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OPTIMAL COST ANALYSIS OF A MULTI-STATE REPAIRABLE SYSTEM WITH REPAIR DELAYS AND INTERRUPTIONS

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Abstract. *This paper deals with the optimal cost analysis for a multi-state repairable system with repair delays and interruptions, in which all states of the system cover normal (N) and abnormal (A) operations, and normal and abnormal failures. Here the normal failure (NF) and abnormal failure (AF) are non-detectable states, and the states N and A are detectable ones. The state N can reach states NF and A. The system cannot enter state AF until it has passed state A. We study the multi-dimension vector case of random diagnostic parameter in detecting whether the system is in state N or A. Further, in our model the impacts of repair delays and interruptions on the system reliability are also considered. The supplementary variable method and the theory of differential equation are applied to obtain the steady-state probability equations and their solutions, through which some important reliability indices of the system are derived. A cost model, developed to determine the optimal diagnostic and detection policy at a minimum cost, is studied. For illustrative purpose two numerical examples are presented.*
Keywords: Multi-state repairable system, Diagnosis and detection, Repair delays and interruptions, Supplementary variable, Reliability indices, Optimal cost

1. **Introduction.** In the classical repairable systems, it is usually assumed that the system has only two states: operation and failure. However, some practical productive equipment usually has four states: normal (N) and abnormal (A) operations, and normal and abnormal failures. Here the normal failure (NF) and abnormal failure (AF) are non-detectable states, and the states N and A are detectable ones. The state N can reach states NF and A. The system cannot enter state AF until it has passed state A. The reason for states A and AF occurrence is the miss of operation or incorrect management. Though the above systems are common in practice, very few researchers have studied them. So it is important to develop their multi-state model and analyze their reliability indices. Due to different failures producing different repair costs, it is necessary to carry out detection measures. In engineering applications, a diagnostic parameter may be used to detect states N and A by its measuring value. This is because the diagnostic parameter, such as the power, amplitude, and frequency, has generally a close relation with the system states and can be observed easily. Thus, the repairable systems with detection and preventive repair has been hot topics of much research, and has been investigated by many researchers. For detailed overviews of the main results and methods, the reader is referred to the papers by Hu et al. [1, 2, 3], Li and Hu [4], Levitin and Lisniansk [5, 6], Levitin [7], Su [8], Su et al. [9], Gu and Li [10], and Di et al. [11].

The existing research has focused mainly on reliability indices [1, 2, 3, 4, 10, 11], detections policy [5, 6, 7], and the combination of these two cases [8, 9]. Further, most of researchers were devoted to single random diagnostic parameter [1-10]. Little work has been conducted to study multi-dimension vector case of random diagnostic parameter.

However, the multi-dimension vector case of random diagnostic parameter is very widespread in detecting the system states. For example, when detecting abnormal failure state of some electronic equipment, the detected diagnostic parameters include the power, the amplitude and the frequency, etc., which forms a diagnostic parameter vector. Also, in most of repairable systems the failed system is assumed to be always repaired immediately whenever the system fails, and the repair is not interrupted, although this assumption is evidently unrealistic. Actually, it is not the case. The repair for the failed system does not usually start immediately due to some particular reasons. For instance, certain essential start-up times are to be taken before repair; the repairman is taking part-time jobs outside for more profits of the system or he (she) is absent due to illness. When the busy repair facility is subjected to lengthy and unpredictable breakdowns or the tired repairman needs a short rest during repair period, etc., the repair for the failed system will be suspended. Therefore, in the reliability theory and application of repairable systems the assumption of repair delays and interruptions is reasonable. Since this kind of repairable system is common in power plants, manufacturing systems, industrial systems and standby systems, etc., it is a class of more general repairable system, and usual repairable systems are its special cases. However, as far as we know, for this kind of repairable system mentioned above no work gives a comprehensive study on multiple states, reliability indices, state detection and diagnosis, and optimal cost analysis. This motivates us to develop an optimal detection and diagnostic policy for minimizing the cost of the above repairable system based on reliability analysis, in which the impacts of repair delays and interruptions on the system reliability are also considered.

