

ADAPTIVE FUZZY BACKSTEPPING CONTROL DESIGN FOR A CLASS OF SWITCHED NONLINEAR SYSTEMS WITH UNKNOWN DEAD-ZONE

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ABSTRACT. *This paper presents an adaptive fuzzy state feedback control design approach for a class of uncertain nonlinear switched systems with unknown dead-zone. The unknown nonlinear functions are approximated by fuzzy logic systems, and the problem of unknown dead-zone is solved by introducing characteristic function, i.e., dead-zone is represented as a simple linear system with a static time-varying gain and bounded disturbance. With the help of backstepping control design principle and common Lyapunov function stability theory, an adaptive fuzzy state feedback controller is developed. It is proved that the proposed control approach can guarantee that all the signals of the closed-loop system are semi-globally uniformly ultimately bounded (SGUUB) and the tracking error remains an adjustable neighborhood of the origin. The results of simulation example illustrate the effectiveness of the proposed approach.*

Keywords: Nonlinear switched systems, Fuzzy adaptive control, Dead-zone, Backstepping technique

1. Introduction. Switched systems have significance in the modeling of many engineering applications, and have attracted more and more attention in the control community. Recently, the analysis and synthesis especially the stability analysis has been studied for several classes of switched nonlinear systems and some design methods have been proposed in [1-3]. The author in [1] developed a novel adaptive fuzzy state feedback control approach based on the common Lyapunov function method for a class of pure-feedback switched nonlinear systems under arbitrary switchings. [2] has proposed an adaptive neural control technique for a class of switched system with unmeasured states based on average dwell time and backstepping. An adaptive neural network control method in [3] has been presented for a class of switched nonlinear systems with switching jumps and uncertainties in both system models and switching signals based on dwell-time property. However, the work in [1-3] did not consider the problem of unknown dead-zone.

In addition, dead-zone is one of very important non-smooth nonlinearities arisen in actuator, which can lead to sever deterioration of the systems performance. In resent years, based on the backstepping control design technique, some methods have been developed to solve the problem of unknown dead-zone [4-6]. The author in [4] introduced smooth inverse function to compensate the effect of the unknown dead-zone in controller design. In [5], a new dead-zone actuator model is proposed which contains a linear-like term, a nonlinear term, and a disturbance-like term. In this paper, we are using the similar method as [6] to solve the problem of unknown dead-zone but [6] proposes an adaptive surface control for pure feedback form and [4-6] are used to control non-switched nonlinear systems and not applied to switched nonlinear systems.

Motivated by the aforementioned observations, this paper proposed an adaptive fuzzy control design method for a class of single input and single out (SISO) switched nonlinear

systems with unknown dead-zone. In the control design, fuzzy logic systems are utilized to model the unknown nonlinear functions, and the problem of unknown dead-zone is solved by introducing characteristic function which is represented as a simple linear system with a static time-varying gain and bounded disturbance. Under the framework of backstepping control design procedure and arbitrary switching rulers, an adaptive fuzzy controller and adaptation laws are developed based on common Lyapunov function. It is proved that all the signals in the closed-loop systems remained SGUUB, and the tracking error converged to a small neighborhood of the origin.

2. Problem Formulations and Preliminaries. Consider the following class of SISO nonlinear switched systems with unknown dead-zone:

$$\begin{cases} \dot{x}_1 = x_2 + f_{\sigma(t),1}(x_1) + \Delta_{\sigma(t),1}(\bar{x}_n, t) \\ \dot{x}_i = x_{i+1} + f_{\sigma(t),i}(\bar{x}_i) + \Delta_{\sigma(t),i}(\bar{x}_n, t), & i = 2, \dots, n-1 \\ \dot{x}_n = u + f_{\sigma(t),n}(\bar{x}_n) + \Delta_{\sigma(t),n}(\bar{x}_n, t) \\ y = x_1 \end{cases} \quad (1)$$

where $\bar{x}_i = [x_1, \dots, x_i]^T \in R^i$ ($i = 1, \dots, n$) are the state vector and $y \in R$ is the output. $\sigma(t) : [0, \infty) \rightarrow \Xi \stackrel{\text{def}}{=} \{1, 2, \dots, N\}$ is a piecewise constant function called switching signal (or law), which takes values in the compact set Ξ . If $\sigma(t) = k$, then we say the k -th subsystem is active and the remaining subsystems are inactive. $f_{k,i}(\bar{x}_i)$ ($k \in \Xi, i = 1, \dots, n$) are unknown smooth function. $\Delta_{k,i}(\bar{x}_n, t)$ are the unknown uncertain disturbances. $u \in R$ is the output of the dead-zone, which is described by:

$$u = D(v) = \begin{cases} g_r(v), & \text{if } v \geq b_r \\ 0, & \text{if } -b_l \leq v \leq b_r \\ g_l(v), & \text{if } v \leq -b_l \end{cases} \quad (2)$$

