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RAPID PLANNING OF RELIABLE REENTRY TRAJECTORY

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Abstract. *In order to satisfy the requirement of reentry guidance of lifting vehicles, an improved method of generating reliable reentry trajectory is proposed in this paper. It is found that, the premise of applying Quasi-Equilibrium Glide Condition (QEGC) does not hold if the rate of path angle is not small enough. The bank angle profile obtained based on QEGC will be non-conservative, and then the generated reentry trajectory maybe cross the reentry corridor. In our study, the reentry trajectory should satisfy path constraints and terminal constraints during the searching process of bank angle. Meanwhile, the rate of path angle must remain a small quality. It is also considered as a constraint during the above searching. The simulation results demonstrate that the proposed method has a favorable adaptability and can improve the reliability of reentry trajectory.*
Keywords: Reentry trajectory planning, Quasi-Equilibrium Glide Condition (QEGC), High-reliability

1. **Introduction.** The purpose of reentry guidance is to make the vehicle follow a predetermined trajectory and arrive at a specified position on ground safely. One typical reentry guidance design includes: to generate a reference trajectory for a specific mission and to design a feedback control law for tracking the reference trajectory. The reference trajectory here is the trajectory from the edge of atmosphere to the entrance point of Heading Alignment Circle (HAC). In order to design the feasible trajectory, path constraints and terminal conditions should be enforced strictly. Path constraints are composed of the heat rate at the stagnation point, overload and dynamic pressure. The terminal conditions are composed of the altitude and velocity. The main purpose of this paper is to generate a reference reentry trajectory which can meet the above requirements.

A lot of studies to obtain feasible reentry trajectory have been made since 1950s, such as direct optimization method represented by Gauss Pseudospectral Method [1,2], the direct shooting method [3], and intelligence algorithm [4,5]. The Quasi-Equilibrium Glide Condition (QEGC) is an ideal solution to plan the reentry trajectory [6-8]. QEGC is used to transform the path constraints to the upper and lower boundary of control variables directly. Qualified control variables can be obtained through a linear search method. This method can ensure the planning efficiency to satisfy the onboard guidance requirement. It is obvious that the precondition of the method above is that both path angle and its rate shall be very small along a major portion of the trajectory. Nevertheless, we found that the above assumed condition is not conservative, which means the path angle variation rate might be rather large without imposing any control. In order to ensure the effectiveness of the above method, the precondition must be valid. During the searching of control variables, not only shall the trajectory satisfy the path constraints and terminal conditions, but also the variation rate of path angle shall be maintained at minimum.

Accordingly, this paper presents an improved method with QEGC to generate the reentry trajectory. Firstly, reentry problem is introduced in Part 2, which gives description of the dynamic equation and path conditions in detail. Part 3 explains the steps of the method based on QEGC to generate reentry trajectory. Then, the problem of the existing method is pointed out in Part 4. Moreover, this paper illustrates the improvement to the above problem through adjusting bank angle to guarantee premise of QEGC. Conclusions are given in Part 5.

2. **Problems Description.**

2.1. **Reentry dynamic equations.** The point-mass dimensionless reentry dynamics equations ignoring self-rotation of Earth are given as Reference [6]. The six dimensionless motion states, r is distance from Earth center to the vehicle; θ and ϕ are longitude and latitude measured in radian, respectively; V is Earth-relative velocity; γ is the flight path angle measured in radian; ψ is velocity azimuth angle in radian.

The range-to-go S_{togo}, which represents the great circle distance from the current location to HAC point, is chosen as the independent variable and given by

$$\dot{S}_{togo} = -\frac{V\cos\gamma\cos\Delta\psi}{r} \tag{1}$$

where $\Delta\psi$ is the angle between the velocity azimuth angle and the angle of the line-of-sight to the HAC point. It is very small during major trajectory and thus $\cos\Delta\psi \approx 1$. The longitudinal dynamic equations with respect to S_{togo} are

