ICIC Express Letters
Part B: Applications
Volume **7**, Number **4**, April **2016**

 ISSN 2185-2766
pp. **765–771**

DESIGN OF AN INTEGRATED ITEM RELATIONAL STRUCTURE THEORY AND APPLICATION WITH Q-MATRIX IN WEB ENVIRONMENT

Hsueh-Fang Hsu[1], Hsiang-Chuan Liu[2,3], Yi-Ting Mai[4,*]
Yen-Kuei Yu[3] and Jing-Ming Ju[5]

[1]Department of Business Administration
National Yunlin University of Science and Technology
No. 123, University Road, Section 3, Douliou, Yunlin 64002, Taiwan
hsuan368@gmail.com

[2]Department of Biomedical Informatics
[5]Department of Psychology
Asia University
No. 500, Lioufeng Road, Wufeng, Taichung 41354, Taiwan
{ lhc; jjm2 }@asia.edu.tw

[3]Graduate Institute of Educational Measurement
National Taichung University of Education
No. 140, Min-Shen Road, Taichung 40306, Taiwan
haikuei@gmail.com

[4]Department of Sport Management
National Taiwan University of Sport
No. 16, Section 1, Shuang-Shih Road, Taichung 40404, Taiwan
*Corresponding author: wkb@wkb.idv.tw; wkb@ieee.org; wkb@ntupes.edu.tw

Received September 2015; accepted November 2015

Abstract. *Testing has always been a traditional form of gauging a student's knowledge level. According to traditional cognitive testing, different level examinees might get the same score from the testing items. To identify or distinguish the examinees' detail ability, the traditional item relational structure theory can be used for detecting the item relational structure of the students. How to construct a useful and efficient test is very important in education, but the traditional item relational structure theory cannot determine the efficiency of tests. For improving the weakness, we try to apply the Q-matrix theory to obtain a validation test with idea items' structure as blueprint. In this paper, we propose an integrated efficient item relational structure to construct the efficient before-test item structure of the test with our proposed Liu's ideal item structure theory. According to examination results of students, we can combine the traditional item relational structure theory to construct an efficient estimated item structure of the students. Moreover, the item relational structure of the test can be evaluated by our proposed Liu's criterion related validity index. Finally, the results could be useful for cognitive diagnosis and remedial instruction. Furthermore, our education examination is designed in our online web environment which is a convenient IT system for interactive purpose and cross-platform.*
Keywords: Web system, Q-matrix, IRS, Integrated IRS, Criterion related validity index, Web education

1. **Introduction.** To identify item relational structure from students is a very important issue for education. *Ordering Theory (OT)* [12] had been proposed by the researchers (*Airasian and Bart*). To improve the OT without the item relationship feature, the *Item Relational Structure (IRS)* [11] had been proposed by Takeya. On the other hand, our previous work [5,7] designed an improved threshold limit value by using the empirical

distribution critical value of all the values of the relational structure indices between any two items. It should be more sensitive and effective than the traditional fixed threshold value for comparing the ordering relation of any two items. However, the OT or IRS might not present efficient items fitting the cognitive structure. The researcher Tatsuoka [9] proposed her cognition diagnosis scheme based on *Q-matrix theory* which is called *Rule Space Model (RSM)* [8], and the Tatsuoka's scheme can posit that exam questions or items can be described by specific cognitive skills; the cognitive skills are called *attributes*. It might identify the ability of an examinee where the examinee could solve a test item according to those attributes. An attributes-items incident matrix can show the relations of all specific attributes and all of the possible items which are called *Q-matrix*. Moreover, Tatsuoka can obtain a useful sub-matrix of Q-matrix, called *reduced Q-matrix*, which contains all of the efficient items fitting in with the requirement of the attributes structure by deleting the inefficient items from the Q-matrix. Our research group has presented a novel validity index to evaluate the performance of item ordering of OT structure based on Q-matrix theory [1,3]. Moreover, the index could be used for cognitive diagnosis and remedial instruction. In this paper, the efficient test items could be constructed according to our proposed *integrated item relational structure scheme with Q-matrix in web environment*. Based on our proposed scheme's index, it can recognize and evaluate the differences from the groups via the efficient items.

