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DESIGN OF MULTI-RATE CONTROL SYSTEM WHICH RESTRAINS THE OUTPUT DISTURBANCE

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Abstract. *We studied a design method for a multi-rate control system, in which the sampling interval of the plant output is twice the hold interval of the control input. In conventional control design method, the pre-designed control law is extended without changing the existing discrete-time control system at a slow-single-rate. In the extended multi-rate control system, the intersample response can be improved without changing the sampled output response if the control system has no disturbance. Therefore, this paper proposes a new multi-rate controller design method that reduces intersample output disturbance while preserving the sampled output of the pre-designed single-rate control system.*
Keywords: Multi-rate control, Disturbance, The sampled output response, The design polynomial vector

1. **Introduction.** In digital control, the output signal of a plant observed by a sensor is discretized at a finite time interval called "sampling interval". Further, the input signal of the plant given through an actuator is held for a finite time interval called "hold interval". These time intervals are constrained by the performance limitations of sensors, actuators, samplers (A/D converter), holders (D/A converter), and so on. Thus, these intervals are not always the same. In such a scenario, the system with a controller that considers these intervals to have the same value is called a "single-rate" control system. In this case, because both intervals are set to the larger value, the hardware performance of the system may not be fully utilized. On the other hand, a system whose controller considers the sampling and hold intervals individually is called a "multi-rate" control system. The multi-rate control system achieves higher performance than the single-rate control system. We studied a design method for a multi-rate control system based on [1], in which the sampling interval of the plant output is twice the hold interval of the control input. As Tangirala et al. point out in [2, 3], when the hold interval is shorter than the sampling interval, multi-rate control systems have an intersample output oscillation problem, even if the sampled output converges to its reference input. To solve this problem, we proposed a multi-rate controller design method using integral compensation or a generalized holder [4]. With this method, the intersample output oscillation of the multi-rate control system can be reduced without changing the sampled output of the pre-designed single-rate control system. This paper considered a multi-rate control system in the presence of output noise. In our previous research, we proposed a multi-rate controller design method that can design noise-output response to attenuate a particular disturbance without changing the closed-loop transfer function [5]. However, this method has a problem in that the sampled output of the obtained multi-rate control system changes from that of a pre-designed single-rate control system, which is the ideal output response. This is because this method reduces not only the intersample output disturbance but also the sampled output disturbance. Therefore, in this paper, we propose

a new multi-rate controller design method that reduces intersample output disturbance while preserving the sampled output of the pre-designed single-rate control system. The effectiveness of our proposed method was confirmed through computer simulations. In the following section, z_1^{-1} denotes the backward shift operator, and $z_j^{-1}y(k) = y(k-j)$ and $z_j^{-1} = z_1^{-j}$. A polynomial with shift operators is described as $A[z_l^{-1}]$, and a polynomial matrix is described as $\boldsymbol{A}[z_l]$.

2. **Design of Multi-rate Control System Which Restrains the Output Disturbance.** In this section, we show that the intersample response is adjustable without changing the sampled output response by applying the conventional design method [4], even if the control system has disturbance. This paper considers a multi-rate control system design for the system that has disturbance $d(k)$ in the output as shown in Figure 1.

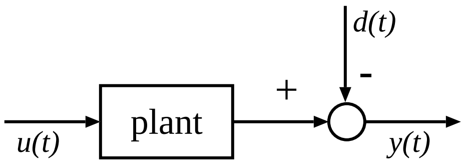

FIGURE 1. Output disturbance error model

A controlled plant of a continuous-time system is transformed into the discrete-time system. Furthermore, it is assumed that control input $u(k)$ is updated every T_u [s], whereas the plant output $y(k)$ is sampled every $T_s = 2T_u$ [s]. Then, the controller is designed as the following two-input single-output discrete-time system by using a lifting technique (see details in [6]).

$$A[z_2^{-1}](y(k) - d(k)) = \boldsymbol{B}[z_2^{-1}]^T \boldsymbol{u}(k-2) \tag{1}$$

$$A[z_2^{-1}] = 1 + a_1 z_2^{-1} + a_2 z_2^{-2} + \ldots + a_n z_2^{-n}$$

$$\boldsymbol{B}[z_2^{-1}] = \begin{bmatrix} B_1[z_2^{-1}] & B_2[z_2^{-1}] \end{bmatrix}^T$$

$$B_1[z_2^{-1}] = b_{1,0} + b_{1,1} z_2^{-1} + \ldots + b_{1,n} z_2^{-n}$$

$$B_2[z_2^{-1}] = b_{2,0} + b_{2,1} z_2^{-1} + \ldots + b_{2,n} z_2^{-n}$$

$$\boldsymbol{u}(k) = \begin{bmatrix} u(k) & u(k+1) \end{bmatrix}^T$$

It is assumed that a stable closed-loop system is achieved by the following multi-rate control law.