The rest of the paper is organized as follows. Section 2 gives the model assumptions. In Sections 3 and 4, we develop differential equations governing the considered repairable system and solve the steady-state state probabilities, which derive important reliability indices of the system. In Section 5 by defining several cost elements we construct a cost function of the system to determine the optimal diagnostic and detection policy at a minimum cost rate. Two numerical examples are presented for illustrative purpose. Finally, in Section 6 some conclusions are drawn.

2. **Model Assumptions.** We consider a multiple-state repairable system with repair delays and interruptions as follows.

(1) All states of the system include normal (N) operation and abnormal (A) operation, normal failure (NF), and abnormal failure (AF). Here the N and A are operating states, and the others are failure ones. Also, the NF and AF are non-detectable states, and the N and A are detectable ones. The operating system in N can transfer to NF or A with constant failure rate λ_1 or λ_2, and the operating system in A only transfers to AF with constant failure rate λ_3.

(2) Whenever the system starts to operate, it will be detected once every a random time T to make sure whether it is in N, or A until it attains NF (AF) or is detected as being in A. Assume that T has distribution function $H(t)$, density function $h(t)$, hazard rate function $\alpha(t)$ and finite mean $E(H)$. By detection the diagnosis parameter value taken by the normal (abnormal) system is measured, where the diagnosis parameter of the normal (abnormal) system is a random vector, denoted by $(X_1, X_2, \cdots, X_m)$. It is assumed that the detected results of the normal system are always accurate. For the abnormal system, let $F_A(t_1, t_2, \cdots, t_m)$ be the distribution function of $(X_1, X_2, \cdots, X_m)$, and $(x_1, x_2, \cdots, x_m)$ and $(u_1, u_2, \cdots, u_m)$ denote the measured and critical values of $(X_1, X_2, \cdots, X_m)$, respectively, and then the diagnostic criterion is as follows: when $x_i > u_i$, $i = 1, 2, \cdots, m$, the abnormal system is deemed to be in A; otherwise it is seen as being in N.

(3) The system detected as being in A is immediately stopped but is not temporarily repaired. For the system in NF (AF), or the system detected as being in A, it needs to wait for a random time V before repair due to some unexpected events, such as, the startup of

repair facility, the invitation of senior repairer, the absence of system repairman and some preparatory work. Suppose that V is generally distributed with the distribution function $V(t)$ $(t \geq 0)$, hazard rate function $r(t)$ and a finite mean $E(V)$. After a random time V, the system in NF (AF), or the stopped system in A is repaired with random repair time Y_1, Y_2 or Y_3, respectively, where the repair time Y_i obeys a general distribution function $G_i(t)$, $t \geq 0$ with hazard rate function $\mu_i(t)$ and a mean repair time $E(Y_i)$, $i = 1, 2, 3$. After repair the system is as good as new, and operates at once.

(4) For the system in NF (AF), or the system detected as being in A, its repair time may be interrupted by some emergencies, such as the repairman's illness, the breakdown of repair facility, and a power cut. The interruption is immediately recovered. It is assumed that the emergencies arrive with Poisson rate a, and the recovery times for repair interruption are independent and identically distributed random variables obeying a general distribution function $B(t)$ $(t \geq 0)$ with hazard rate function $b(t)$ and a mean $E(B)$. Once the recovery is terminated, the repair will restart until the system is repaired and begins to operate. The repair time for the system is cumulative, that is, the repaired time of the system is still valid.

(5) Initially, the system in N begins to operate. All random variables are mutually independent.