$v \in R$ is the input of the dead-zone, b_r and b_l are unknown parameters of the dead-zone.

The control objective of this paper is to design an adaptive fuzzy controller for switched system with unknown dead-zone (1), under arbitrary switching rulers such that all signals in the closed-loop system remain SGUUB and the tracking error $z_1 = y - y_r$ (y_r is the given reference signal) can converge to a small neighborhood of the origin.

Throughout the paper, the following assumptions and lemma are needed.

Assumption 2.1. *There exist unknown positive constants $p_{k,i}$, such that $\forall (\bar{x}_n, t) \in R^n \times R_+$, $|\Delta_{k,i}(\bar{x}_n, t)| \leq p_{k,i} \rho_{k,i}(\bar{x}_i)$, with $\rho_{k,i}(\bar{x}_i)$ being unknown positive smooth functions, $i = 1, \dots, n$.*

Assumption 2.2. [6]: *The dead-zone can be rewritten as follows:*

$$u = D(v) = k^T(t)\phi(t)v + d(v) \quad (3)$$

where $|d(v)| \leq \tau$, $\tau = (k_{r1} + k_{l1}) \max\{b_r, -b_l\}$ is an unknown positive constant. The assumptions about the dead-zone are the same with Assumptions 1-3 in [6]. From [6], we obtain that $k^T(t)\phi(t) \in [\beta_0, k_{l1} + k_{r1}]$, in which $\beta_0 \leq \min\{k_{l0}, k_{r0}\}$ is a positive constant.

Lemma 2.1. [7]: *For any given real continuous function $F(\bar{x})$, on a compact set $\Omega \in R^N$, there exists a fuzzy logic system $\hat{F}(\bar{x}|\theta) = \theta^T \varphi(\bar{x})$ such that $\forall \varepsilon > 0$*

$$|F(\bar{x}) - \theta^T \varphi(\bar{x})| < \varepsilon \quad (4)$$

where $\theta = (\theta_1, \dots, \theta_M)^T$ is the estimate parameter vector, and $\varphi(\bar{x}) = (\varphi_1(\bar{x}), \dots, \varphi_M(\bar{x}))^T$ is the vector of fuzzy basis functions, M is the number of fuzzy rulers.

Define the optimal parameter vector θ^* as:

$$\theta^* := \arg \min_{\theta \in R^N} \left\{ \sup_{\bar{x} \in \Omega} |F(\bar{x}) - \theta^T \varphi(\bar{x})| \right\} \quad (5)$$

Assumption 2.3. *There exists a parameter vector θ^* such that $|\varepsilon| \leq \varepsilon^*$ with constant $\varepsilon^* > 0$ for all $\bar{x} \in \Omega$.*

3. Adaptive Fuzzy Control Design. The n -step adaptive fuzzy backstepping control design is based on the following change of coordinates:

$$z_1 = x_1 - y_r \tag{6}$$

$$z_i = x_i - \alpha_{i-1}, \quad i = 2, \dots, n \tag{7}$$

where z_i ($i = 1, \dots, n$) is called the virtual error, α_{i-1} is the intermediate function, which will be designed later.

Step 1: The time derivative of z_1 is

$$\dot{z}_1 = x_2 + f_{k,1}(x_1) + \Delta_{k,1}(\bar{x}_n, t) - \dot{y}_r \tag{8}$$

Using Young's inequality and Assumption 2.1, one has

$$|z_1 \Delta_{k,1}(\bar{x}_n, t)| \leq |z_1| p_{k,1} \rho_{k,1}(x_1) \leq \frac{z_1^2 \rho_{k,1}^2(x_1)}{2} + \frac{1}{2} p_1^2 \tag{9}$$

where $p_1 = \max_{k \in \Xi} p_{k,1}$ is an unknown positive constant.