$$\boldsymbol{Y}[z_2^{-1}]\boldsymbol{u}(k) = \boldsymbol{K}[z_2^{-1}]r(k) - \boldsymbol{X}[z_2^{-1}]y(k) \tag{2}$$

where $r(k)$ is reference input and

$$\boldsymbol{Y}[z_2^{-1}] = \begin{bmatrix} Y_1[z_2^{-1}] & 0 \\ 0 & Y_2[z_2^{-1}] \end{bmatrix}$$

$$\boldsymbol{K}[z_2^{-1}] = \begin{bmatrix} K_1[z_2^{-1}] \\ K_2[z_2^{-1}] \end{bmatrix}$$

$$\boldsymbol{X}[z_2^{-1}] = \begin{bmatrix} X_1[z_2^{-1}] \\ X_2[z_2^{-1}] \end{bmatrix}$$

Using (1) and (2), the closed-loop system is calculated as

$$y(k) = \frac{z_2^{-1}\boldsymbol{Y}_B[z_2^{-1}]^T \boldsymbol{K}[z_2^{-1}]}{T[z_2^{-1}]} r(k) + \frac{A[z_2^{-1}]Y_p[z_2^{-1}]}{T[z_2^{-1}]} d(k) \tag{3}$$

where

$$T[z_2^{-1}] = A[z_2^{-1}]Y_p[z_2^{-1}] + z_2^{-1}\boldsymbol{Y}_B[z_2^{-1}]^T\boldsymbol{X}[z_2^{-1}] \quad (4)$$

$$\boldsymbol{Y}_B = \left[\begin{array}{cc} B_1[z_2^{-1}]Y_2[z_2^{-1}] & B_2[z_2^{-1}]Y_1[z_2^{-1}] \end{array}\right]^T \quad (5)$$

$$Y_p[z_2^{-1}] = Y_1[z_2^{-1}]Y_2[z_2^{-1}] \quad (6)$$

The control law (2) is extended as the following to adjust the intersample response without changing the sampled output response. Then, $U_u[z_2^{-1}]$ and $U_y[z_2^{-1}]$ are determined not to change the sampled output response [4]. The design polynomial vectors $U_1[z_2^{-1}]$ and $U_2[z_2^{-1}]$ are introduced.

$$\boldsymbol{Y}_e[z_2^{-1}]\boldsymbol{u}(k) = \boldsymbol{K}[z_2^{-1}]r(k) - \boldsymbol{X}_e[z_2^{-1}]y(k) \quad (7)$$

where

$$\boldsymbol{Y}_e[z_2^{-1}] = \boldsymbol{Y}[z_2^{-1}] - z_2^{-1}\boldsymbol{U}_u[z_2^{-1}]\boldsymbol{B}[z_2^{-1}]^T \quad (8)$$

$$\boldsymbol{X}_e[z_2^{-1}] = \boldsymbol{X}[z_2^{-1}] + \boldsymbol{U}_y[z_2^{-1}]A[z_2^{-1}] \quad (9)$$

$$\boldsymbol{U}_u[z_2^{-1}] = \left[\begin{array}{c} U_1[z_2^{-1}]B_2[z_2^{-1}]Y_1[z_2^{-1}] \\ U_2[z_2^{-1}]B_1[z_2^{-1}]Y_2[z_2^{-1}] \end{array}\right] \quad (10)$$

$$\boldsymbol{U}_y[z_2^{-1}] = \left[\begin{array}{c} U_2[z_2^{-1}]B_2[z_2^{-1}]Y_1[z_2^{-1}] \\ U_1[z_2^{-1}]B_1[z_2^{-1}]Y_2[z_2^{-1}] \end{array}\right] \quad (11)$$