The remainder of the paper is organized as follows. A brief survey of item relational theory and Q-matrix is presented in Section 2. The integrated item relational structure approach is presented in Section 3. Performance analysis with web system is presented in Section 4. Finally, Section 5 concludes this paper.

2. **Related Work.**

2.1. **Ordering theory and item relational structure theory.** The *Ordering Theory (OT)* [12] method can determine the hierarchical test structure in education research area. *Item Relational Structure (IRS)* [11] analysis scheme is frequently used to analyze test data. IRS scheme should count the frequency of examinees who answered correctly one item or question and incorrectly another item or question respectively, and it could also build a two-D test structure.

2.2. **Item relational structure theory with dynamic threshold.** Both OT and IRS need to use the threshold of two items' difference probability to identify the items' dependence (e.g., OT's ε and IRS's 0.5). Since the threshold of dependence is arbitrary and the results depend on the threshold value, the flexible threshold is an important issue for those schemes. The threshold values of the two methods (OT and IRS) are fixed quantity and lack of statistical meaning. An improved IRS theory algorithm based on empirical distribution critical value was proposed by our previous article [5] as shown below.

Let n be number of items, m be number of examinees, and let the ordering coefficient r^*_{jk} be defined from [11], hence the number of all ordering coefficients is $n\,(n-1)$. We can get a distribution of all ordering coefficients and let the dynamic threshold limit value of IRS be as Formula (1).

$$r_c = \arg_x \left[1 - \int_{-\infty}^{x} f\left(r^*_{jk}\right) dr^*_{jk} = 0.05 \right] \tag{1}$$

2.3. **Q-matrix theory.** To identify the study ability from exam, the researcher Tatsuoka [9] has proposed her cognition diagnosis method based on *Q-matrix theory* which is called *Rule Space Model (RSM)*. Based on the approach, the testing questions can be described by specific cognitive learn skills. They assumed these cognitive skills as attributes for an examinee to solve a test item. The relations of all specific attributes and all of the possible items can be represented by an attributes-items incident matrix, called *Q-matrix*; moreover, Tatsuoka obtains a useful sub-matrix of *Q-matrix*, called reduced *Q-matrix*, which contains all of the efficient items fitting in with the requirement of the attributes structure by cutting the inefficient items from the *Q-matrix*. *Q-matrix theory* is based on after-test items. However, the test blueprint for before-test is better than the after-test. Furthermore, Leighton, Gier, and Hunka [8] tried to emphasize the before-test *Attribute Hierarchy Method (AHM)*.

3. **Integrated Efficient Item Relational Structure Approach.** Either IRS or OT can construct a hierarchical test structure. Why then do we need to use Q-matrix to build another test structure? Because Q-matrix theory can determine the test structure before testing, so the test structure has the feature of independence of the sample. So our proposed *integrated efficient item relational structure approach* includes three features: *ideal item relational structure theory for efficient items*, *estimated item relational structure theory*, *and Liu's criterion related validity index*. Our proposed integrated approach can provide useful cognitive diagnosis information for instructors. How to integrate these three features can be shown by examples as Sections 3.1, 3.2 and Section 4.1.

3.1. **Ideal item relational structure theory for efficient items.** According to the test structure of Q-matrix, the ideal item relational structure could be identified accordingly. To combine the IRS test structure (after-test), our proposed integrated efficient item relational structure approach could provide the overall information about items and attributes for student's study analysis. We create an example to show our proposed approach. The ideal item relational structure theory for efficient items was proposed by our previous work [2], the example of the multiplication of fractions, and its efficient item matrix as structure-(2). From [2,9], we can show example's each item set of required attributes as structure-(3).

