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IMPROVED DELAY-DEPENDENT STABILITY CRITERIA FOR T-S FUZZY SINGULAR SYSTEMS WITH INTERVAL TIME-VARYING DELAY

Chao Sun[1,2,*], Fuli Wang[1] and Xiqin He[2]

[1]College of Information Science and Engineering
Northeastern University
No. 3-11, Wenhua Road, Heping District, Shenyang 110819, P. R. China
wangfuli@ise.neu.edu.cn

[2]School of Science
University of Science and Technology Liaoning
No. 185, Middle Qianshan Road, Gaoxin District, Anshan 114051, P. R. China
[*]Corresponding author: chaosun@ustl.edu.cn

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Abstract. *This paper deals with the admissible problem for a class of Takagi-Sugeno (T-S) fuzzy singular systems with interval time-varying delay. By decomposing the delay interval into two unequal subintervals, a simple Lyapunov-Krasovskii functional (LKF) is constructed and a tighter upper bound of the derivative of LKF can be obtained. Several new delay-dependent criteria are derived in terms of linear matrix inequalities (LMIs) to guarantee that the fuzzy singular system is regular, impulse free and asymptotically stable. Compared with some existing results, the proposed ones give the result with less conservatism. Finally, two examples are given to show the effectiveness and the improvement of the proposed method.*

Keywords: Fuzzy singular system, Interval time-varying delay, Stability, LMIs

1. **Introduction.** Over the past few decades, a wider class of fuzzy systems that are described by the singular form have been studied, where the model is the extension of T-S fuzzy model [1]. T-S fuzzy singular model provides a new way to the analysis and synthesis of the nonlinear singular system and the time-varying singular system. Meanwhile, time delays always exist in many dynamical systems and delays are sources of poor stability and performance of a system [2, 3]. Therefore, lots of stability analysis results [4-16] have been reported for T-S fuzzy systems or fuzzy singular systems with time-delay. It should be pointed out that for all of the aforementioned results, the maximum allowable delay serves as performance index for measuring the conservatism of the conditions obtained.

To reduce the conservativeness of the delay-dependent criteria, the delay-partitioning method [4-7], convex combination technique [8, 9] and free weighting matrices method [10, 11] were well used for delayed T-S fuzzy systems. Recently, some work has been extended to the stability analysis for T-S fuzzy singular systems with time-varying delay. In [12], the problems of delay-dependent stability were discussed utilizing model transformation techniques. Using free-weight matrix method, [13] discussed the problems of delay-dependent stability and $L_2 - L_\infty$ control. Based on delay partitioning approach, some less conservative stability criteria for fuzzy singular systems with time-varying delay have been investigated in [14, 15]. By using quadratic method, sufficient conditions on stability and stabilization are proposed in [16] for uncertain T-S fuzzy singular systems.

Inspired by the methods mentioned above, the objective of this paper is to revisit the delay-dependent stability analysis for T-S fuzzy singular systems with interval time-varying delays. Different from [12-16], we decompose the constant part of time-varying delay $[0, \tau_1]$ into N segments, and the delay interval $[\tau_1, \tau_2]$ is divided into two subintervals with an unequal width as $[\tau_1, \tau_\rho]$ and $[\tau_\rho, \tau_2]$, where $\tau_\rho = \tau_1 + \rho\delta$, $\delta = \tau_2 - \tau_1$, $0 < \rho < 1$. A simple LKF is constructed on the intervals, which is with less number of decision variables, but with more information of the delay. The newly developed conditions are expected to be less conservative than the previous ones. The rest of this paper is organized as follows. Section 2 formulates the system descriptions and problem under consideration. Stability analysis is presented in Section 3. Finally, two numerical examples are given in Section 4 to demonstrate the effectiveness and less conservatism over the existing results. Some conclusions are made in Section 5.