Using (7), the closed-loop system is calculated as

$$y(k) = \frac{z_2^{-1}\boldsymbol{Y}_B[z_2^{-1}]^T\boldsymbol{K}[z_2^{-1}]}{T[z_2^{-1}]}r(k) + \frac{A[z_2^{-1}]\left(Y_p[z_2^{-1}] - z_2^{-1}\boldsymbol{Y}_B[z_2^{-1}]^T\boldsymbol{U}_u[z_2^{-1}]\right)}{T[z_2^{-1}]}d(k) \quad (12)$$

So the sampled output of the closed-loop system (3) is not changed, and (12) must be equal to (3). However, because the influence of disturbance $d(k)$ is not considered, it cannot be performed by the conventional design method. Therefore, it is necessary to find the following constraint to preserve the disturbance response of (3):

$$A[z_2^{-1}]Y_p[z_2^{-1}] = A[z_2^{-1}]\left(Y_p[z_2^{-1}] - z_2^{-1}\boldsymbol{Y}_B[z_2^{-1}]^T\boldsymbol{U}_u[z_2^{-1}]\right) \quad (13)$$

To satisfy (13), it is necessary to satisfy the following condition

$$z_2^{-1}\boldsymbol{Y}_B[z_2^{-1}]^T\boldsymbol{U}_u[z_2^{-1}] = 0 \quad (14)$$

Substituting (5) and (10) into (14), we have

$$z_2^{-1}\boldsymbol{Y}_B[z_2^{-1}]^T\boldsymbol{U}_u[z_2^{-1}] = z_2^{-1}B_1[z_2^{-1}]Y_1[z_2^{-1}]B_2[z_2^{-1}]Y_2[z_2^{-1}]\left(U_1[z_2^{-1}] + U_2[z_2^{-1}]\right) \quad (15)$$

From (15), it is determined that (14) is satisfied with $U_1[z_2^{-1}] + U_2[z_2^{-1}] = 0$. Therefore, we define the following condition for $U_1[z_2^{-1}]$ and $U_2[z_2^{-1}]$.

$$U_1[z_2^{-1}] = -U_2[z_2^{-1}] \quad (16)$$

Using (16), we can adjust the intersample response without changing the sampled output response when the control system has disturbance. In the case that $U_1[z_2^{-1}] = U_2[z_2^{-1}]$, because the extended control law does not change with the non-extended control law, the output does not change.

3. **Numerical Example.** A numerical example demonstrates the effectiveness of the proposed method. In this simulation, a controlled plant $G(s)$, reference input $r(k)$, and disturbance $d(k)$ are expressed as

$$G(s) = \frac{1}{s^2 + 3.2s + 1} \quad (17)$$

$$r(k) = 1 \quad (18)$$

$$d(t) = -0.2\sin(18t) \quad (19)$$

In the following discussion, the disturbance $d(t)$ in (19) is transformed into the discrete time form given by $d(k)$. It is assumed that control input $u(k)$ is updated every T_u [s] but the plant output $y(k)$ is sampled every $T_s = 2T_u$ [s]. Then, the controller is designed as the following two-input single-output discrete-time system by using the lifting technique.

$$A[z_2^{-1}](y(k) - d(k)) = \begin{bmatrix} B1[z_2^{-1}] & B2[z_2^{-1}] \end{bmatrix} \boldsymbol{u}(k-2) \tag{20}$$

where

$$\begin{aligned} A &= 1 - 0.50z_2^{-1} + 1.7 \times 10^{-3} z_2^{-2} \\ B1 &= 0.23 + 3.0 \times 10^{-3} z_2^{-1} \\ B2 &= 0.21 + 6.5 \times 10^{-2} z_2^{-1} \end{aligned}$$

It is assumed that a stable control of the plant is achieved by using the following multi-rate control law.

$$\begin{bmatrix} 1 + 9.7 \times 10^{-3} z_2^{-1} & 0 \\ 0 & 1 \end{bmatrix} \boldsymbol{u}(k) = \begin{bmatrix} 0.89 \\ 0.89 \end{bmatrix} r(k) - \begin{bmatrix} -1.37 - 0.21z_2^{-1} \\ 1 \end{bmatrix} y(k) \tag{21}$$

The steady-state gains of (10) and (11) are calculated as follows using (20) and (21).

$$\boldsymbol{U}_u[1] = \begin{bmatrix} 0.2723U_1[1] \\ 0.2330U_2[1] \end{bmatrix} \tag{22}$$

$$\boldsymbol{U}_y[1] = \begin{bmatrix} 0.2723U_2[1] \\ 0.2330U_1[1] \end{bmatrix} \tag{23}$$