We can get the ideal items relational structure without slipping and guessing as Figure 1. The ideal efficient item matrix $M_R\left(I_{\{E\}}\right)$ is shown in Table 1.

Developing real efficient items set according to Q_R or $Q_{\{R\}}$. According to the test blueprint, $Q_{\{R\}}$, we could build the real test of the efficient items set. As an example of multiplication of fractions, we can get the real test of the efficient items from structure-(4).

$$Q_{\{R\}} = \left[\overset{I_1}{\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}}, \overset{I_2}{\begin{bmatrix}1\\1\\0\\0\\0\end{bmatrix}}, \overset{I_3}{\begin{bmatrix}1\\1\\1\\0\\0\end{bmatrix}}, \overset{I_4}{\begin{bmatrix}1\\0\\0\\1\\0\end{bmatrix}}, \overset{I_5}{\begin{bmatrix}1\\0\\0\\1\\1\end{bmatrix}}, \overset{I_6}{\begin{bmatrix}1\\1\\0\\1\\0\end{bmatrix}}, \overset{I_7}{\begin{bmatrix}1\\1\\0\\1\\1\end{bmatrix}}, \overset{I_8}{\begin{bmatrix}1\\1\\1\\1\\0\end{bmatrix}}, \overset{I_9}{\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}} \right] \tag{2}$$

$$\begin{array}{l} I_{(1)} \subset I_{(1)}, I_{(2)}, I_{(3)}, I_{(4)}, I_{(5)}, I_{(6)}, I_{(7)}, I_{(8)}, I_{(9)} \\ I_{(2)} \subset I_{(2)}, I_{(3)}, I_{(6)}, I_{(7)}, I_{(8)}, I_{(9)} \\ I_{(4)} \subset I_{(4)}, I_{(5)}, I_{(6)}, I_{(7)}, I_{(8)}, I_{(9)} \\ I_{(6)} \subset I_{(6)}, I_{(7)}, I_{(8)}, I_{(9)} \\ I_{(3)} \subset I_{(3)}, I_{(8)}, I_{(9)} \\ I_{(5)} \subset I_{(5)}, I_{(7)}, I_{(9)} \\ I_{(7)} \subset I_{(7)}, I_{(9)} \\ I_{(8)} \subset I_{(8)}, I_{(9)} \\ I_{(9)} \subset I_{(9)} \end{array} \tag{3}$$

$$\begin{array}{lll} I_1:\ 12,321\times 3=? & I_2:\ 12,231\times 4=? & I_3:\ 15,414\times 5=? \\ I_4:\ 12,102\times 3=? & I_5:\ 10,010\times 5=? & I_6:\ 12,041\times 4=? \\ I_7:\ 10,040\times 6=? & I_8:\ 13,401\times 7=? & I_9:\ 1,340,040\times 4=? \end{array} \quad (4)$$

