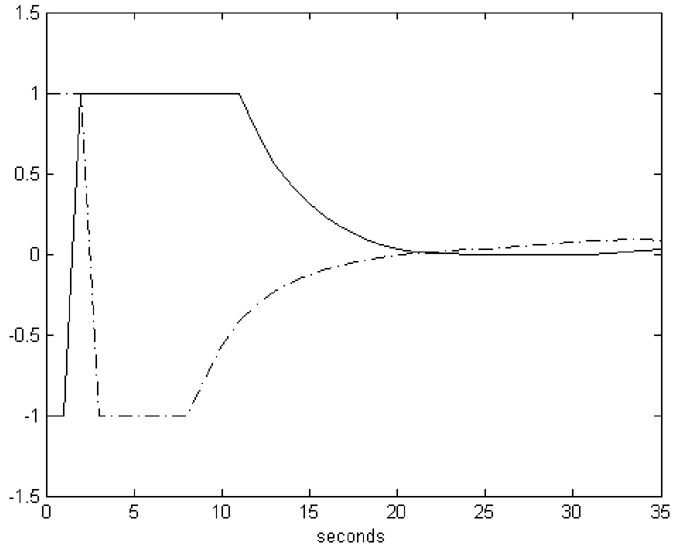

Figure 3. u_{11} (solid line) and u_{12} (dotted line)

$i = 1, 2)$; Figure 3 and Figure 4 show the trajectories of $u_{\sigma_l i}$ ($\sigma_l = 1, 2$, $i = 1, 2$). From the simulation results, it is clear that the fuzzy decentralized controller with the decentralized switching law guarantee the stability of switched T-S fuzzy interconnected system with actuator saturation.

5. **Conclusions.** In this paper, the decentralized control problem has been investigated for a class of switched T-S fuzzy interconnected system with actuator saturation. The fuzzy model has been described by a class of fuzzy IF-THEN rules. By using the PDC design principle, a state feedback decentralized controller has been developed. Based on multiple Lyapunov function method and LMIs, the decentralized switching law and sufficient conditions of ensuring the system stability have been obtained. It has been proved that the proposed fuzzy control method can guarantee the closed-loop system to be asymptotically stable. Future research will extend the results of this paper to switched stochastic systems with actuator saturation.

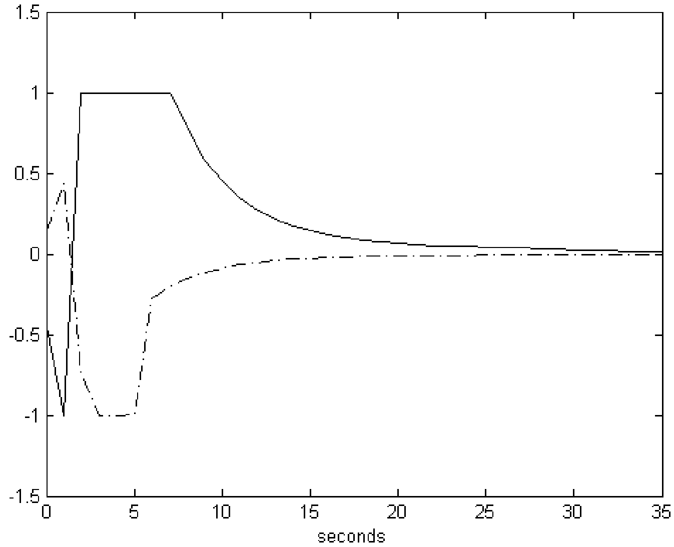

FIGURE 4. u_{21} (solid line) and u_{22} (dotted line)

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