$$\dot{r} = -r\tan\gamma \tag{2}$$

$$\dot{V} = \frac{rD}{V\cos\gamma} + \frac{\tan\gamma}{rV} \tag{3}$$

$$\dot{\gamma} = -\frac{r}{V^2}\left[\frac{L\ \cos\sigma}{\cos\gamma} + \left(V^2 - \frac{1}{r}\right)\frac{1}{r}\right] \tag{4}$$

where $D = \rho v^2 SC_D/(2mg_0)$ and $L = \rho v^2 SC_L/(2mg_0)$ represent the non-dimensional drag and lift accelerations. The magnitude of gravity $g_0 = 9.81$ m/s^2. ρ is the atmospheric density related with altitude h from sea level. v is the flight velocity. S is the reference area of the entry vehicle and m is the mass. The lift and drag coefficients C_L and C_D are presented as functions of angle of attack α and velocity v. Additionally, two control variables are bank angle σ and attack angle α.

2.2. **Reentry path constraints.** The reentry path constraints for the vehicle include: heat rate, dynamic pressure and overload.

$$\dot{Q} = \frac{C}{\sqrt{R}}\rho^{1/2}V^3 \leq \dot{Q}_{\max} \tag{5}$$

$$q = \frac{1}{2}\rho V^2 \leq q_{\max} \tag{6}$$

$$n = \sqrt{L^2 + D^2} \leq n_{\max} \tag{7}$$

where C is a constant and R is radius of the stagnation point. $C = 30.5$, $R = 0.15$ m. The right terms $\dot{Q}_{\max}$, $q_{\max}$ and $n_{\max}$ of the above inequalities represent the maximum allowable heating rate of stagnation point, dynamic pressure and overload respectively. Set $\dot{Q}_{\max} = 2.8 \times 10^7$ W/m^2, $q_{\max} = 1.0 \times 10^5$ N/m^2, $n_{\max} = 2.15$. These inequalities illustrate the constraint relationship between altitude and flight velocity and they can form the lower boundary of flight corridor in $h - v$ space.

The QEGC is considered as a constraint to make the flight trajectory as smooth as possible.

$$\left(\frac{1}{r} - V^2\right)\frac{1}{r} - L\cos\sigma_{EQ} \leq 0 \tag{8}$$

where the term σ_{EQ} denotes a constant bank angle. During the flight from the initial reentry point to the entrance point of HAC, both flight path angle γ and its rate $\dot{\gamma}$ should be remained small. The altitude and flight velocity in Equation (8) forms the upper boundary of flight corridor. The corridor will be given during the process of trajectory planning.

3. **Reentry Trajectory Planning.** Reentry trajectory planning is to generate reentry trajectory satisfying the above path constraints and terminal conditions by seeking suitable control variables. The specific steps to generate the trajectory are given in this section. Table 1 shows the initial and terminal conditions for reentry.

Table 1. The initial and terminal conditions for reentry

Variables	Initial conditions	Terminal conditions
Altitude, km	120	18-20
Longitude, degree	0	55
Latitude, degree	0	-32
Velocity, m/s	7350	1360
Flight path angle, degree	-1	≤ 5
Azimuth angle, degree	90	≤ 8

The following equation can be obtained by assuming that both flight path angle γ and its rate $\dot{\gamma}$ are zero in Equation (4):

$$0 = L\cos\sigma + \frac{1}{r}\left(V^2 - \frac{1}{r}\right) \tag{9}$$

Equation (9) is the so-called Quasi-Equilibrium Glide Condition (QEGC). Using the QEGC, once the profile of attack angle is given, the relationship among altitude, velocity and bank angle is determined. In other words, along a trajectory where QEGC holds, the corresponding σ can be calculated if altitude and velocity are given. In this paper, the profile of attack angle α is predetermined by off-line optimization through the genetic algorithm in Reference [9].

As a result, the reentry boundary in $h - v$ space is transformed into the boundary of bank angle σ through Equation (9). In addition, we obtain the required terminal bank angle σ_f with terminal altitude r_f and velocity V_f. With initial value σ_0 and terminal value σ_f, profile of bank angle is designed as piecewise linear function of velocity, setting $\sigma = \sigma_{mid}$ at the turning point. Bank angle profile is the very appropriate one if the integral result of dynamic equations satisfies terminal conditions. During the search, the above boundary constraint on σ should be met. In case that the profile of bank angle σ exceeded the limit, boundary value will be taken as the current bank angle. With the above steps, a feasible profile of bank angle can be determined as shown in Figure 1.