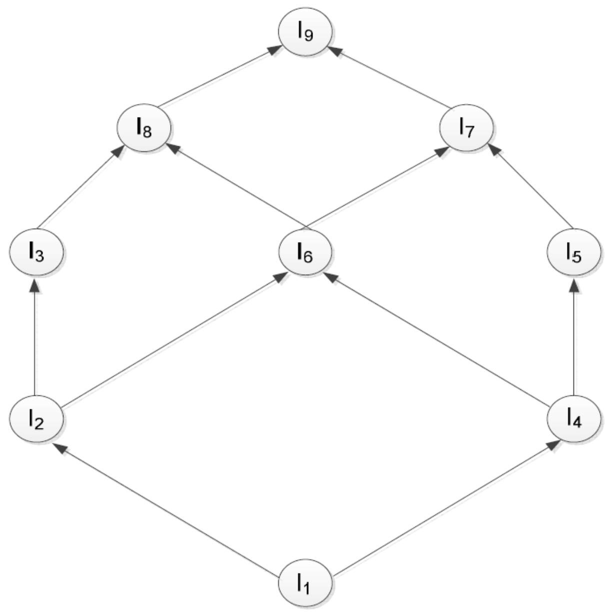

FIGURE 1. The ideal item relational structure

TABLE 1. Ideal efficient item matrix

	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9
I_1	1	1	1	1	1	1	1	1	1
I_2	0	1	1	0	0	1	1	1	1
I_3	0	0	1	0	0	0	0	1	1
I_4	0	0	0	1	1	1	1	1	1
I_5	0	0	0	0	1	0	1	0	1
I_6	0	0	0	0	0	1	1	1	1
I_7	0	0	0	0	0	0	1	0	1
I_8	0	0	0	0	0	0	0	1	1
I_9	0	0	0	0	0	0	0	0	1

3.2. **Estimated item relational structure theory.** After testing the students, an estimated item relational structure could be identified by IRS theory [12]. If all of the items in that given test are efficient, then the estimated item relational structure is efficient. For a given efficient test, $I'_{\{E\}} = \{I'_1, I'_2, \ldots, I'_n\}$, after testing the students, by using *Liu's generalized polytomous item relational structure theory* [6], L,

(I) The estimated item relational structure matrix of $I'_{\{E\}} = \{I'_1, I'_2, \ldots, I'_n\}$ by L is $M_{E'}\left(I_{\{E\}}, S\right) = \left[e'_{ij}\right]_{n\times n}$, where $e'_{ij} = \begin{cases} 1 & \text{if } I_i \overset{L}{\rightarrow} I_j \\ 0 & \text{if } I_i \overset{L}{\nrightarrow} I_j \end{cases}$.

(II) The graph of the estimated item relational structure matrix is denoted by $G(I_{\{E\}}, S)$.

4. **Performance Analysis.**

4.1. **Liu's criterion related validity index.** To date, there is no criterion about related validity index for comparing the performance of different item relational structures by a given item ordering structure theory. For evaluating the performance of an estimated efficient item relational structure of an efficient item set by a given item ordering structure theory, our research group proposed a useful validity index [1] as follows.

Definition 4.1. *Liu's criterion related validity index.*

Let $M_E\left(I_{\{E\}}\right) = [e_{ij}]_{n\times n}$ be the ideal item structure matrix of the efficient item set $I_{\{E\}}$, if $M_{E'}\left(I'_{\{E\}}, S\right) = \left[e'_{ij}\right]_{n\times n}$ is an estimated item structure matrix of $I_{\{E\}}$ by using an item ordering structure theory, S, then the criterion related validity index of $I_{\{E\}}$ by S, is denoted as $Val\left(I'_{\{E\}}, S|I_{\{E\}}\right)$, and is defined as follows:

$$Val\left(I'_{\{E\}}, S|I_{\{E\}}\right) = \frac{1}{2}\left[1 + \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(e_{ij}-\bar{e}\right)\left(e'_{ij}-\overline{e'}\right)}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(e_{ij}-\bar{e}\right)^2}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(e'_{ij}-\overline{e'}\right)^2}}\right] \tag{5}$$

where

$$\bar{e} = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}e_{ij}, \quad \bar{e}' = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}e'_{ij}, \tag{6}$$

$$Val\left(I'_{\{E\}}, S|I_{\{E\}}\right) \in [0, 1] \tag{7}$$

4.2. **Results analysis.** According to our proposed integrated efficient item relational structure approach, both the ideal test structure and real test structure can be constructed. Since the real test structure is from the testing of real examinees, the ideal test structure could be assigned as a criterion for this specific test. Furthermore, the real testing results from different examinee groups might be measured and analyzed by using Liu's criterion related validity index. It is a creative and important index to estimate the performance of our proposed integrated efficient item relational structure approach. Moreover, the index can help us to evaluate those real testing or after-test results more accurately. Based on our multi-function web system [4,10], we have designed an online web testing system (as Figure 2) for examinees to immediately answer test questions (as Figure 3), and then we can get the analysis results according to our proposed integrated scheme online. There are a total of 128 students from five classes ($S1 \sim S5$), and the analysis results from our online web site are shown in Table 2. By using our proposed criterion related validity index, the results can show the quantitative difference from the five classes. The higher the index value is, the better the performance of after-test result is. So the class $S1$ has the best performance due to the highest index value (0.7164) from our proposed Liu's criterion related validity index.