Substituting (9) into (8) results in

$$z_1 \dot{z}_1 \leq z_1 \left[x_2 + f_{k,1}(x_1) - \dot{y}_r + \frac{z_1 \rho_{k,1}^2(x_1)}{2} \right] + \frac{1}{2} p_1^2 \tag{10}$$

Fuzzy logic systems $\theta_{k,1}^{*T} \varphi_{k,1}(x_1, y_r)$ are used to approximate $h_{k,1}(x_1, y_r)$, i.e.,

$$h_{k,1}(x_1, y_r) = f_{k,1}(x_1) + \frac{z_1 \rho_{k,1}^2(x_1)}{2} = \theta_{k,1}^{*T} \varphi_{k,1}(x_1, y_r) + \varepsilon_{k,1}(x_1, y_r) \tag{11}$$

From (7) and substituting (11) into (10) results in

$$z_1 \dot{z}_1 \leq z_1 \left[z_2 + \alpha_1 + \theta_{k,1}^{*T} \varphi_{k,1}(x_1, y_r) + \varepsilon_{k,1}(x_1, y_r) - \dot{y}_r \right] + \frac{1}{2} p_1^2 \tag{12}$$

Applying Young's inequality again and using Assumption 2.3, one has

$$\begin{aligned} & z_1 \left[z_2 + \alpha_1 + \theta_{k,1}^{*T} \varphi_{k,1}(x_1, y_r) + \varepsilon_{k,1}(x_1, y_r) - \dot{y}_r \right] \\ & \leq z_1 \left(\alpha_1 - \dot{y}_r + z_1 + \frac{z_1 \Theta_1^*}{2a_1^2} \right) + \frac{\varepsilon_1^{*2}}{2} + \frac{a_1^2}{2} + \frac{z_2^2}{2} \end{aligned} \tag{13}$$

where $a \neq 0$, $\Theta_1^* = \max_{k \in \Xi} \|\theta_{k,1}^*\|^2$, $\varepsilon_1^* = \max_{k \in \Xi} \varepsilon_{k,1}^*$ is an unknown constant.

Consider the following Lyapunov function candidate:

$$V_1 = \frac{z_1^2}{2} + \frac{1}{2\gamma_1} \tilde{\Theta}_1^2 \tag{14}$$

where $\gamma_1 > 0$ is a design parameter, Θ_1 is the estimate of Θ_1^* and $\tilde{\Theta}_1 = \Theta_1^* - \Theta_1$ is the parameter estimate error.

From (13) and take the time derivative of V_1 as

$$\dot{V}_1 \leq z_1 \left[\alpha_1 + z_1 + \frac{z_1 \Theta_1}{2a_1^2} - \dot{y}_r \right] + \frac{a_1^2}{2} + \frac{\varepsilon_1^{*2}}{2} + \frac{z_2^2}{2} + \frac{\tilde{\Theta}_1}{\gamma_1} \left(\frac{z_1^2 \gamma_1}{2a_1^2} - \dot{\Theta}_1 \right) + \frac{p_1^2}{2} \tag{15}$$

Choose the intermediate control function α_1 and parameter adaptive law Θ_1 as:

$$\alpha_1 = -c_1 z_1 - z_1 - \frac{z_1}{2a_1^2} \Theta_1 + \dot{y}_r \tag{16}$$

$$\dot{\Theta}_1 = \frac{z_1^2 \gamma_1}{2a_1^2} - \sigma_1 \Theta_1 \tag{17}$$

where $c_1 > 0$ and $\sigma_1 > 0$ are design constants.

Substituting (16) and (17) into (15) results in

$$\dot{V}_1 \leq -c_1 z_1^2 + \frac{\sigma_1 \tilde{\Theta}_1 \Theta_1}{\gamma_1} + \frac{1}{2} z_2^2 + \frac{a_1^2}{2} + \frac{\varepsilon_1^{*2}}{2} + \frac{1}{2} p_1^2 \tag{18}$$

By completion of squares, we have

$$\frac{\sigma_1 \tilde{\Theta}_1 \Theta_1}{\gamma_1} \leq -\frac{\sigma_1 \tilde{\Theta}_1^2}{2\gamma_1} + \frac{\sigma_1 \Theta_1^{*2}}{2\gamma_1} \tag{19}$$

