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A STUDY ON THE TRACKING CONTROL OF OIL WELL'S WORKING FLUID LEVEL BASED ON THE STATE-DEPENDENT SWITCHING TECHNIQUE

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Abstract. *In this paper, in order to track the reference working fluid level signal, a switching controller is introduced, whose parameters can be adjusted appropriately according to the variation of the working fluid level. By the aid of single Lyapunov function method, the stability of the switched system can be analyzed, and a switching controller can be designed such that the tracking error converges to be an arbitrary small neighborhood of zero by tuning design parameters. An experiment is given to illustrate the effectiveness of the switching controller.*

Keywords: Working fluid level, Switching control system, Oil well

1. **Introduction.** An oil well needs the highest possible submergence depth on the premise that the liquid production and the effective stroke of pump keep a high level. The submergence depth of pump must be in a reasonable range in the production of oil well [1]. It is very significant that the working fluid level of oil well is measurable and controllable to guarantee the submergence depth in a reasonable range based on closed-loop control [2,3]. According to the technical requirements of optimal operation of oil wells, in this paper, a closed-loop control system of working fluid level and submergence depth is presented to solve the common problems that bottom hole flowing pressure changes and formation pressure is instable. The switching control technique is adopted to solve the problem of working fluid level model changes which is caused by bottom hole flowing pressure changes. Meanwhile, this control method could reduce the influence of irregular formation pressure disturbance on control quality. The relationship between the submergence depth and the rate of pump will be established by the Inflow Performance Relationship Curve, pressure gradient equations, and parameter combination of sucker rods. A closed-loop feedback switched control system is established. The rate of pump to control the dynamic oil level such that the submergence depth of pump changes in a reasonable range.

2. Control System Design.

2.1. The mathematical model of working fluid level.

Inflow performance represents the fluid supply capacity from oil reservoir to the well. It is the basis of well productivity prediction and optimization of pumping systems. Inflow Performance Relationship Curve (IPR Curve) is the most common method to describe the well inflow performance.

At the beginning of exploitation, most of the oil wells were exploited through the method of natural energy mining because of the high flowing pressure. When bottom hole flowing pressure (BHFP) P_{wf}, is higher than saturation pressure P_b, oil flows in single phase in the reservoir. When the flow pressure is lower than the saturation pressure, dissolved gas around the well strata is separated from oil. The oil and gas flow to the bottom of the

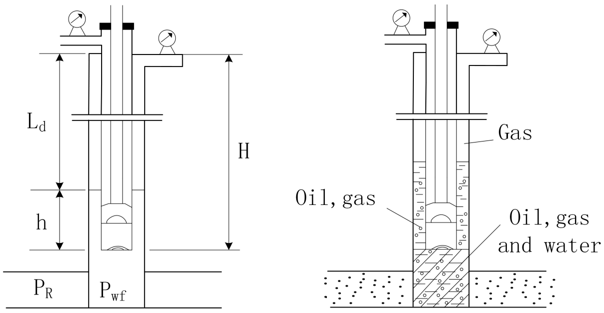

FIGURE 1. The distribution schematic of oil, water and gas in well bores

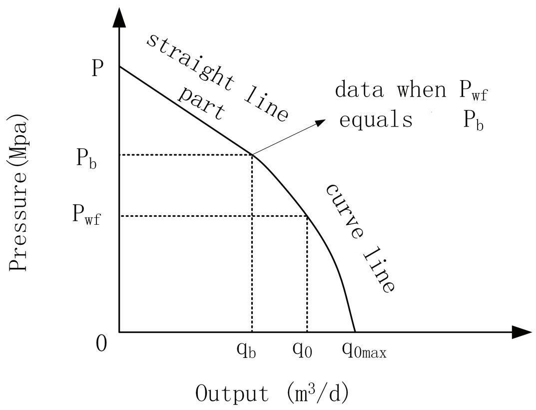

FIGURE 2. IPR curve

well under the pressure. Especially in the oil field developed by water injection, when the pressure is under the saturation pressure, oil, gas and water will flow three-phase simultaneously around the oil wells. For the well in which the average formation pressure, P_R is greater than the saturation pressure, P_b, IPR curve of the well is divided into two parts (Figure 2).

$q_{0\max}$ is the maximum theoretic oil-leakage under the condition that P_{wf} equals zero in m^3/d.