2. **Problem Statement and Preliminaries.** Consider a nonlinear singular system with time delay, which can be represented by the following extended T-S fuzzy singular model:

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\{A_i x(t) + A_{\tau i} x(t-\tau(t))\} = A(t)x(t) + A_\tau(t)x(t-\tau(t)) \\ x(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\phi_i(t) = \phi(t), \quad \forall t \in [-\tau_2, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, and $\phi_i(t)$ is a vector-valued initial continuous function defined on the interval $[-\tau_2, 0]$. The fuzzy basis functions are given by $\mu_i(\xi(t)) = \beta_i(\xi(t))/\sum_{j=1}^{r}\beta_j(\xi(t))$, $\beta_i(\xi(t)) = \prod_{i=1}^{p} M_{ij}(\xi(t))$ with $M_{ij}(\xi_j(t))$ representing the grade of membership of $\xi_j(t)$ in M_{ij}, where $\xi_j(t)$ is the premise variable. $E \in \mathbb{R}^{n\times n}$ is a constant matrix, which may be singular, that is, $\mathrm{rank}(E) = g \le n$. A_i, $A_{\tau i}$ are constant real matrices of appropriate dimensions. The delay $\tau(t)$ is time varying and satisfies $\tau_1 \le \tau(t) \le \tau_2$, $\dot{\tau}(t) \le d$, where τ_1, τ_2 and d are constants. Next, we will introduce some lemmas to be needed in the development of main results throughout this paper.

Lemma 2.1. [17] *For any positive semi-definite matrices $X = (X_{ij})_{3\times 3} \ge 0$, the following integral inequality holds:*

$$-\int_{t-\tau(t)}^{t} \dot{x}^T(s) X_{33} \dot{x}(s) ds \le \int_{t-\tau(t)}^{t} \beta(t,s) \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix} \beta^T(t,s) ds$$

where $\beta(t,s) = \left[x^T(t)\ x^T(t-\tau(t))\ \dot{x}^T(s)\right]$.

Lemma 2.2. [18] *If a functional $V : C_n[-\tau, 0] \to \mathbb{R}$ is continuous and $x(t,\phi)$ is a solution to (1), we define $\dot{V}(\phi) = \lim_{h\to 0^+} sup\frac{1}{h}(V(x(t+h,\phi) - V(\phi)))$. Denote the system parameters of* (1) *as* $(E,\ A,\ A_\tau) = \left(\begin{bmatrix} I_g & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \begin{bmatrix} A_{\tau 11} & A_{\tau 12} \\ A_{\tau 21} & A_{\tau 22} \end{bmatrix}\right)$. *Assume that the singular system* (1) *is regular and impulse free, A_{22} is invertible, $\rho\left(A_{22}^{-1}A_{\tau 22}\right) < 1$. Then, the system* (1) *is stable if there exist positive numbers α, μ, ν and a continuous function, $V : C_n[-\tau, 0] \to \mathbb{R}$, such that $\mu\|\phi_1(0)\|^2 \le V(\phi) \le \nu\|\phi\|^2$, $\dot{V}(x_t) \le -\alpha\|x_t\|^2$, where $x_t = x(t+\theta)$ with $\theta \in [-\tau, 0]$ and $\phi = [\phi_1^T\ \phi_2^T]$ with $\phi_1 \in \mathbb{R}^q$.*

3. **Main Results.** Based on the Lyapunov-Krasovskii stability theorem, the following result is obtained.

Theorem 3.1. *For the given scalars τ_1, τ_2, d and ρ, system (1) is regular, impulse-free and asymptotically stable for any time-varying delay $\tau(t)$, if there exist matrices $P > 0$, $Q_n > 0$, $W_n > 0$ $(n = 1, 2, \ldots, N)$, $\Lambda^T(Y_{ij})_{3\times 3}\Lambda = \hat{Y} \ge 0$, $\Lambda^T(Z_{ij})_{3\times 3}\Lambda = \hat{Z} \ge 0$, $\Lambda = diag\{E, E, E\}$, $S_1 > 0$, $S_2 > 0$, $S_3 > 0$, $R_1 > 0$, $R_2 > 0$, such that the following set of conditions holds:*

$$E^T P = P^T E \ge 0,\ \Theta^i = \begin{bmatrix} \Theta_{11}^i & \Theta_{12}^i \\ * & \Theta_{22}^i \end{bmatrix} < 0,\ R_1 - Y_{33} \ge 0,\ R_2 - Z_{33} \ge 0 \tag{2}$$

where

$$\Theta_{11}^{i}=\begin{bmatrix}\Theta_{1,1}^{i} & E^TW_1E & \cdots & 0\\ * & \Theta_{2,2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ * & * & \cdots & \Theta_{n,n}\end{bmatrix},\ \Theta_{12}^{i}=\begin{bmatrix}0 & 0 & \Theta_{1,(N3)}^{i} & 0 & \Theta_{1,(N5)}^{i}\\ 0 & 0 & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ E^TW_NE & 0 & 0 & 0 & 0\end{bmatrix} \tag{3}$$