(18) can be rewritten as

$$\dot{V}_1 \leq -c_1 z_1^2 - \frac{\sigma_1 \tilde{\Theta}_1^2}{2\gamma_1} + \frac{1}{2} z_2^2 + \frac{\sigma_1 \Theta_1^{*2}}{2\gamma_1} + \frac{a_1^2}{2} + \frac{1}{2} \varepsilon_1^{*2} + \frac{1}{2} p_1^2 \tag{20}$$

Step i ($2 \leq i \leq n - 1$): The time derivative of z_i is

$$\dot{z}_i = x_{i+1} + f_{k,i}(\bar{x}_i) + \Delta_{k,i}(\bar{x}_n, t) - \dot{\alpha}_{i-1} \tag{21}$$

As the similar procedure in Step 1, and using mathematical induction, we can get the intermediate control function α_i and parameter adaptive law Θ_i as the following:

$$\alpha_i = -c_i z_i - \frac{3}{2} z_i - \frac{z_i \Theta_i}{2a_i^2} \tag{22}$$

$$\dot{\Theta}_i = \frac{z_i^2 \gamma_i}{2a_i^2} - \sigma_i \Theta_i \tag{23}$$

where $a_i \neq 0$, $\gamma_i > 0$, $c_i > 0$ and $\sigma_i > 0$ are design constants.

Consider the following Lyapunov function candidate:

$$V_i = V_{i-1} + \frac{z_i^2}{2} + \frac{1}{2\gamma_i} \tilde{\Theta}_i^2 \tag{24}$$

The following expression can be obtained

$$\dot{V}_i \leq -\sum_{j=1}^i c_j z_j^2 - \sum_{j=1}^i \frac{\sigma_j \tilde{\Theta}_j^2}{2\gamma_j} + \sum_{j=1}^i \frac{a_j^2}{2} + \sum_{j=1}^i \frac{\varepsilon_j^{*2}}{2} + \sum_{j=1}^i \frac{\sigma_j \Theta_j^{*2}}{2\gamma_j} + \frac{z_{i+1}^2}{2} + \sum_{j=1}^i \frac{p_j^2}{2} \tag{25}$$

Step n: The time derivative of z_n is

$$\dot{z}_n = f_{k,n}(\bar{x}_n) + u + \Delta_{k,n} - \dot{\alpha}_{n-1} \tag{26}$$

As the similar procedure in Step 1, and using mathematical induction, we can get the control law v and parameter adaptive law Θ_n as the following:

$$v = -\frac{1}{\beta_0} \left[c_n z_n + \frac{3}{2} z_n + \frac{z_n \Theta_n}{2a_n^2} \right] \tag{27}$$

$$\dot{\Theta}_n = \frac{z_n^2 \gamma_n}{2a_n^2} - \sigma_n \Theta_n \tag{28}$$

where Θ_n is the estimate of $\Theta_n^* \left(\Theta_n^* = \max_{k \in \Xi} \|\theta_{k,n}^*\|^2 \right)$, $a_n \neq 0$, $c_n > 0$, $\gamma_n > 0$ and $\sigma_n > 0$ are design constants.

4. Stability Analysis. The aforementioned design procedures can be summarized as the following theorem, and the correctness of the theorem will be verified later.

Theorem 4.1. *For nonlinear uncertain switched system with unknown dead-zone (1), under Assumptions 2.1-2.3, Lemma 2.1 and arbitrary switching rulers, the controller (27) and together with the intermediate control (16) and (22), parameter adaptive laws (17), (23) and (28), can guarantee our control objective by choosing the appropriate design parameters.*

Proof: Consider the total Lyapunov candidate functions

$$V = V_n = V_{n-1} + \frac{z_n^2}{2} + \frac{1}{2\gamma_n} \tilde{\Theta}_n^2 \tag{29}$$