Then, we select $U_1[1]$ and $U_2[1]$ satisfying (16). In this numerical example, we use simple values that were chosen by trial and error.

$$U_1[1] = -10.0 \tag{24}$$

$$U_2[1] = 10.0 \tag{25}$$

Using (24) and (25), the extended control law is calculated as

$$\begin{bmatrix} Y_e11 & Y_e12 \\ Y_e21 & Y_e22 \end{bmatrix} \boldsymbol{u}(k) = \begin{bmatrix} K \\ K \end{bmatrix} r(k) - \begin{bmatrix} X_e1 \\ X_e2 \end{bmatrix} y(k) \tag{26}$$

where

$$\begin{aligned} Y_e11 &= 1 + 0.48z_2^{-1} + 0.16z_2^{-2} + 3.4 \times 10^{-3} z_2^{-3} + 1.9 \times 10^{-5} z_2^{-4} \\ Y_e12 &= 0.42z_2^{-1} + 0.27z_2^{-2} + 4.4 \times 10^{-2} z_2^{-3} + 4.0 \times 10^{-4} z_2^{-4} \\ Y_e21 &= -0.53z_2^{-1} - 1.4 \times 10^{-2} z_2^{-2} - 9.0 \times 10^{-5} z_2^{-3} \\ Y_e22 &= 1 - 0.47z_2^{-1} - 0.15z_2^{-2} - 1.9 \times 10^{-3} z_2^{-3} \\ K &= 0.89 \\ X_e1 &= 0.68 - 0.57z_2^{-1} - 0.32z_2^{-2} - 2.0 \times 10^{-3} z_2^{-3} + 1.0 \times 10^{-5} z_2^{-4} \\ X_e2 &= -1.30 + 1.12z_2^{-1} + 1.1 \times 10^{-2} z_2^{-2} - 5.0 \times 10^{-5} z_2^{-3} \end{aligned}$$

The numerical simulation result of the non-extended input given by (21) is shown in Figure 2, and that of the extended input given by (26) is shown in Figure 3. Furthermore, outputs that correspond with each of the non-extended and extended inputs are shown in Figure 4, where the dotted line, solid line, and the circle points indicate the non-extended intersample output, the extended intersample output, and the sampled output, respectively. This shows that we can adjust the intersample response without changing the sampled output response.

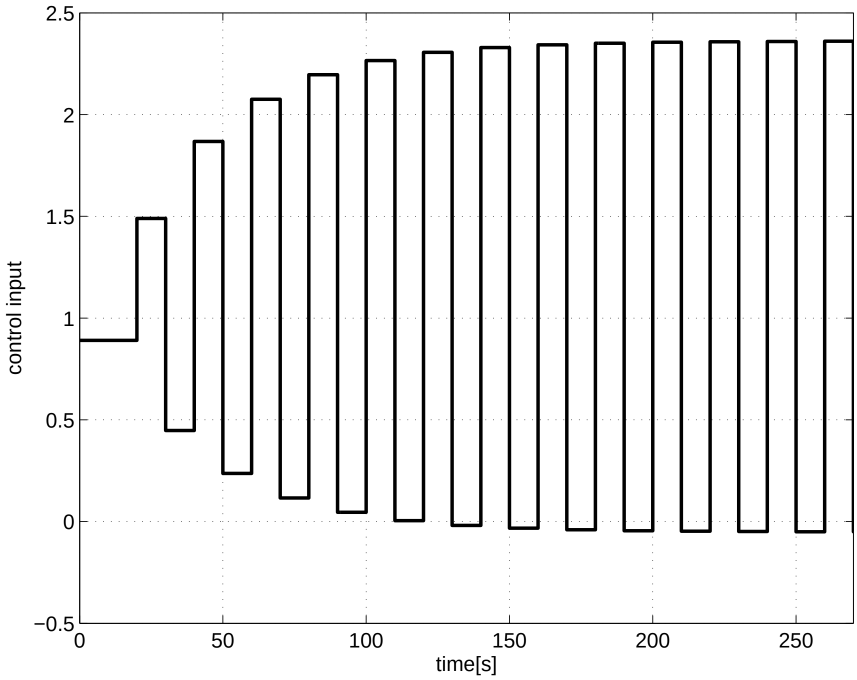

Figure 2. Non-extended input given by (21) for the multi-rate control system with disturbance