Remark 2.1. *Let $q = F_A(u_1, u_2, \cdots, u_m)$, $p = 1 - q$, then from assumption (2), $p(q)$ is the probability that the detected result of abnormal system is right (wrong).*

3. **Model Analysis.** Let $k = 1, 2, 3$ represent the system is in NF, AF and $\tilde{\text{A}}$ (the system has accurately been detected as being in A), respectively. We define the possible states of the system as follows:

State $(0, n)$: the system in N is operating, and has been detected n times, $n = 0, 1, 2, \cdots$;

State $(1, n)$: the system in A is operating, and has been detected n times, $n = 0, 1, 2, \cdots$;

State $(2, k)$: the system is waiting for repair due to some unexpected events, $k = 1, 2, 3$;

State $(3, k)$: the system is under repair, $k = 1, 2, 3$;

State $(4, k)$: the system is waiting for remaining repair due to some emergencies, $k = 1, 2, 3$.

Let $S(t)$ be the system state at time t. For $t \geq 0$, we define $\xi_1(t)$ as the elapsed detection time at time t when $S(t) = (0, n), (1, n)$ $(n = 1, 2, \cdots)$, $\xi_2(t)$ the elapsed repair time of the failed system at time t when $S(t) = (3, k), (4, k)$, $(k = 1, 2, 3)$, $\xi_3(t)$ the elapsed delay time due to unexpected events at time t when $S(t) = (2, k)$, $(k = 1, 2, 3)$, and $\xi_4(t)$ the elapsed repair recovery time due to emergencies at time t when $S(t) = (4, k)$, $(k = 1, 2, 3)$. Then $\{S(t), \xi_1(t), \xi_2(t), \xi_3(t), \xi_4(t), t \geq 0\}$ is a vector Markov process.

At time t, the state probabilities of the system are defined as:

$$p_{in}(t, x)\mathrm{d}x = pr\{S(t) = (i, n),\ x \leq \xi_1(t) < x + \mathrm{d}x\},\ x > 0,\ i = 0, 1,\ n = 0, 1, 2, \cdots;$$

$$p_{2k}(t, w)\mathrm{d}w = pr\{S(t) = (2, k),\ w \leq \xi_3(t) < w + \mathrm{d}w\},\ w > 0,\ k = 1, 2, 3;$$

$$p_{3k}(t, y)\mathrm{d}y = pr\{S(t) = (3, k),\ y \leq \xi_2(t) < y + \mathrm{d}y\},\ y > 0,\ k = 1, 2, 3;$$

$$p_{4k}(t, y, z)\mathrm{d}z = pr\{S(t) = (4, k), \xi_2(t) = y, z \leq \xi_4(t) < z + \mathrm{d}z\},\ y > 0,\ z > 0,\ k = 1, 2, 3.$$

In steady-state, we define

$$p_{in}(x) = \lim_{t\to\infty} p_{in}(t, x),\ i = 0, 1, \quad p_{jk}(w) = \lim_{t\to\infty} p_{jk}(t, w),\ j = 2, 3,$$
$$p_{4k}(y, z) = \lim_{t\to\infty} p_{4k}(t, y, z),\ k = 1, 2, 3.$$

According to Cox [12], the steady-state Kolmogorov forward equations that govern the system can be written as follows:

$$\left(\frac{\mathrm{d}}{\mathrm{d}x} + \lambda_1 + \lambda_2 + \alpha(x)\right) p_{0n}(x) = 0, \ n = 0, 1, 2, \cdots, \tag{1}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}x} + \lambda_3 + \alpha(x)\right) p_{10}(x) = \sum_{n=0}^{\infty} \lambda_2 p_{0n}(x)\mathrm{d}x, \tag{2}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}x} + \lambda_3 + \alpha(x)\right) p_{1n}(x) = 0, \ n = 0, 1, 2, \cdots, \tag{3}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}w} + r(w)\right) p_{2k}(w) = 0, \ k = 1, 2, 3, \tag{4}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}y} + \alpha + \mu_k(y)\right) p_{3k}(y) = \int_0^{\infty} b(z) p_{4k}(y, z)\mathrm{d}z, \ k = 1, 2, 3, \tag{5}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}z} + b(z)\right) p_{4k}(y, z) = 0, \ k = 1, 2, 3. \tag{6}$$