$$\Theta_{22}^{i}=\begin{bmatrix}\Theta_{(N1),(N1)} & \Theta_{(N1),(N2)} & \Theta_{(N1),(N3)} & 0 & 0\\ * & \Theta_{(N2),(N2)} & \Theta_{(N2),(N3)} & \Theta_{(N2),(N4)} & 0\\ * & * & \Theta_{(N3),(N3)} & \Theta_{(N3),(N4)} & \Theta_{(N3),(N5)}^{i}\\ * & * & * & \Theta_{(N4),(N4)} & 0\\ * & * & * & * & \Theta_{(N5),(N5)}\end{bmatrix} \tag{4}$$

with

$$\begin{aligned}
&\Theta_{1,1}^{i}=P^TA_i+A_i^TP+Q_1+S_1-E^TW_1E,\ \Theta_{1,(N3)}^{i}=P^TA_{\tau i},\\
&\Theta_{1,(N5)}^{i}=A_i^T\Delta,\ \Theta_{(N3),(N5)}^{i}=A_{\tau i}^T\Delta,\ \Delta=\sum_{n=1}^{N}h^2W_n+\rho\delta R_1+(1-\rho)\delta R_2\\
&\Theta_{n,n}=-Q_{n-1}-E^TW_{n-1}E+Q_n-E^TW_nE,\\
&\Theta_{(N1),(N1)}=-Q_N-W_N+S_2+\rho\delta\hat{Y}_{11}+\hat{Y}_{13}+\hat{Y}_{13}^T\\
&\Theta_{(N2),(N2)}=S_3-S_2+\rho\delta\hat{Y}_{22}-\hat{Y}_{23}-\hat{Y}_{23}^T+(1-\rho)\delta\hat{Z}_{11}+\hat{Z}_{13}+\hat{Z}_{13}^T\\
&\Theta_{(N4),(N4)}=-S_3+(1-\rho)\delta\hat{Z}_{22}-\hat{Z}_{23}-\hat{Z}_{23}^T,\ \Theta_{(N5),(N5)}=-\Delta
\end{aligned} \tag{5}$$

Case 1: when $\tau_1\le\tau(t)\le\tau_\rho$

$$\begin{aligned}
&\Theta_{(N1),(N3)}=\Theta_{(N2),(N3)}^T=\rho\delta\hat{Y}_{12}-\hat{Y}_{13}+\hat{Y}_{23}^T\\
&\Theta_{(N2),(N4)}=(1-\rho)\delta\hat{Z}_{12}-\hat{Z}_{13}+\hat{Z}_{23}^T,\quad \Theta_{(N3),(N4)}=\Theta_{(N1),(N2)}=0\\
&\Theta_{(N3),(N3)}=-(1-d)S_1+\rho\delta\hat{Y}_{11}+\hat{Y}_{13}+\hat{Y}_{13}^T+\rho\delta\hat{Y}_{22}-\hat{Y}_{23}-\hat{Y}_{23}^T
\end{aligned} \tag{6}$$

Case 2: when $\tau_\rho\le\tau(t)\le\tau_2$

$$\begin{aligned}
&\Theta_{(N1),(N2)}=\rho\delta\hat{Y}_{12}-\hat{Y}_{13}+\hat{Y}_{23}^T,\ \Theta_{(N2),(N4)}=\Theta_{(N1),(N3)}=0\\
&\Theta_{(N2),(N3)}=\Theta_{(N3),(N4)}=(1-\rho)\delta\hat{Z}_{12}-\hat{Z}_{13}+\hat{Z}_{23}^T\\
&\Theta_{(N3),(N3)}=-(1-d)S_1+(1-\rho)\delta\left(\hat{Z}_{11}+\hat{Z}_{22}\right)+\hat{Z}_{13}+\hat{Z}_{13}^T-\hat{Z}_{23}-\hat{Z}_{23}^T
\end{aligned} \tag{7}$$