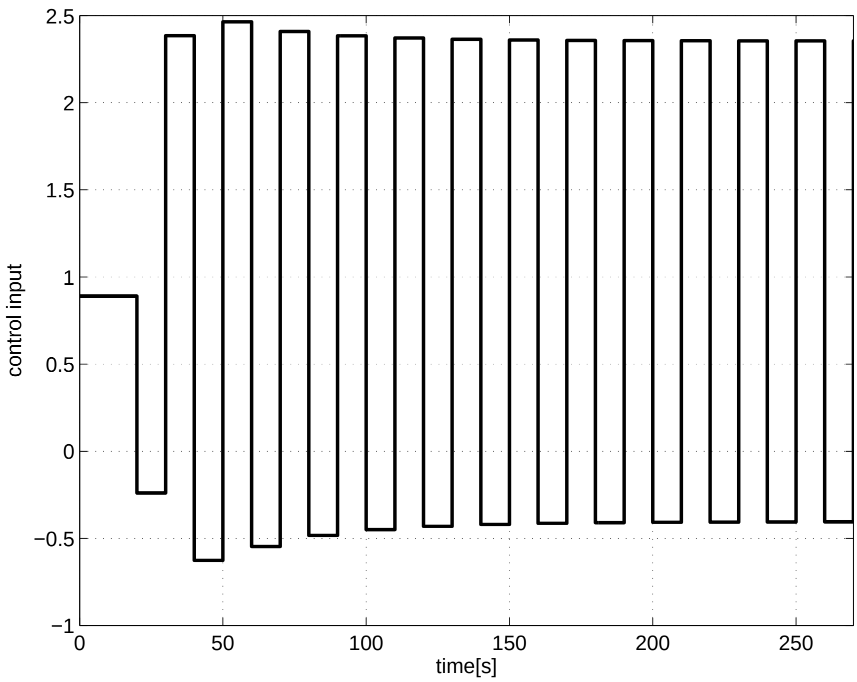

Figure 3. Extended input given by (26) for the multi-rate control system with disturbance

4. **Conclusions.** This paper considered a multi-rate control system in the presence of output noise. We showed that the intersample response is adjustable without changing the sampled output response by adding a constraint to the design polynomial vectors, even when the system has disturbance. However, this study determines the design polynomial vectors. Therefore, these may not be the optimal parameter values. In our future work, we plan to include the selection of the most suitable parameter values.

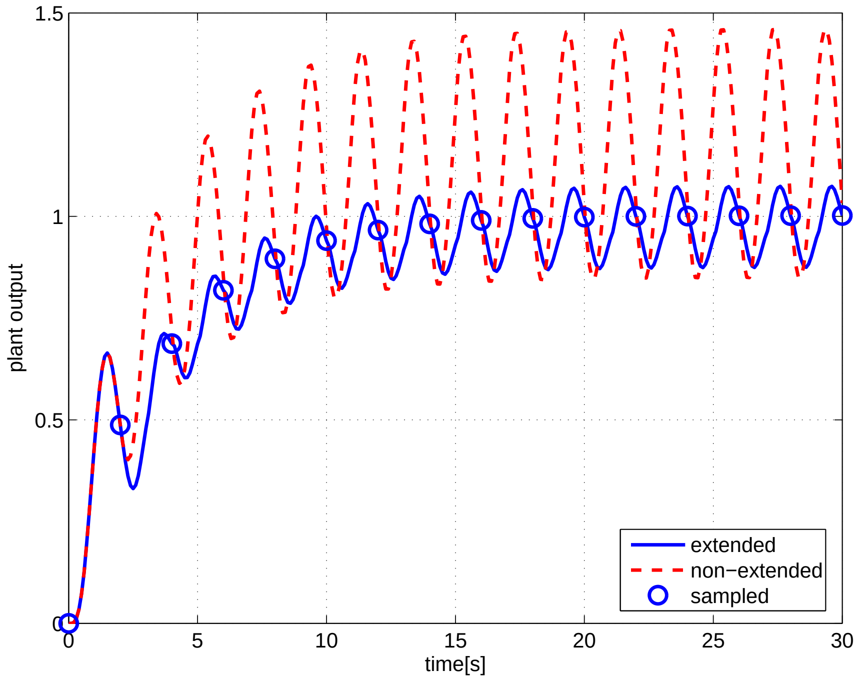

FIGURE 4. Comparison of non-extended and extended outputs

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