IPR Curve equation could be expressed as the following form:

1. Single-phase flow part

$$q_0 = \frac{q_b(P_R - P_{wf})}{P_R - P_b} \tag{1}$$

2. Multiphase flow part

$$q_0 = q_b \left[1 + \frac{q_b(P_R - P_{wf})}{P_R - P_b} + B\frac{(P_b - P_{wf})^2}{P_b(P_R - P_b)}\right] \tag{2}$$

where q_0 is formation permeability oil in m^3/d, P_{wf} is bottom hole flowing pressure in Pa, P_R is average formation pressure in Pa, P_b is saturation pressure of gas dissolving in Pa, B is a coefficient of the well which is constant for one well, q_b is flow rate in m^3/d when saturation pressure equals P_b.

According to the calculation of pressure in annular space, the result is

$$P_{wf} = P_c + (\rho_0 g - \rho_g g)h + \rho_g gH + P_L \tag{3}$$

where P_c is casing pressure in Pa, ρ_g is relative density of the gas phase in kg/m^3, ρ_0 is density of the cruel oil in kg/m^3, g is gravitational acceleration, H is depth of pump in m, and P_L is pressure from oil layer to the entrance of the pump in Pa. The above parameters are given by the well data. And h is submergence depth, $h = H - L_d$, and L_d is dynamic liquid level height in m measured by the method of indicator diagrams.

According to the oil well production formula [4].

$$q_{pump} = 1440 A_p S_e N \eta_p B_v \tag{4}$$

where η_p is discharge coefficient obtained by looking up table, N is rate of pump in $\min^{(-1)}$, A_p is the sectional area of plunger in m^2, S_e is effective stroke in m, and B_v is volume coefficient which refers to the ratio of volume of reservoir oil and volume of reservoir oil after degassing on ground, and usually equals 1.

According to the material balance principle in the well bore, the following equation is obtained.

$$q_0 \Delta t - q_{pump} \Delta t = \pi \left(d_{ci}^2 - d_{ie}^2\right) \Delta h \tag{5}$$

where d_{ci} and d_{ie} refer to inner casing diameters and outer tubing diameter respectively in m, and Δh is the variation of the height of submergence depth in the well bore in m within Δt times.

When Δt is small, the type can be written as the following differential form

$$d_h/d_t = (q_0 - q_{pump})/\pi \left(d_{ci}^2 - d_{ie}^2\right) \tag{6}$$

2.2. **A switching model of the system.** The system is affected by the step disturbance, $\left(P_R = \overline{P_R} + \varepsilon_R\right)$, caused by formation pressure fluctuation because of the existence of unpredictable changes of formation pressure. In order to simplify mathematical model, all parameters obtained from oil well measurement data are merged into constant coefficients.

1) Single-phase flow part

There is simplified equation:

$$d_h/d_t = K_{in1}(C_{in1} - h)/(P_R + \delta_R - P_b) - K_{out1} N \tag{7}$$

where K_{in1}, C_{in1}, K_{out1} and P_b are constant coefficients obtained by above formulas and given by the well data, N is rate of pump, δ_R is step disturbance, P_R is average formation pressure, and h is corresponding submergence depth measured by indicator diagram method.

2) Multiphase flow part

There is simplified equation:

$$d_h/d_t = K_{in2}\left(-h^2 + b_{in} h + C_{in2}\right)/(P_R + \delta_R - P_b) - K_{out1} N \tag{8}$$

where K_{in2}, C_{in2} and b_{in} are constant coefficients obtained by above formulas and given by the well data.

Establish switched system, the state $x(t) = h$, input $u_{\sigma(h)} = N$, and the output is $y(t)$. We obtain

$$\dot{x}(t) = A_{\sigma(h)} + B_{\sigma(h)} x(t) + C_{\sigma(h)} + D_{\sigma(h)} u_{\sigma(h)}(t) \tag{9}$$

$$y(t) = x(t) \tag{10}$$

According to Equations (9) and (10), when the subsystem is Z_1 (single-phase flow part), we have $A_1 = 0$, $B_1 = -K_{in1}/(P_R + \delta_R - P_b) < 0$, $C_1 = c_{in1} K_{in1}/(P_R + \delta_R - P_b)$, $D_1 = -K_{out1} < 0$; when the subsystem is Z_2 (multiphase flow part), we have $A_2 = -a_{in} K_{in2}/(P_R + \delta_R - P_b) < 0$, $B_2 = b_{in} K_{in2}/(P_R + \delta_R - P_b)$, $C_2 = c_{in2} K_{in1}/(P_R + \delta_R - P_b)$, $D_2 = -K_{out1} < 0$.