Proof: Since rank$(E)=g\le n$, there must exist two invertible matrices $G\in\mathbb{R}^{n\times n}$ and $H\in\mathbb{R}^{n\times n}$ such that $\tilde{E}=GEH=\begin{bmatrix}I_g & 0\\ 0 & 0\end{bmatrix}$. Similarly, we define $\tilde{A}_i=GA_iH=\begin{bmatrix}\tilde{A}_{i11} & \tilde{A}_{i12}\\ \tilde{A}_{i21} & \tilde{A}_{i22}\end{bmatrix}$, and $\tilde{P}=G^{-T}PH=\begin{bmatrix}\tilde{P}_{11} & \tilde{P}_{12}\\ \tilde{P}_{21} & \tilde{P}_{22}\end{bmatrix}$. Since $\Theta^i<0$ and $Q_1>0$, $S_1>0$, we can formulate the following inequality easily:

$$\Upsilon_i=A_i^TP+P^TA_i-E^TW_1E<0 \tag{8}$$

Then, pre- and post-multiplying $\Upsilon_i<0$ by H^T and H, respectively, (8) yields

$$\tilde{\Upsilon}_i=\tilde{A}_i^T\tilde{P}+\tilde{P}^T\tilde{A}_i-H^TE^TW_1EH=\begin{bmatrix}\tilde{\Upsilon}_{11} & \tilde{\Upsilon}_{12}\\ * & \tilde{A}_{i22}^T\tilde{P}_{22}+\tilde{P}_{22}^T\tilde{A}_{i22}\end{bmatrix}<0 \tag{9}$$

Since $\tilde{\Upsilon}_{11}$ and $\tilde{\Upsilon}_{12}$ are irrelevant to the results of the following discussion, the real expressions of these two variables are omitted here. From Equation (9), it is easy to see that $\tilde{A}_{i22}^T\tilde{P}_{22}+\tilde{P}_{22}^T\tilde{A}_{i22}<0$. Since $\mu_i(\xi(t))\ge 0$ and $\sum_{i=1}^{r}\mu_i(\xi(t))=1$, we have

$\sum_{i=1}^{r} \mu_i(\xi(t)) \left(\tilde{A}_{i22}^T \tilde{P}_{22} + \tilde{P}_{22}^T \tilde{A}_{i22}\right) < 0$. This implies that $\sum_{i=1}^{r} \mu_i(\xi(t)) \tilde{A}_{i22}$ is nonsingular. Therefore, the unforced fuzzy singular system (1) is regular and impulse free.

Next, we will show the stability of the system (1). If conditions (2) hold, we have

$$\begin{bmatrix} \sum_{i=1}^{r} \mu_i \left(\tilde{P}_{22}^T \tilde{A}_{i22} + \tilde{A}_{i22}^T \tilde{P}_{22}\right) + \tilde{S}_{1,22} & \tilde{P}_{22}^T \sum_{i=1}^{r} \mu_i \tilde{A}_{\tau i,22} \\ \sum_{i=1}^{r} \mu_i \tilde{A}_{\tau i,22}^T \tilde{P}_{22} & -(1-d)\tilde{S}_{1,22} \end{bmatrix} < 0 \tag{10}$$

Then, pre-multiplying and post-multiplying (10) by $\begin{bmatrix} -\vartheta^T & I \end{bmatrix}$ and its transpose, respectively, (10) yields $\vartheta^T \tilde{S}_{1,22} \vartheta - (1-d)\tilde{S}_{1,22} < 0$, which shows that $\rho(\vartheta) < 1$ holds for all allowable μ_i with $\vartheta = \left(\sum_{i=1}^{r} \mu_i \tilde{A}_{i22}\right)^{-1} \left(\sum_{i=1}^{r} \mu_i \tilde{A}_{\tau i22}\right)$. Now, we define the following Lyapunov-Krasovskii functional for the unforced fuzzy singular system (1),

$$\begin{aligned} V_1(x_t, t) = {} & x^T(t) E^T P x(t) + \sum_{n=1}^{N} \int_{t-nh}^{t-(n-1)h} x^T(s) Q_n x(s) ds + \int_{t-\tau(t)}^{t} x^T(s) S_1 x(s) ds \\ & + \int_{t-\tau_\rho}^{t-\tau_1} x^T(s) S_2 x(s) ds + \int_{t-\tau_2}^{t-\tau_\rho} x^T(s) S_3 x(s) ds \\ & + \int_{-\tau_\rho}^{-\tau_1} \int_{t+\theta}^{t} \dot{x}^T(s) E^T R_1 E \dot{x}(s) ds d\theta \\ & + \sum_{n=1}^{N} \int_{-nh}^{-(n-1)h} \int_{t+\theta}^{t} \dot{x}^T(s) h E^T W_n E \dot{x}(s) ds d\theta \\ & + \int_{-\tau_2}^{-\tau_\rho} \int_{t+\theta}^{t} \dot{x}^T(s) E^T R_2 E \dot{x}(s) ds d\theta \end{aligned}$$