The boundary conditions are

$$p_{00}(0) = \sum_{k=1}^{3} \int_0^{\infty} \mu_k(w) p_{3k}(y)\mathrm{d}y, \tag{7}$$

$$p_{0n}(0) = \int_0^{\infty} \alpha(x) p_{0,n-1}(x)\mathrm{d}x, \ n = 1, 2, 3, \cdots, \tag{8}$$

$$p_{10}(0) = 0, \tag{9}$$

$$p_{1n}(0) = q \int_0^{\infty} \alpha(x) p_{1,n-1}(x)\mathrm{d}x, \ n = 1, 2, 3, \cdots, \tag{10}$$

$$p_{21}(0) = \sum_{n=0}^{\infty} \int_0^{\infty} \lambda_1 p_{0n}(x)\mathrm{d}x, \tag{11}$$

$$p_{22}(0) = \sum_{n=0}^{\infty} \int_0^{\infty} \lambda_3 p_{1n}(x)\mathrm{d}x, \tag{12}$$

$$p_{23}(0) = p \sum_{n=0}^{\infty} \int_0^{\infty} \alpha(x) p_{1n}(x)\mathrm{d}x, \tag{13}$$

$$p_{3k}(0) = \int_0^{\infty} r(w) p_{2k}(w)\mathrm{d}w, \ k = 1, 2, 3, \tag{14}$$

$$p_{4k}(y, 0) = a p_{3k}(y), \ k = 1, 2, 3. \tag{15}$$

The normalization condition is

$$\sum_{n=0}^{\infty} \int_0^{\infty} [p_{0n}(x) + p_{1n}(x)]\mathrm{d}x + \sum_{k=1}^{3} \left[\int_0^{\infty} p_{2k}(w)\mathrm{d}w + \int_0^{\infty} p_{3k}(y)\mathrm{d}y + \int_0^{\infty}\int_0^{\infty} p_{4k}(y, z)\mathrm{d}y\mathrm{d}z\right] = 1. \tag{16}$$

To derive interesting reliability indices of the system, we define

$$\bar{H}(t) = 1 - H(t), \quad h^*(s) = \int_0^{\infty} \mathrm{e}^{-st} h(t)\mathrm{d}t,$$

$$\bar{H}^*(s) = \int_0^{\infty} \mathrm{e}^{-st} \bar{H}(t)\mathrm{d}t, \ s > 0, \quad \Lambda = \lambda_1 + \lambda_2 - \lambda_3.$$

By the theory of first-order, linear, ordinary differential equations, we can obtain the solution of the above Equations (1)-(16) as follows:

$$p_{0n}(x) = p_{0n}(0)\mathrm{e}^{-(\lambda_1+\lambda_2)x}\bar{H}(x),\ n = 0, 1, 2, \cdots,$$
$$p_{10}(x) = \frac{\lambda_2 \mathrm{e}^{-\lambda_3 x}\bar{H}(x)p_{00}(0)\left(1 - \mathrm{e}^{-(\lambda_1+\lambda_2-\lambda_3)x}\right)}{(\lambda_1+\lambda_2-\lambda_3)\left(1-h^*(\lambda_1+\lambda_2)\right)},$$
$$p_{1n}(x) = p_{1n}(0)\mathrm{e}^{-\lambda_3 x}\bar{H}(x),\ n = 1, 2, 3, \cdots, \quad p_{2k}(w) = p_{2k}(0)\bar{V}(w),\ k = 1, 2, 3,$$
$$p_{3k}(y) = p_{3k}(0)\bar{G}_k(y),\ k = 1, 2, 3, \quad p_{4k}(y, z) = p_{3k}(0)a\bar{B}(z)\bar{G}_k(y),\ k = 1, 2, 3.$$