Due to the pressure gauge measuring precision and formation pressure disturbance factors, there are errors with P_{wf} that the flowing bottom hole pressure. Switch the system when P_{wf} is in the range between $P_b - \delta_P$ and $P_b + \delta_P$. Set

$$\psi_1^d = \{h : P_{wf}(h) < P_b - \Delta P\} \quad \psi_2^d = \{h : P_{wf}(h) > P_b - \Delta P\} \quad \partial = R^n / \left(\psi_1^d \cup \psi_2^d\right)$$

Obtain $R^n = \psi_1^d \cup \psi_2^d \cup \partial$, and the switching signal can be defined as

$$\sigma(t) = \begin{cases} 1, & h \in \psi_1^d \\ 2, & h \in \psi_2^d \\ \sigma(t^-), & h \in \partial \end{cases} \tag{11}$$

$\sigma(t^-)$ denotes the left limit of σ at time t. Without loss of generality, set $x_0 \in \psi_1^d$.

For any $I > 0$, use the method of recursive definition switching times which are as follows.

$\tau_0 = t_0$, $\eta_{1,l} = inf\{t : \tau_0 \le t, | \phi_1(t, \tau_0) * x_0 | \ge l\}$, $\eta_{1,\infty} = \lim_{l\to 0} \eta_{1,l}$

$\tau_1 = inf\left\{t : \tau_0 \le \eta_{1,l}, \phi_1(t, \tau_0) * x_0 \in \psi_2^d\right\}, \cdots,$

$\eta_{1,l} = inf\left\{t : \tau_{j-1} \le t, | \phi_{\frac{3+(-1)j}{2}}(t, \tau_{j-1}) * \cdots * \phi_1(t, \tau_0) * x_0 | \ge l\right\}$

$\eta_{1,\infty} = \lim_{l\to 0} \eta_{j,l}$

$\tau_j = inf\left\{t : \tau_{j-1} \le \eta_{1,\infty}, \phi_{\frac{3+(-1)j}{2}}(t, \tau_{j-1}) * \cdots * \phi_1(t, \tau_0) * x_0 \in \psi_{\frac{3+(-1)(j+1)}{2}}^{\;d}\right\}$

When the definition of $\eta_{(j,l)}$ contains a special situation of $inf\phi = \infty$, and the definition of τ_j contains a special situation of $inf\Phi = \eta_{j,\infty}$, $\Phi_i(t_2, t_1)$ is the system flow from t_1 to t_2. The definition of $1 \le j \le j^*$, j^* as follows

$$j^* = \begin{cases} j_0, & \text{if there is a limited integer } j_0, \text{ make } \tau_{j0} = \eta_{j0,\infty} < \infty \\ \infty, & \text{other situation} \end{cases} \tag{12}$$

means that

$$\tau_j^* = \begin{cases} \tau_{j0}, & \text{if there is a limited integer } j_0, \text{ make } \tau_{j0} = \eta_{j0,\infty} < \infty \\ \infty, & \text{other situation} \end{cases} \tag{13}$$

So the maximum switching times is $j^* - 1$, for the switching system (12) the largest interval is $\left[t_0, \tau_j^*\right)$. From what has been discussed above

$$x(t) = \phi_{\frac{3+(-1)j}{2}}(t, \tau_{j-1}) * \cdots * \phi_1(t, \tau_0) * x_0, \ \forall t \in [\tau_{i-1}, \tau_j), \ j = 1, \cdots, j^* \tag{14}$$

2.3. **Stability analysis.** Consider the switched systems containing disturbance

$$\dot{x}(t) = f_\sigma(x, u) + g_\sigma(x, u)\xi(t), \quad x(t) \in \psi_\sigma \tag{15}$$

where $x \in R^n$, $u \in R^m$ is the system input, $f_\sigma, g_\sigma : R^n \times R^m \to R^n$ are locally Lipschitz functions, and $\xi(t)$ is disturbance.