Table 2. Criterion related validity index for students of five classes

Class ID	S_1	S_2	S_3	S_4	S_5
Val	*0.7164*	*0.6912*	*0.6685*	*0.5680*	*0.6461*

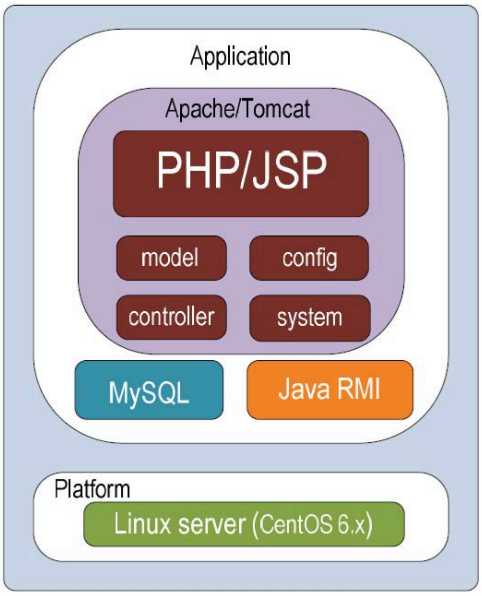

FIGURE 2. Online web testing system

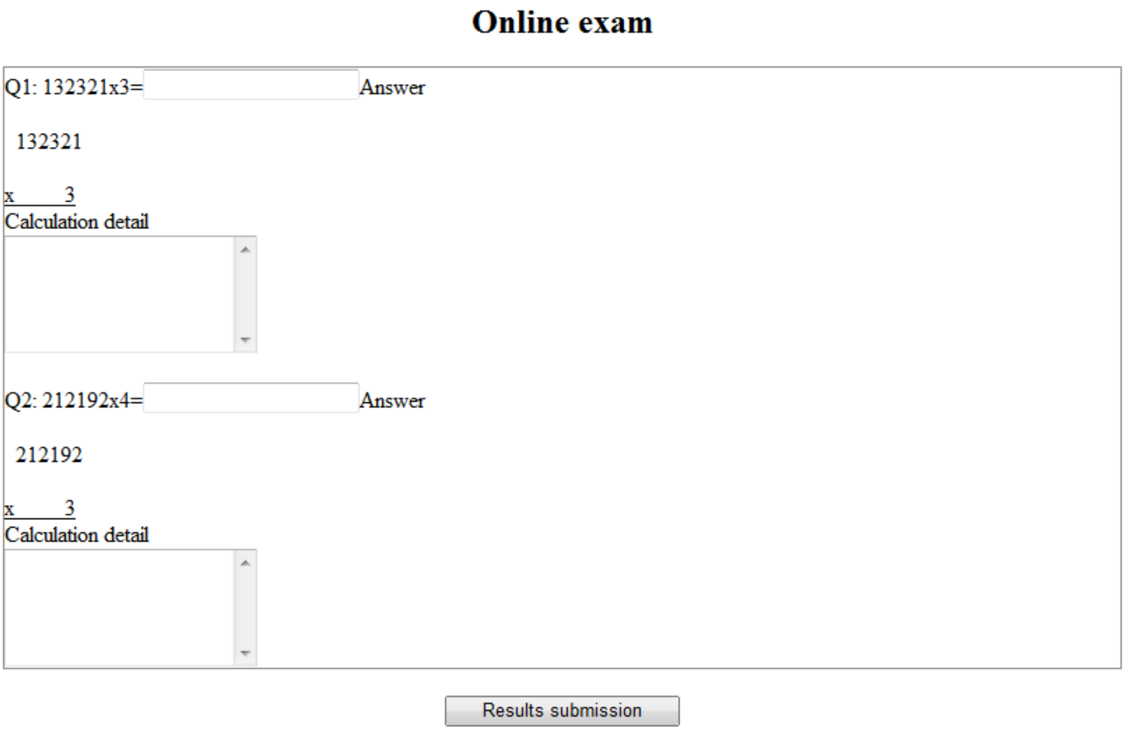

FIGURE 3. Example of online math exam

5. **Conclusions.** For a given test, we do not know which item of the test is efficient or not for skill of students or examinees. To solve and improve the drawback, we firstly use the Q-matrix theory to obtain a validation test with all items which are efficient. So the efficient items of the example of multiplications of fractions could be constructed accordingly. Based on our proposed integrated efficient item relational structure approach, the useful testing structure can be built easily. Furthermore, to apply our proposed online web system with Liu's criterion related validity index in our proposed online web system, we can evaluate the item relational structure of the test, and the results could be useful for cognitive diagnosis and remedial instruction in education conveniently.

Acknowledgment. This work is partially supported by the National Science Council grant and Ministry of Science and Technology, Taiwan, under grants (No. NSC-100-2511-S-468-001, MOST-103-2221-E-028-001).

REFERENCES

[1] H.-C. Liu, A novel validity index for evaluating the item ordering structure based on Q-matrix theory, *Proc. of the 2nd International Conference on Mechatronics and Materials Engineering*, Hangzhou, China, 2012.

[2] H.-C. Liu, *Before-Test Ideal Efficient Item Relational Structure Theory*, Asia University, 2006.

[3] H.-C. Liu, Theoretical approach to reduced Q-matrix for cognition diagnosis, *Proc. of the 2nd International Conference on Engineering and Technology Innovation*, Koahsung, Taiwan, 2012.