Then, the time derivatives of $V(x_t, t)$ along the trajectories of the system (1) satisfy

$$\begin{aligned} \dot{V}(x_t, t) = {} & x^T(t) \left[P^T A(t) + A^T(t) P\right] x(t) + 2x^T(t) P^T A_\tau(t) x(t-\tau(t)) \\ & + \sum_{n=1}^{N} x^T(t-(n-1)h) Q_n x(t-(n-1)h) - \sum_{n=1}^{N} x^T(t-nh) Q_n x(t-nh) \\ & + x^T(t) S_1 x(t) - (1-\dot{\tau}(t)) x^T(t-\tau(t)) S_1 x(t-\tau(t)) + x^T(t-\tau_1) S_2 x(t-\tau_1) \\ & - x^T(t-\tau_\rho) S_2 x(t-\tau_\rho) + x^T(t-\tau_\rho) S_3 x(t-\tau_\rho) - x^T(t-\tau_2) S_3 x(t-\tau_2) \\ & + \dot{x}^T(t) E^T \left(\sum_{n=1}^{N} h^2 W_n + \rho\delta R_1 + (1-\rho)\delta R_2\right) E \dot{x}(t) \\ & - \sum_{n=1}^{N} \int_{t-nh}^{t-(n-1)h} \dot{x}^T(s) h E^T W_n E \dot{x}(s) ds \\ & - \int_{t-\tau_\rho}^{t-\tau_1} \dot{x}^T(s) E^T (R_1 - Y_{33}) E \dot{x}(s) ds - \int_{t-\tau_\rho}^{t-\tau_1} \dot{x}^T(s) E^T Y_{33} E \dot{x}(s) ds \\ & - \int_{t-\tau_2}^{t-\tau_\rho} \dot{x}^T(s) E^T (R_2 - Z_{33}) E \dot{x}(s) ds - \int_{t-\tau_2}^{t-\tau_\rho} \dot{x}^T(s) E^T Z_{33} E \dot{x}(s) ds \end{aligned} \tag{11}$$

For the case 1, when $\tau_1 \le \tau(t) \le \tau_\rho$, the following equations are true:

$$\begin{aligned} & - \int_{t-\tau_\rho}^{t-\tau_1} \dot{x}^T(s) \hat{Y}_{33} \dot{x}(s) ds - \int_{t-\tau_2}^{t-\tau_\rho} \dot{x}^T(s) \hat{Z}_{33} \dot{x}(s) ds \\ = {} & - \int_{t-\tau_\rho}^{t-\tau(t)} \dot{x}^T(s) \hat{Y}_{33} \dot{x}(s) ds - \int_{t-\tau(t)}^{t-\tau_1} \dot{x}^T(s) \hat{Y}_{33} \dot{x}(s) ds - \int_{t-\tau_2}^{t-\tau_\rho} \dot{x}^T(s) \hat{Z}_{33} \dot{x}(s) ds \end{aligned} \tag{12}$$

By utilizing Lemma 2.1 and the Leibniz-Newton formula, we have

$$\begin{aligned} -\int_{t-\tau_\rho}^{t-\tau(t)} \dot{x}^T(s)\hat{Y}_{33}\dot{x}(s)ds \leq{} & x^T(t-\tau(t))\left[\rho\delta\hat{Y}_{11}+\hat{Y}_{13}+\hat{Y}_{13}^T\right]x(t-\tau(t)) \\ & + 2x^T(t-\tau(t))\left[\rho\delta\hat{Y}_{12}-\hat{Y}_{13}+\hat{Y}_{23}^T\right]x(t-\tau_\rho) \\ & + x^T(t-\tau_\rho)\left[\rho\delta\hat{Y}_{22}-\hat{Y}_{23}-\hat{Y}_{23}^T\right]x(t-\tau_\rho) \end{aligned} \tag{13}$$