Definition 2.1. *If there exists a KL-function β, a K-function γ and a nonnegative constant number d, for any bounded input $u(t)$ whose initial state is $x(0)$ and is defined in $[0, \infty)$, Solution $x(t)$ of system (15) exists for any $t \ge 0$ and meets*

$$|x(t)| \le \beta(|x(0)|, t) + \gamma\left(sup_{t\ge 0} |\xi(t)|\right) + d, \ \forall t \ge 0 \tag{16}$$

The system (15) is input-to-state practically stable (ISpS) [4].

Theorem 2.1. *For system (15) if there exists function $V_i \in C^1$, K_∞ like function α_1, α_2, α, γ and constant number $d \ge 0$, ξ and $V(x)$ meets*

$$\alpha_1(|x|) \le V(x) \le \alpha_2(|x|) \tag{17}$$

$$\dot{V}|_i = \frac{\partial V}{\partial x} f_i(x) + d\left|\frac{\partial V}{\partial x} g_i(x, t)\right|^2 \le -\alpha(|x|) + \gamma(|\xi(t)|), \ \forall x(t) \in \psi_i, \ i \in I \tag{18}$$

The system is ISpS.

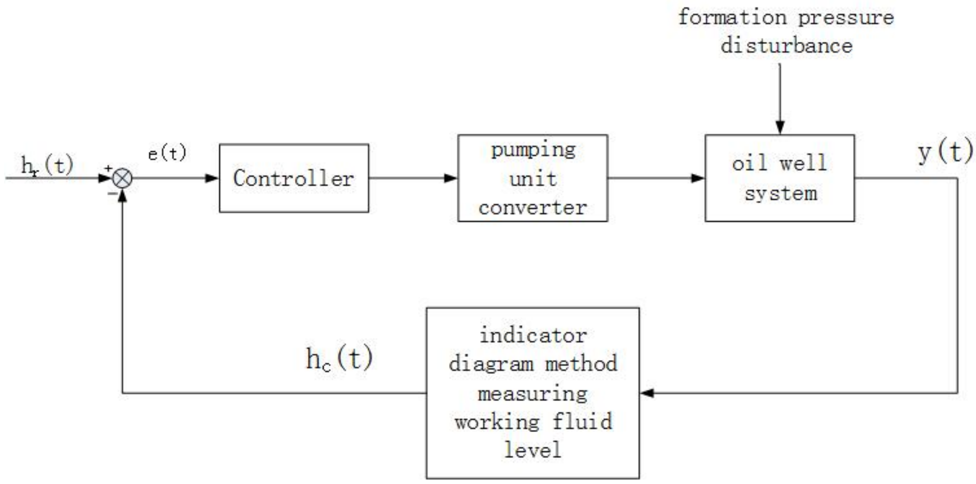

Figure 3. Working fluid level control system

2.4. **Controller design.** As proved above, a stable controller can be designed for closed loop feedback system of well working fluid level. Set the reference input signal as $h_r(t)$, and get working fluid level height $h_c(t)$ by indicator diagrams, with error

$$e(t) = h_r(t) - h_c(t) = h_r(t) - y(t) = h_r(t) - x(t) \tag{19}$$

$$\dot{e} = \dot{e}(t) = \dot{h}_r(t) - \left(A_{\sigma(h)}x(t)^2 + B_{\sigma(h)}x(t) + C_{\sigma(h)} + D_{\sigma(h)}u_{\sigma(h)}(t)\right) \tag{20}$$

Instead $x(t) = h_r(t) - e(t)$.
Reform as

$$\begin{aligned} \dot{e} &= \dot{e}(t) \\ &= \dot{h}_r(t) - A_{\sigma(h)}h_r^2(t) - A_{\sigma(h)}e^2(t) + 2A_{\sigma(h)}h_r(t)e(t) \\ &\quad - B_{\sigma(h)}h_r(t) + B_{\sigma(h)}e(t) - C_{\sigma(h)} - D_{\sigma(h)}u_{\sigma(h)}(t) \end{aligned} \tag{21}$$

Choose Lyapunov function [5] as

$$V(e) = \frac{1}{2}e^2 \tag{22}$$

$$\dot{V}(e) = e\dot{e} \tag{23}$$

Plug (19) into Equation (22), and sort as (23).

$$\begin{cases} \dot{V}_1 = B_1e^2 + e\left(\dot{h}_r - B_1h_r - C_1 - D_1u_1(t)\right) \\ \dot{V}_2 = 2A_2h_re^2 + e\left(\dot{h}_r - A_2h_r^2 - A_2e^2 - B_2h_r + B_2h_r + B_2 - C_2 - D_2u_2(t)\right) \end{cases} \tag{24}$$