The reentry trajectory shown in Figure 2 can be generated through numerical integration with attack angle profile and bank angle profile. The space formed by lines of heat rate, overload, dynamic pressure and QEGC in Figure 2 is the so-called "reentry corridor" defined by Equations (5)-(8).

4. **Problems Analysis and Improvement.**

4.1. **Problems analysis.** The reentry trajectory has crossed the reentry corridor which is obvious in Figure 2. That means this trajectory is infeasible. The reason for this phenomenon might be caused by setting $\gamma = 0$ and $\dot{\gamma} = 0$ in Equations (4) and (9). It is found that the flight path angle is small during the whole reentry and can be considered as zero, while its rate $\dot{\gamma}$ as Figure 3 varies from -0.6 to 1.1, equivalent to the value of the

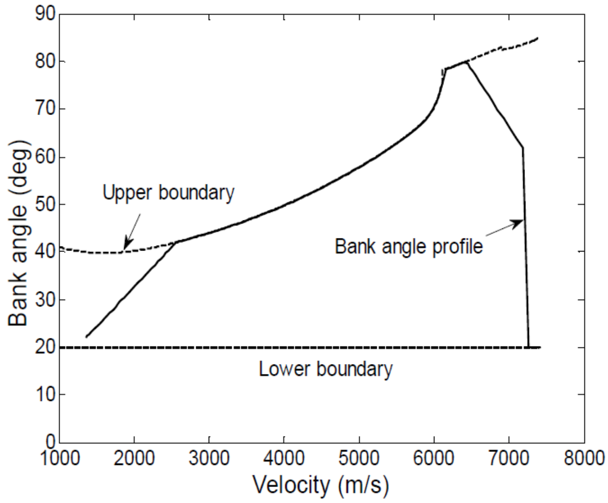

FIGURE 1. Boundary and profile of bank angle

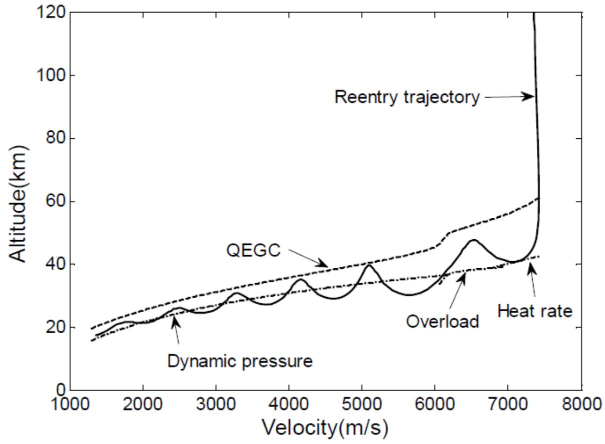

FIGURE 2. Reentry corridor and reentry trajectory

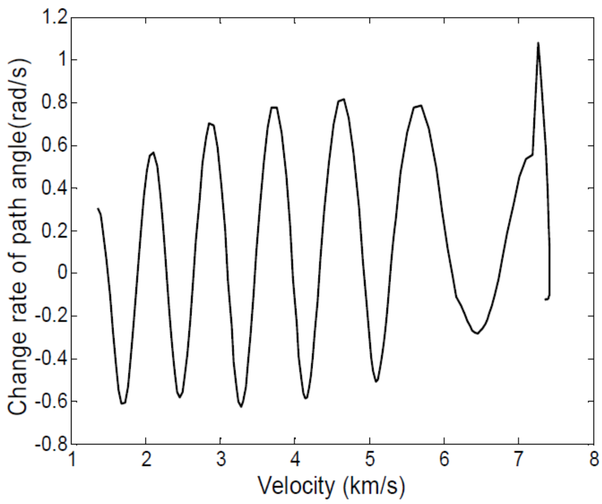

FIGURE 3. Rate of flight path angle

dimensionless state variable, especially at the later Quasi-Equilibrium Glide stage. It is obvious that the $\dot{\gamma}$ here cannot be treated as zero.