From (3), the time derivative of V is

$$\begin{aligned} \dot{V} = \dot{V}_n &\leq \dot{V}_{n-1} + z_n [u + \theta_{k,n}^{*\top} \varphi_{k,n}(\bar{x}_n, y_r) + \varepsilon_{k,n}(\bar{x}_n, y_r)] + \frac{1}{2} p_n^2 - \frac{\tilde{\Theta}_n \dot{\Theta}_n}{\gamma_n} \\ &\leq - \sum_{j=1}^{n-1} c_j z_j^2 - \sum_{j=1}^{n-1} \frac{\sigma_j \tilde{\Theta}_j^2}{2\gamma_j} + \sum_{j=1}^{n-1} \frac{1}{2} a_j^2 + \sum_{j=1}^{n-1} \frac{1}{2} \varepsilon_j^{*2} + \sum_{j=1}^{n-1} \frac{\sigma_1 \Theta_1^{*2}}{2\gamma_1} + \sum_{j=1}^{n-1} \frac{p_j^2}{2} \\ &\quad - \frac{\tilde{\Theta}_n \dot{\Theta}_n}{\gamma_n} + z_n [k^\top(t) \phi(t) v + d(v) + \theta_{k,n}^{*\top} \varphi_{k,n}(\bar{x}_n, y_r) + \varepsilon_{k,n}(\bar{x}_n, y_r)] + \frac{p_n^2}{2} \end{aligned} \tag{30}$$

Applying Young’s inequality, and applying $k^\top(t) \phi(t) \in [\beta_0, k_{l1} + k_{r1}]$ and $|d(v)| \leq \tau$, we have

$$\begin{aligned} &z_n [k^\top(t) \phi(t) v + \theta_{k,n}^{*\top} \varphi_{k,n}(\bar{x}_n, y_r) + \varepsilon_{k,n}(\bar{x}_n, y_r) + d(v)] \\ &\leq z_n \left(\beta_0 v + z_n + \frac{z_n \Theta_n^*}{2a_n^2} \right) + \frac{a_n^2}{2} + \frac{1}{2} \varepsilon_n^{*2} + \frac{1}{2} \tau^2 \end{aligned} \tag{31}$$

where $\Theta_n^* = \max_{k \in \Xi} \|\theta_{k,n}^*\|^2$, $\varepsilon_n^* = \max_{k \in \Xi} \varepsilon_{k,n}(\bar{x}_n, y_r)$ is an unknown constant.

Substituting (27), (28) and (31) into (3), we obtain

$$\dot{V} \leq \dot{V}_n \leq - \sum_{j=1}^n c_j z_j^2 - \sum_{j=1}^n \frac{\sigma_j \tilde{\Theta}_j^2}{2\gamma_j} + D \tag{32}$$

where $D = \sum_{j=1}^n \frac{a_j^2}{2} + \sum_{j=1}^n \frac{1}{2} \varepsilon_j^{*2} + \frac{1}{2} \tau^2 + \sum_{j=1}^n \frac{\sigma_j \Theta_j^{*2}}{2\gamma_j} + \sum_{j=1}^n \frac{p_j^2}{2}$.

(32) can be further rewritten as

$$\dot{V}_n \leq -\alpha V_n + \beta \tag{33}$$

where $\alpha = \min\{2c_1, \dots, 2c_n, \sigma_1, \dots, \sigma_n\}$ and $\beta = D$.

Integrating the differential inequality (33), we have

$$V = V_n \leq V_n(0) e^{-\alpha t} + \frac{\beta}{\alpha} (1 - e^{-\alpha t}) \tag{34}$$

From (34), we can conclude that all the signals in the closed-loop system remain bounded and the tracking error can converge to a small neighborhood of the origin.

5. Simulation Study. Consider the following second-order nonlinear switched systems with unknown dead-zone:

$$\begin{cases} \dot{x}_1 = x_2 + f_{1,1}(x_1) + \Delta_{1,1}(\bar{x}_2, t) \\ \dot{x}_2 = u + f_{1,2}(\bar{x}_2) + \Delta_{1,2}(\bar{x}_2, t) \\ y = x_1 \end{cases} \tag{35}$$

$$\begin{cases} \dot{x}_1 = x_2 + f_{2,1}(x_1) + \Delta_{2,1}(\bar{x}_2, t) \\ \dot{x}_2 = u + f_{2,2}(\bar{x}_2) + \Delta_{2,2}(\bar{x}_2, t) \\ y = x_1 \end{cases} \tag{36}$$

where $f_{1,1}(x_1) = 0.5x_1$, $f_{1,2}(\bar{x}_2) = x_1 x_2 \sin(x_1)$, $f_{2,1}(x_1) = x_1 \sin(x_1)$, $f_{2,2}(\bar{x}_2) = \sin(x_1 x_2)$. The reference signal is given as $y_r(t) = \sin(0.5t)$.