Because of

$$\frac{1}{\overline{P}_R + \delta_R - P_b} = \frac{1}{\overline{P}_R - P_b} - \frac{\delta_R}{\left(\overline{P}_R + \delta_R - P_b\right)\left(\overline{P}_R - P_b\right)}$$

the real coefficients M_1, M_2, T_1, T_2 are introduced into the model, which are always greater than 0, then

$$\begin{cases} \dot{V}_1 = -M_1e^2 + e\left(M_1e + h_r + K_{in1}(h_r - e - c_{in1})/\left(\overline{P}_R - P_b\right) + K_{out1}u_1(t)\right) \\ \qquad + eT_1K_{in1}(h_r - e - c_{in1}) * \delta_R/\left[T_1\left(\overline{P}_R + \delta_R - P_b\right)\left(\overline{P}_R - P_b\right)\right] \\ \dot{V}_2 = -M_2e^2 + e(M_2e + h_r + K_{in2}\left(a_{in}(h_r - e)^2\right) - b_{in}(h_r - e \\ \qquad - c_{in2})/\left(\overline{P}_R - P_b\right) + K_{out1}u_1(t)) + eT_2K_{in2}\left(a_{in}(h_r - e)^2 - b_{in}(h_r - e)\right. \\ \qquad \left. -\ c_{in2}\right) * \delta_R/\left[T_2\left(\overline{P}_R + \delta_R - P_b\right)\left(\overline{P}_R - P_b\right)\right] \end{cases} \tag{25}$$

Known from the analysis of stability, the controller design needs set $\dot{V}_{\sigma(h)}(e) < 0 + \delta$ under the situation of disturbance. We set

$$\begin{cases} u_1(t) = \left[M_1 e + h_r + K_{in1}(h_r - e - c_{in1})/\left(\overline{P}_R - P_b\right) + 0.5T_1^2 K_{in1}^2 e(h_r - e - c_{in1})^2\right]/(-K_{out1}) \\ u_2(t) = \left(\begin{array}{c} M_2 e + h_r + K_{in2}\left[a_{in}(h_r - e)^2 - b_{in}(h_r - e) - c_{in2}\right]/\left(\overline{P}_R - P_b\right) \\ +0.5T_2^2 K_{in2}^2 e\left[a_{in}(h_r - e)^2 - b_{in}(h_r - e) - c_{in2}\right]^2 \end{array}\right)/(-K_{out1}) \end{cases} \tag{26}$$

and get

$$\begin{cases} \dot{V}_1 = -M_1 e^2 + 0.5\dfrac{1}{T_1^2}\left(\dfrac{\delta_R}{\left(\overline{P}_R + \delta_R - P_b\right)\left(\overline{P}_R - P_b\right)}\right)^2 < 0.5\dfrac{1}{T_1^2}\left(\dfrac{\delta_R}{\left(\overline{P}_R + \delta_R - P_b\right)\left(\overline{P}_R - P_b\right)}\right)^2 \\ \dot{V}_2 = -M_2 e^2 + 0.5\dfrac{1}{T_2^2}\left(\dfrac{\delta_R}{\left(\overline{P}_R + \delta_R - P_b\right)\left(\overline{P}_R - P_b\right)}\right)^2 < 0.5\dfrac{1}{T_2^2}\left(\dfrac{\delta_R}{\left(\overline{P}_R + \delta_R - P_b\right)\left(\overline{P}_R - P_b\right)}\right)^2 \end{cases} \tag{27}$$

Theorem 2.2. [6] *For the disturbance system (15), we design a controller u_σ to make the output y gradual tracking to a reference signal γ_r and tune the parameters to make tracking error converge to a small neighborhood of zero, namely $e \to 0 \pm \varepsilon$, $t \to \infty$.*

The value adjustment of M_1, M_2, T_1, T_2 can adjust the performance of the controller and the influence of the size of the errors. Make the system input to actual stability of the state.