Similarly, we obtain

$$\begin{aligned} -\int_{t-\tau(t)}^{t-\tau_1} \dot{x}^T(s)\hat{Y}_{33}\dot{x}(s)ds \leq{} & x^T(t-\tau_1)\left[\rho\delta\hat{Y}_{11}+\hat{Y}_{13}+\hat{Y}_{13}^T\right]x(t-\tau_1)+2x^T(t-\tau_1) \\ & \times\left[\rho\delta\hat{Y}_{12}-\hat{Y}_{13}+\hat{Y}_{23}^T\right]x(t-\tau(t)) \\ & + x^T(t-\tau(t))\left[\rho\delta\hat{Y}_{22}-\hat{Y}_{23}-\hat{Y}_{23}^T\right]x(t-\tau(t)) \end{aligned} \tag{14}$$

$$\begin{aligned} -\int_{t-\tau_2}^{t-\tau_\rho} \dot{x}^T(s)\hat{Z}_{33}\dot{x}(s)ds \leq{} & x^T(t-\tau_\rho)\left[(1-\rho)\delta\hat{Z}_{11}+\hat{Z}_{13}+\hat{Z}_{13}^T\right]x(t-\tau_\rho) \\ & + 2x^T(t-\tau_\rho)\times\left[(1-\rho)\delta\hat{Z}_{12}-\hat{Z}_{13}+\hat{Z}_{23}^T\right]x(t-\tau_2) \\ & + x^T(t-\tau_2)\left[(1-\rho)\delta\hat{Z}_{22}-\hat{Z}_{23}-\hat{Z}_{23}^T\right]x(t-\tau_2) \end{aligned} \tag{15}$$

Substituting (12)-(15) into (11), a straightforward computation gives

$$\begin{aligned} \dot{V}(t) \leq{} & \zeta^T(t)\Theta(t)\zeta(t) - \int_{t-\tau_\rho}^{t-\tau_1} \dot{x}^T(s)E^T(R_1-Y_{33})E\dot{x}(s)ds \\ & - \int_{t-\tau_2}^{t-\tau_\rho} \dot{x}^T(s)E^T(R_2-Z_{33})E\dot{x}(s)ds \end{aligned} \tag{16}$$

where $\zeta^T(t) = [x^T(t)\ x^T(t-h)\ \cdots\ x^T(t-\tau_1)\ x^T(t-\tau_\rho)\ x^T(t-\tau(t))\ x^T(t-\tau_2)]$. When $R_1 - Y_{33} \geq 0$, $R_2 - Z_{33} \geq 0$, and $\tau_1 \leq \tau(t) \leq \tau_\rho$, the last two terms in (16) are all less than 0. Therefore, if the conditions (2)-(7) hold, there exists $\alpha > 0$ such that $\dot{V}(x_t) < \alpha\|x_t\|$. By Lemma 2.2, we conclude that the unforced fuzzy singular system (1) is stable.

For the case 2, when $\tau_\rho \leq \tau(t) \leq \tau_2$, the proof can be completed in a similar formulation to case 1 and is omitted here for simplification. This completes the proof.

Remark 3.1. *By dividing the constant part of time-varying delay $[0, \tau_1]$ into N segments, and the interval $[\tau_1, \tau_2]$ into two unequal variable subintervals $[\tau_1, \tau_\rho]$ and $[\tau_\rho, \tau_2]$, in which ρ is a tunable parameter, a more general and simple Lyapunov-Krasovskii functional is constructed. Different from [5-7, 9, 11] and [12-14], we define different energy functional Q_n in each different segment, and by seeking an appropriate parameter ρ, both the information of delayed state $x\left(t-\frac{n}{N}\tau_1\right)$ $(n = 1, 2, 3, \ldots, N)$ and $x(t-\tau_\rho)$ can be taken into account; therefore, the result can further reduce the analysis and synthesis conservatism.*

Remark 3.2. *In the case when the information of the time-derivative of delay is unknown or the time-delay is not differentiable, and systems are nonsingular systems, just let $S_1 = 0$, $E = I_{n\times n}$ and proceed in a similar manner as the previous proof, the criteria can be obtained from Theorem 3.1. Due to limited space, no more tautology here.*

4. **Numerical Examples.**

In this section, two well-known examples are presented to show the usefulness and effectiveness of the proposed results.

Example 4.1. *Consider a nominal T-S delayed system with two rules as* [5], *and the system matrices are*

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\ A_{\tau 1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},\ A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix},\ A_{\tau 2} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}$$

The upper delay bounds τ_2 derived from [5-7,9,11] and the method proposed in this paper are tabulated in Table 1 under different values of τ_1. It is seen from Table 1 that the results obtained from Theorem 3.1 (d is unknown, set $S_1 = 0$) are significantly better than those obtained from other methods. Moreover, the conservatism is gradually reduced with the increase of N while guaranteeing stability of the considered systems.