[4] H.-C. Liu, B.-C. Jeng, Y.-T. Mai, Y.-D. Jheng and H.-T. Lin, Design of online survey system with an advanced IPA discrimination index for customer satisfaction assessment, *Electronic Commerce Research*, vol.14, no.3, pp.223-243, 2014.

[5] H.-C. Liu, S.-N. Wu and C.-C. Chen, Item relational structure algorithm based on empirical distribution critical value, *Journal of Software*, vol.6, no.11, pp.2106-2113, 2011.

[6] H.-C. Liu, W.-S. Chen and H.-C. Tsai, Polytomous item relational structure theory based on empirical sistribution critical value, *Advanced Materials Research*, vols.472-475, pp.1329-1332, 2012.

[7] H.-C. Liu, Y.-K. Yu, H.-C. Tsai, Y.-T. Huang and Y.-T. Mai, Optimized item relational structure theory: Using the differentiations of multinomial functions as an example, *ICIC Express Letters*, vol.8, no.5, pp.1425-1431, 2014.

[8] J. P. Leighton, M. J. Gier and S. M. Hunka, The attribute hierarchy method for cognitive assessment: A variation on Tatsuoka's rule space approach, *Journal of Educational Measurement*, vol.41, no.3, pp.205-237, 2004.

[9] K. K. Tatsuoka, Rule space: An approach for dealing with misconceptions based on item response theory, *Journal of Educational Measurement*, vol.20, no.4, pp.345-354, 1983.

[10] M. Chen, S.-J. Lee, Y.-T. Mai, Y.-D. Jheng and H.-C. Liu, A novel multiple-option web-based questionnaire system with IPA and IPA discrimination index, *International Journal of Digital Content Technology and Its Applications*, vol.6, no.3, pp.295-303, 2012.

[11] M. Takeya, Construction and utilization of item relational structure graphs for use in test analysis, *Japan Journal of Educational Technology*, vol.5, pp.93-103, 1980.

[12] P. W. Airasian and W. M. Bart, Ordering theory: A new and useful measurement model, *Journal of Education Technology*, vol.5, pp.56-60, 1973.