3. **Simulation.**

The simulation parameters are selected as follows:

$A_p = 0.057$; $S_e = 1.5$; $\eta_p = 0.6$; $B_V = 1$; $\pi = 3.14$; $d_{ci} = 0.1778$; $d_{ie} = 0.107$; $q_b = 52.6$; $q_{0\max} = 176.9$; $\rho_0 = 771$; $\rho_g = 1$; $g = 9.8$; $B = -0.25$; $h_0 = 400$; $H = 1500$; $P_R = 9.65 * 10^6$; $P_b = 6.89 * 10^6$; $P_C = 1.0 * 10^6$; $P_L = 4.02 * 10^6$; $\triangle p = 0.15 * 10^6$.

And select $M_1 = 310$; $M_2 = 280$; $T_1 = 0.12 * 10^{(-8)}$; $T_2 = 0.76 * 10^{(-7)}$.

The simulation results are shown in the following figures.

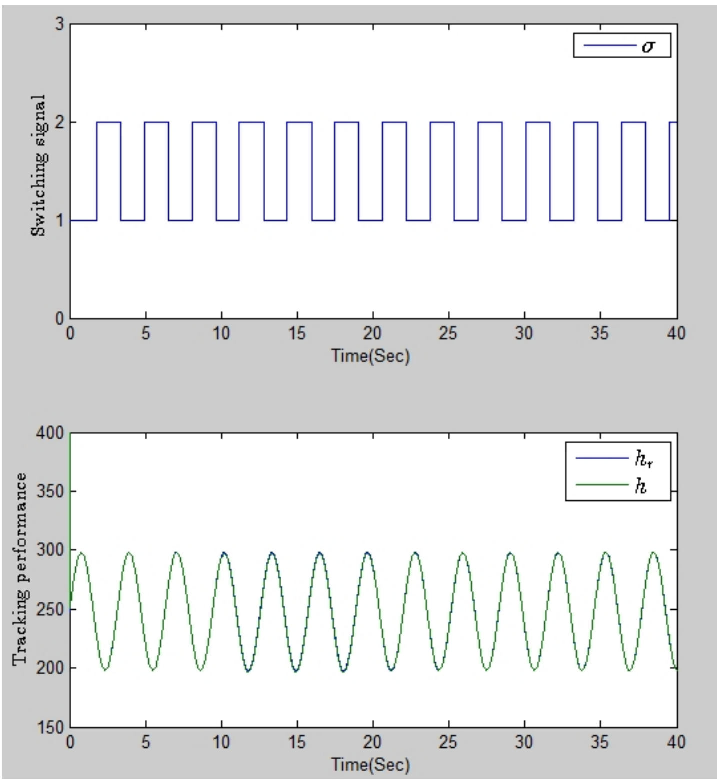

FIGURE 4. Switching signal diagram and the reference level compared with the actual level diagram