Table 1. Comparisons of maximum allowed delay τ_1 for Example 4.1 (d unknown)

Method $\backslash\tau_1$	0	0.4	0.8	1.0	1.2
[7] Corollary 1	–	1.2647	1.3032	1.3528	1.4214
[5] Theorem 1 ($N = 3$)	1.2780	1.3030	1.3160	1.3610	1.4250
[11] Corollary 4	–	1.2836	1.3394	1.4009	1.4815
[9] Theorem 1 ($N = 3$)	1.3800	1.3900	1.4300	–	1.5700
[6] Theorem 4	–	1.5274	1.5361	1.5762	1.6340
Ours $C2$ ($N = 1, \rho = 0.7$)	1.4841	1.6743	1.7794	1.7965	1.7805
Ours $C2$ ($N = 2, \rho = 0.7$)	1.4839	1.6761	1.8001	1.8403	1.8699
Ours $C1$ ($N = 1, \rho = 0.3$)	3.2721	2.5582	2.0346	1.8698	1.7495
Ours $C1$ ($N = 2, \rho = 0.3$)	3.2712	2.6034	2.1798	2.0577	1.9769

Example 4.2. *Consider a continuous fuzzy singular system composed of two rules and the following system matrices:*

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ A_1 = \begin{bmatrix} -3 & 0 & 0 & 0.2 \\ 0 & -4 & 0.1 & 0 \\ 0 & 0 & -0.1 & 0 \\ 0.1 & 0.1 & -0.2 & -0.2 \end{bmatrix},\ A_2 = \begin{bmatrix} -2 & 0 & 0 & -0.2 \\ 0 & -2.5 & -0.1 & 0 \\ 0 & -0.2 & -0.3 & 0 \\ 0.1 & 0.1 & -0.2 & -0.2 \end{bmatrix}$$

$$A_{\tau 1} = \begin{bmatrix} -0.5 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0.1 & -0.2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ A_{\tau 2} = \begin{bmatrix} -0.5 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0.1 & -0.5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Supposing that $\tau(t)$ satisfies $\tau_1 \leq \tau(t) \leq \tau_2$ and with $\tau_1 = 2$. We apply Theorem 3.1 to calculating the maximal allowable value τ_2 that guarantees the asymptotical stability of the considered system in Cases 1 and 2, respectively. Since the proposed analysis used a delay-central point method as well as tighter bounding on the time derivative of LKF.

Table 2. Comparisons of maximum allowed delay τ_2 for Example 4.2 ($\tau_1 = 2$)

Method $\backslash d$	0.1	0.35	0.6	0.85	0.9	0.95
[12] Theorem 1	3.3623	2.9810	2.6010	1.8330	1.0380	–
[13] Theorem 3	3.3685	3.1560	3.1510	3.0760	2.6750	2.0780
[14] Theorem 1 ($N = 2$)	3.6761	3.4755	3.3580	3.2425	3.0737	2.8257
Ours $C2$ ($N = 1, \rho = 0.95$)	3.7445	3.7553	3.7566	3.7573	3.7550	3.7546
Ours $C2$ ($N = 2, \rho = 0.95$)	3.8070	3.8066	3.8064	3.8045	3.8065	3.8072
Ours $C1$ ($N = 1, \rho = 0.45$)	5.2484	4.4255	4.1246	4.0834	4.0663	4.0552
Ours $C1$ ($N = 2, \rho = 0.45$)	5.3596	4.5894	4.3146	4.2472	4.2140	4.1870

Therefore, from the comparison results with various d in Table 2, it is easy to see that the condition in Theorem 3.1 gives less conservative results than those in [12-14].

5. **Conclusions.** This paper has investigated the asymptotical stability problem for T-S fuzzy singular systems with interval time-varying delays. Based on the improved delay partitioning approach, several new stability criteria have been derived by constructing an appropriate LKF. Finally, some examples have been given to demonstrate the effectiveness and less conservatism of the proposed method. Further, the delay partitioning method in this paper can also be used to solve some other interesting issues such as H_∞ control, and fault-tolerant control. Our future work will focus on improving the proposed method to deal with the above mentioned problems.

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