To verify the validity of the above conclusion, the upper boundary of bank angle is recalculated with the value of $\dot{\gamma}$ shown in Figure 4. As expected, the previously planned

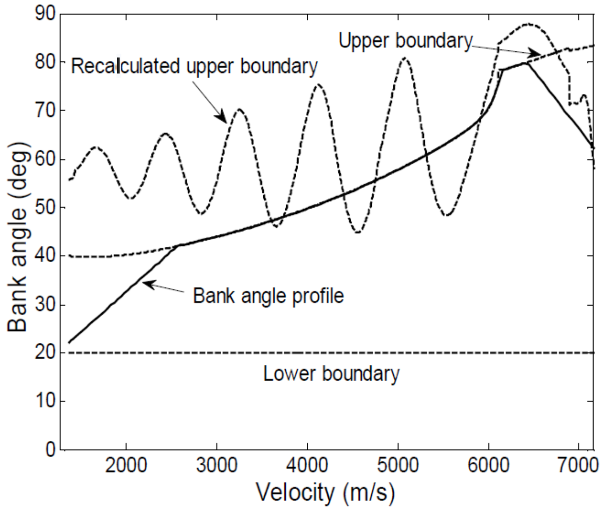

Figure 4. Recalculated upper boundary of bank angle

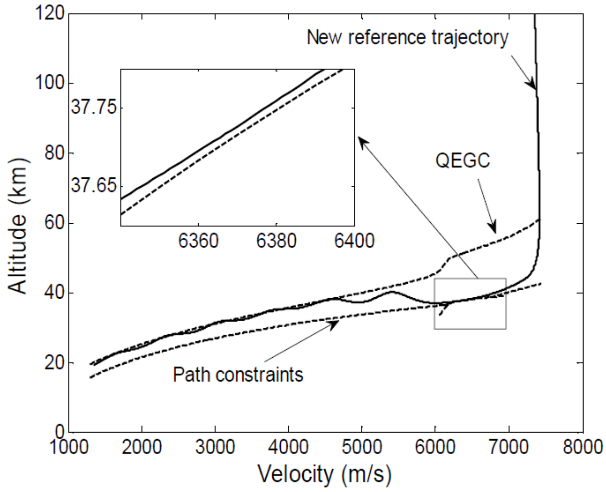

Figure 5. The new reentry trajectory

bank angle profile crosses the upper boundary, which coincides with the phenomenon in Figure 2. It can be concluded that the reentry trajectory obtained by the above method is not conservative. In other words, to obtain a reliable profile of bank angle, the $\dot{\gamma}$ must be remained small. Next part, the improvement is proposed aiming at the inappropriateness in this method.

4.2. **Improvement.** The vehicle enters the Quasi-Equilibrium Glide stage after transition point. If we set $\dot{\gamma} = 0$ at transition point, the corresponding bank angle σ_0 can be obtained and taken as the initial value for subsequent searching. During the linear search, δ is defined as a small constant and will be the upper limit of path angle variation rate, i.e., $\dot{\gamma} \leq \delta$. Once the path angle rate $\dot{\gamma}$ reaches δ, reset $\dot{\gamma} = 0$ and solve σ according to Equation (9). Then take the solved bank angle as the initial value for later searching. Otherwise, the searching will continue. Repeat the above searching process until the terminal requirements are met.

According to the above method, the trajectory is obtained by numerical integration with bank angle and attack angle as shown in Figure 5. It is gratifying that the reentry trajectory does not cross the flight corridor. Furthermore, the terminal conditions can be satisfied simultaneously.

5. Conclusions. The planning method of reentry trajectory based on QEGC is studied in this paper. It should be noted that the reentry trajectory generated by this method is non-conservative and might cross the reentry corridor. An improved method of generating reliable reentry trajectory is proposed aiming at the above problem. In this method, the rate of path angle also is considered as a constraint and remains a small quality during searching the bank angle. The simulation results illustrate that the method proposed in this paper can improve the reliability of reentry trajectory.

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