where

$$p_{0n}(0) = [h^*(\lambda_1+\lambda_2)]^n p_{00}(0),\ n = 1, 2, \cdots, \quad p_{11}(0) = \frac{q\lambda_2 p_{00}(0)\left(h^*(\lambda_3) - h^*(\lambda_1+\lambda_2)\right)}{\Lambda\left(1-h^*(\lambda_1+\lambda_2)\right)},$$
$$p_{1n}(0) = [qh^*(\lambda_3)]^{n-1}p_{11}(0),\ n = 2, 3, \cdots, \quad p_{10}(0) = 0, \quad p_{3k}(0) = p_{2k}(0),\ k = 1, 2, 3,$$
$$p_{21}(0) = \frac{\lambda_1\bar{H}^*(\lambda_1+\lambda_2)}{1-h^*(\lambda_1+\lambda_2)}p_{00}(0), \quad p_{23}(0) = \frac{q\lambda_2\left(h^*(\lambda_3)-h^*(\lambda_1+\lambda_2)\right)}{\Lambda\left(1-qh^*(\lambda_3)\right)\left(1-h^*(\lambda_1+\lambda_2)\right)}p_{00}(0),$$
$$p_{22}(0) = \frac{\lambda_2\lambda_3 p_{00}(0)}{\Lambda\left(1-h^*(\lambda_1+\lambda_2)\right)}\left[\bar{H}^*(\lambda_3) - \bar{H}^*(\lambda_1+\lambda_2) + \frac{q\bar{H}^*(\lambda_3)}{1-qh^*(\lambda_3)}\left(h^*(\lambda_3)-h^*(\lambda_1+\lambda_2)\right)\right],$$
$$\begin{aligned} p_{00}^{-1}(0) = {} & \frac{\lambda_2}{\Lambda\left(1-h^*(\lambda_1+\lambda_2)\right)}\left[\bar{H}^*(\lambda_3) - \bar{H}^*(\lambda_1+\lambda_2) + \frac{q\bar{H}^*(\lambda_3)\left(h^*(\lambda_3)-h^*(\lambda_1+\lambda_2)\right)}{1-qh^*(\lambda_3)}\right] + \frac{1}{\lambda_1+\lambda_2} + E(V) \\ & + (1+aE(B))\left\{\frac{\lambda_1 E(G_1)}{\lambda_1+\lambda_2} + \frac{p\lambda_2 E(G_3)\left(h^*(\lambda_3)-h^*(\lambda_1+\lambda_2)\right)}{\Lambda\left(1-qh^*(\lambda_3)\right)\left(1-h^*(\lambda_1+\lambda_2)\right)}\right. \\ & + \frac{\lambda_2\lambda_3 E(G_2)}{\Lambda\left(1-h^*(\lambda_1+\lambda_2)\right)}\left[\bar{H}^*(\lambda_3) - \bar{H}^*(\lambda_1+\lambda_2)\right. \\ & \left.\left. + \frac{q\bar{H}^*(\lambda_3)\left(h^*(\lambda_3)-h^*(\lambda_1+\lambda_2)\right)}{1-qh^*(\lambda_3)}\right]\right\}. \end{aligned}$$

4. **Reliability Indices.** Based on the system assumptions and the results obtained in Section 3, we easily derive some important reliability indices of the system as follows.

Theorem 4.1. *(1) In steady state, let A_0 (A_1) denote the normal (abnormal) availability of system, i.e., the probability that the system in N (A) is operating, then*

$$A_0 = \frac{p_{00}(0)}{\lambda_1+\lambda_2}, \tag{17}$$

$$A_1 = \frac{\lambda_2 p_{00}(0)}{\Lambda\left(1-h^*(\lambda_1+\lambda_2)\right)}\left[\bar{H}^*(\lambda_3) - \bar{H}^*(\lambda_1+\lambda_2) + \frac{q\bar{H}^*(\lambda_3)\left(h^*(\lambda_3)-h^*(\lambda_1+\lambda_2)\right)}{1-qh^*(\lambda_3)}\right]. \tag{18}$$

(2) In steady state, denote p_d and p_I as the probability that the repair of system is delayed and interrupted, respectively, then

$$p_d = E(V)p_{00}(0), \quad p_I = aE(B)\sum_{k=1}^{3}E(G_k)p_{2k}(0). \tag{19}$$