Parameters in controller and adaptive laws are chosen as $c_1 = 15$, $c_2 = 20$, $a_1 = 5$, $a_2 = 5$, $\gamma_1 = 0.2$, $\gamma_2 = 0.2$, $\sigma_1 = 0.2$, $\sigma_2 = 0.2$.

Choose the initial conditions as $x_1(0) = 1$, $x_2(0) = -1$, and the other initial values are chosen as zeros. We choose $D(v)$ as follows:

$$u = D(v) = \begin{cases} (1 - 0.3 \sin(v))(v - 0.2) & v \geq 0.2 \\ 0 & -0.2 < v < 0.2 \\ (0.8 - 0.2 \cos(v))(v + 0.2) & v \leq -0.2 \end{cases}$$

The simulation results are shown in Figures 1 and 2, where Figure 1 expresses the trajectories of the output and tracking signal; Figure 2 exhibits the trajectory of u . From Figures 1 and 2, it can be concluded that under arbitrary switching rulers the proposed adaptive fuzzy controller can guarantee all signals in the closed-loop system remain SGUUB and the tracking error $z_1 = y - y_r$ converges to a small neighborhood of the origin.

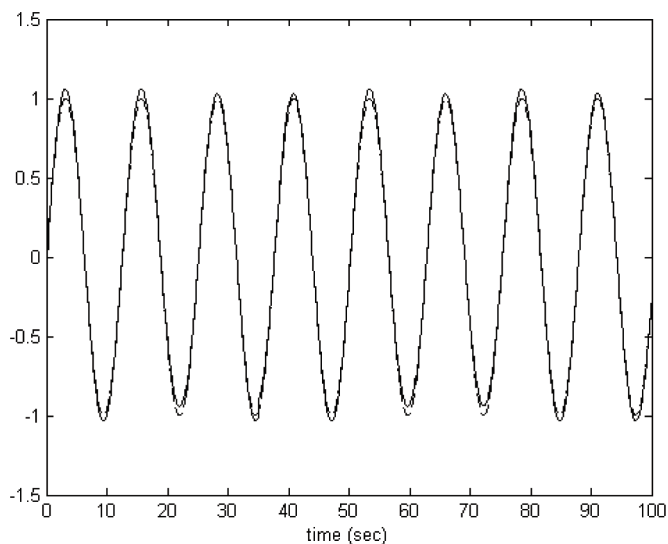


FIGURE 1. y (solid line) and y_r (dashed line)

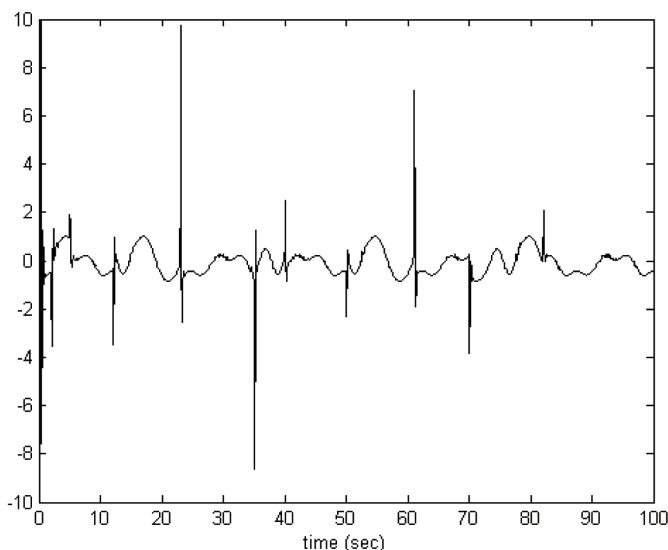


FIGURE 2. The trajectory of u

6. Conclusions. This paper has investigated the problem of tracking control design for a class of SISO switched nonlinear systems with unknown dead-zone. With the help of backstepping control design principle and common Lyapunov function stability theory, an adaptive fuzzy controller and adaptation laws are developed under arbitrary switchings.

The proposed control approach can not only solve the control design problem for a class of switched nonlinear system with unknown dead-zone, but also can guarantee the control performance. Future research will concentrate on the adaptive fuzzy backstepping control design for uncertain nonlinear MIMO systems with unmeasured states and stochastic systems based on the results of this paper.

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