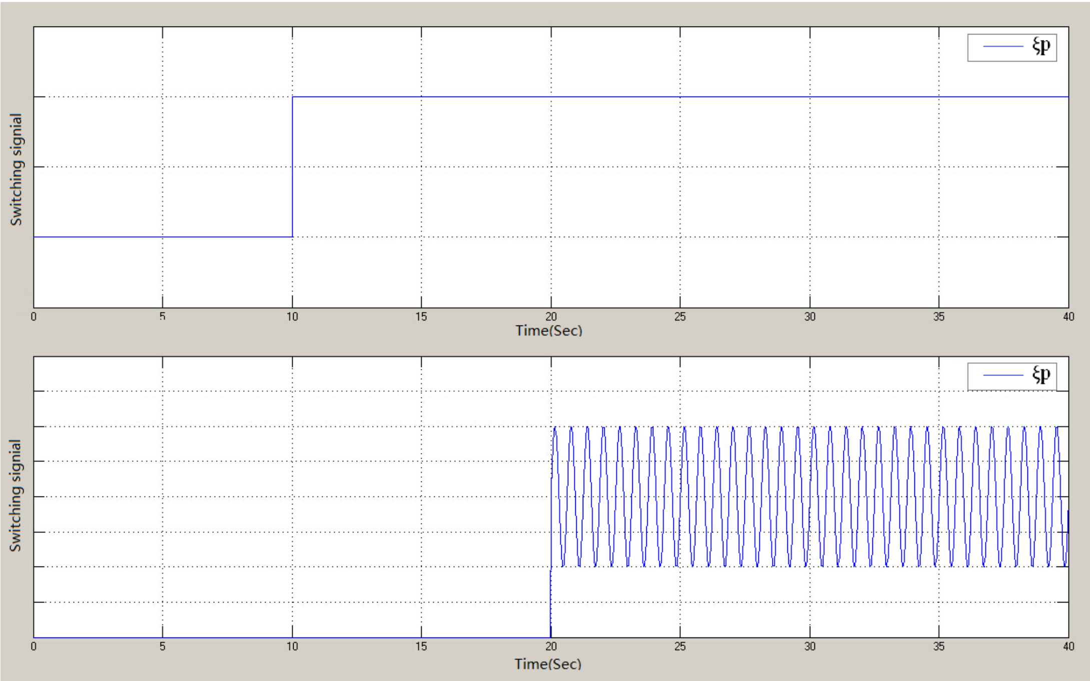

Figure 5. The disturbance diagram

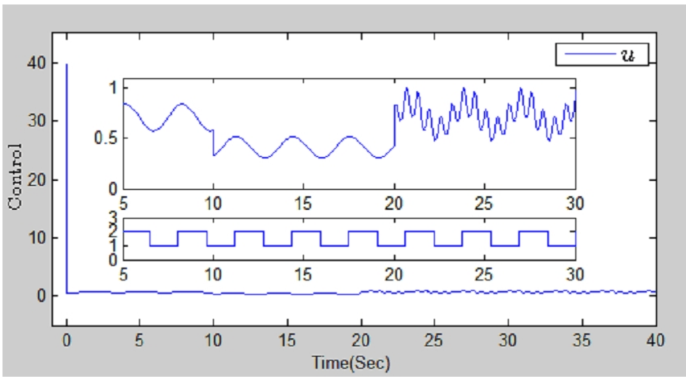

Figure 6. The controller u simulation diagram

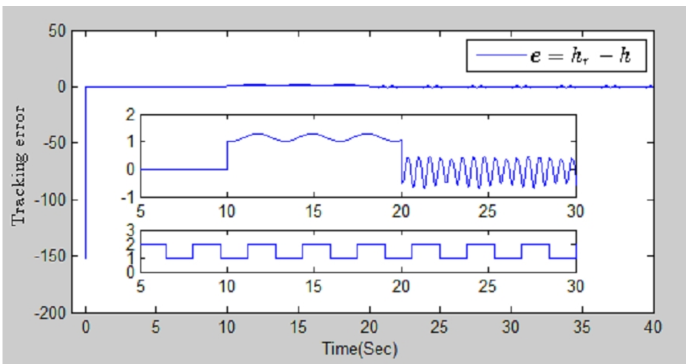

Figure 7. The error e simulation diagram

As shown in Figure 4, for inspecting the signal tracking ability of switching system controller, the reference signal is chosen as sine signal. The initial value is 248m, frequency is 2rad/s, amplitude is 50m; the actual level of initial value is 400m, the simulation time is 40s. The system will switch from subsystem 1 to subsystem 2 when dynamic liquid level height declines to 227.8m; and switch from subsystem 2 to subsystem 1 when working liquid level height rises to 667.8m.

Figure 4 shows that actual working fluid level can follow the reference value.

As shown in Figure 5, we bring in with a 1MPa step disturbance to simulate the step change of the formation pressure in the 10th second; when it comes to the 20th second, we simulate the unstable disturbances by bringing in a mixed disturbance composed by sinusoidal signal 10rad/s, 0.5MPa and a 2MPa step signal.

It shows the changes of controller u and error e under the condition of disturbance in Figures 6 and 7; what is more, we magnify the data between 5th second and 30th second. As we can see from the figure that our controller enables good mobility, and it can guarantee following the reference value of the working fluid level on the condition of disturbance. Importantly, it also limits the error within a small neighborhood.

4. **Conclusion.** In this paper, we established the switched system of the oil well working fluid level-submergence depth close-loop controller. Due to the variation of formation pressure, the fluid phase in the ground settlement will change between single-phase flow and multi-phase flow. By introducing the switching signal, we established a system with the state-dependent switching. In this paper, we did the stability analysis for the system with disturbances. Based on the theory of stability, we designed a switching controller that can make the height of the dynamic liquid level follow the considered dynamic liquid level which is given. When having disturbances, we can limit the tracking error within a small neighborhood by regulating the controller parameters. Aiming at complex nonlinear system (or disturbances), we designed the controller which can meet demand in oil production field.

The system input and the state delay will be considered in future work.

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