(3) In steady state, denote p_{3l}, $l = 1, 2, 3$ as the repair probability of the system in NF, AF and A (the system has accurately been detected as being in A), respectively, then

$$p_{3l} = E(G_l)p_{2l}(0),\ l = 1, 2, 3. \tag{20}$$

(4) In steady state, the inspection frequency of system in N or A, i.e., the rate of occurrence of inspections of the system in N or A, denoted by Θ, is

$$\Theta = \frac{h^*(\lambda_1+\lambda_2)p_{00}(0)}{1-h^*(\lambda_1+\lambda_2)} + \frac{\lambda_2 p_{00}(0)\left(h^*(\lambda_3)-h^*(\lambda_1+\lambda_2)\right)}{\Lambda\left(1-qh^*(\lambda_3)\right)\left(1-h^*(\lambda_1+\lambda_2)\right)}, \tag{21}$$

where $p_{2l}(0)$, $l = 1, 2, 3$, and $p_{00}(0)$ are given by Section 3.

Proof: (i) By the definitions of A_0, A_1, p_d, p_I and p_{3l}, $l = 1, 2, 3$, we have

$$A_i = \sum_{n=0}^{\infty}\int_0^{\infty} p_{in}(x)\mathrm{d}x,\ i = 0, 1, \quad p_d = \sum_{k=1}^{3}\int_0^{\infty} p_{2k}(w)\mathrm{d}w,$$

$$p_I = \sum_{k=1}^{3}\int_0^{\infty}\int_0^{\infty} p_{4k}(y,z)\mathrm{d}y\mathrm{d}z, \quad p_{3l} = \int_0^{\infty} p_{3l}(y)\mathrm{d}y,\ l = 1, 2, 3.$$

Substituting the results in Section 3 into the above equations, we complete the proofs of (1), (2) and (3).

(ii) By the system assumptions and the steady-state frequency formula in [13], it follows that

$$\Theta = \sum_{n=0}^{\infty}\int_0^{\infty}\alpha(x)[p_{0n}(x)+p_{1n}(x)]\mathrm{d}x.$$

Again by means of the obtained results in Section 3, we can complete the proof.

5. **Optimal Cost Analysis.** In this section, we construct the long-run expected cost function per unit time for the multi-state repairable system presented by this paper, in which the detection and diagnostic parameters are decision variables. Our objective is to numerically analyze the optimal detection and diagnostic policy of the system while maintaining a minimum cost. Let us define the following cost elements:

$C_\theta \equiv$ expected cost of each inspection;

C_N $(C_A) \equiv$ detection cost per unit time when the system in N(A) is detected;

$C_{3k} \equiv$ repair cost per unit time when the system is in states $(3, k)$, $k = 1, 2, 3$, respectively;

$C_d \equiv$ repair delay cost per unit time due to some unexpected events;

$C_r \equiv$ repair recovery cost per unit time due to emergencies;

then the long-run expected cost function of the system per unit time is given by

$$C = C_\theta\Theta + C_N A_0 + C_A A_1 + C_d p_d + \sum_{k=1}^{3} C_{3k}p_{3k} + C_r p_I,$$

where Θ, A_0, A_1, p_d, p_{3k} and p_I are given by Theorem 4.1.

For illustrative purpose, we set the values of system parameters and cost elements as follows: $\lambda_1 = \frac{1}{660}$, $\lambda_2 = \frac{1}{620}$, $\lambda_3 = \frac{1}{65}$, $E(G_1) = 25$, $E(G_2) = 72$, $E(G_3) = 30$, $E(V) = 28$, $C_T = 50000$, $C_N = 10000$, $C_A = 15000$, $C_d = 12000$, $C_{31} = 20000$, $C_{32} = 25000$, $C_{33} = 24000$, $C_r = 16000$. Also, let us assume that the inter-detection time of the system is deterministic with the mean v. Thus, we have

$$H(t) = \begin{cases} 0, & t < v, \\ 1, & t \geq v, \end{cases} \quad \bar{H}^*(\lambda_1+\lambda_2) = \frac{1-e^{-(\lambda_1+\lambda_2)v}}{\lambda_1+\lambda_2}, \quad \bar{H}^*(\lambda_3) = \frac{1-e^{-\lambda_3 v}}{\lambda_3},$$

$$h^*(\lambda_3) = 1-\lambda_3\bar{H}^*(\lambda_3), \quad h^*(\lambda_1+\lambda_2) = 1-(\lambda_1+\lambda_2)\bar{H}^*(\lambda_1+\lambda_2).$$

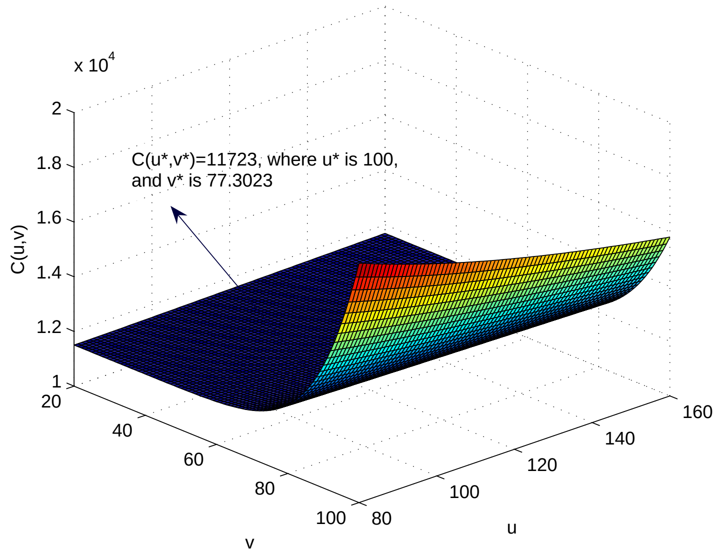

FIGURE 1. The expected cost for different diagnostic and detection parameters u and v

In the first numerical experiment, we consider the case of the diagnosis parameter having a distribution function $F_A(x) = \int_{-\infty}^{x} \frac{1}{20\sqrt{2\pi}} e^{-\frac{(t-120)^2}{2\times 20^2}} \mathrm{d}t$, which gives $q = F_A(u)$. With Matlab 7.0 and the above given parameter values the cost $C(u, v)$ for this case is shown in Figure 1, in which we select the values of v from 20 to 100, and vary u from 80 to 160. It is observed that a minimum cost value per unit time of 11723 is achieved at $u^* = 100$ and $v^* = 77.3023$. So we know that in this case the optimal critical values of diagnostic parameter and optimal expected detection period are 100 and 77.3023, respectively.

For the second numerical example we suppose that the distribution function of diagnosis parameter is given by

$$F_A(x_1, x_2) = \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-\delta(x,y)} \mathrm{d}x\mathrm{d}y,$$

where

$$\delta(x, y) = \frac{1}{2(1-\rho^2)} \left[\frac{(x-\mu_1)^2}{\sigma_1^2} + \frac{(y-\mu_2)^2}{\sigma_2^2} + 2\rho \frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2} \right],$$

$$\mu_1 = 110, \quad \mu_2 = 140, \quad \sigma_1 = \sqrt{477}, \quad \sigma_2 = \sqrt{426}, \quad \rho = \frac{73}{\sigma_1\sigma_2},$$

then we get $q = F_A(u_1, u_2)$. The selection for other parameter values is same as the first example.

In this case, the optimal critical values of diagnostic parameter and optimal expected detection period are $(u_1^*, u_2^*) = (82.4123, 113.3671)$ and $v^* = 65.5237$, respectively, and the minimum cost per unit time is $C\left(u_1^*, u_2^*, v^*\right) = 13624$.

6. **Conclusions.** In this paper, we consider the optimal cost analysis for a multi-state repairable system with repair delays and interruptions, in which the system has two operating states and two failure states. By means of supplementary variable and the theory of differential equation some important reliability indices are derived. We develop a cost function for searching the joint suitable values of diagnostic and detection parameters. Some examples have numerically illustrated the optimization cost. For future research, one could consider the optimal diagnostic and detection policy for some complicated

repairable systems. The results are planned to be reported later in some journals in the field of reliability theory.

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