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STABILITY ANALYSIS OF NETWORKED CONTROL SYSTEMS WITH NETWORK-INDUCED DELAY AND CHANNEL NOISE CONSTRAINT

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Abstract. *This paper investigates the stability issue of single-input single-output (SISO) networked control systems (NCSs) with consideration of the network-induced delay and channel noise. The communication network is characterized by network-induced delay and the channel noise. The minimal value of signal-to-noise ratio (SNR) to stabilize the networked system is obtained by frequency-domain approach. It is shown that the minimal value of SNR is dependent on the network-induced delay, the nonminimum phase zeros and unstable poles of the given plant. The obtained result shows the relationship among the stability of NCSs, structural characteristics of the given plant (non-minimum phase zeros and unstable poles) and communication parameters (network-induced delay). Finally, typical examples are given to illustrate the result.*
Keywords: Network-induced delay, Signal-to-noise ratio (SNR), Networked control systems, Unstable poles

1. **Introduction.** The development of the communication technology combined the communication network and the traditional control system, formed the network control systems (NCSs), using the communication channel to connect the sensor, controller and actuator, and formed a real time closed loop spatial distribution feedback control system [1, 2, 3]. Many advantages appeared with the introduction of the communication channel, for example easy for maintenance and to expand, high reliability and safety. Just because of this, networked control systems have been applied into many areas such as aerospace, military, medical and automatic control [4, 5]. For the traditional control systems, the nonminimun phase zeros and unstable poles could influence the performance limits, but in networked control systems, the performance limits are not only constrained by these factors but also influenced by the communication parameters. Therefore, at present, the performance limit of NCSs has been an important research topic [6].

However, every thing has two sides; because of the realistic characteristics of the communication network, some new constraints are generated, and networked control systems have some disadvantages. In realistic situation, the bandwidth of the communication network is not unlimited, called bandwidth constraint [7]; furthermore, data dropouts [8] and network-induced delay [3, 9] will occur when the information transmitted through bandwidth-limited communication channel. In addition, channel noise could also exist in communication channel [10]. These constraints inevitably affect the performance of NCSs, and even cause NCSs instability. The performance issue of SISO discrete-time NCSs with network-induced delay was studied in [11]. The tracking performance of multiple-input multiple-output (MIMO) NCSs with additive white Gaussian noise (AWGN) and the scaling factor was studied in [12]. The performance issue of SISO networked control systems with the constraint of SNR was studied in [13]. The obtained result shows that the optimal tracking performance is constrained by communication network factors and the internal structure of NCSs.

We study the stability issue of SISO NCSs with the network-induced delay and channel noise. The stability condition of SISO NCSs is attained by using one-parameter compensator structure and applying the methods of co-prime factorization, partial factorization and spectral decomposition. It is shown that the minimal value of SNR is dependent on the network-induced delay, the nonminimum phase zeros and unstable poles of the given plant. The obtained result shows relationship among the stability of NCSs, structural characteristics of given plant (non-minimum phase zeros and unstable poles) and communication parameters (network-induced delay). The results also show that the stability condition is determined by the plant's internal structure and networked parameters, no matter what compensator is adopted. These results will provide a guidance to the design of networked systems.

The proposed model will be applied in lots of realistic systems, such as remote monitoring robot surgery, in which patient is considered as the plant, and robot is considered as the controller. The remote expert obtains information by the network transmission, and information from experts will be returned to the robot by the network transmission. Thus, we study the relationship among the tracking performance, structural characteristics of plant and communication parameters (network-induced delay and channel noise in this paper).

The rest of the paper is organized as follows. Section 2 introduces some preliminaries. In Section 3, we study stability analysis of networked control systems with the network-induced delay and channel noise. Typical examples are given to illustrate the obtained results in Section 4. The paper conclusions and future research directions are presented in Section 5.

2. Preliminaries.

The symbols used in this paper are standard. $\overline{z}$ denotes the conjugate of a complex number z; for any vector u, define u^T and u^H as its transpose and conjugate transpose, respectively. Let the open right-half plane be denoted $\mathbb{C}_+ := \{s : Re(s) > 0\}$, and the open left-half plane is $\mathbb{C}_- := \{s : Re(s) < 0\}$. Denote the Euclidean vector norm as $\| \cdot \|_2$, $\mathcal{L}_2$ is called the Lebesgue spaces standard frequency range and it has the inner product $< f, g >:= (1/2\pi) \int_{-\infty}^{+\infty} tr \left[f^H(jw) g(jw) \right] dw$. $\mathcal{L}_2$ can be decomposed into two orthogonal subspace, and they are defined as $\mathcal{H}_2$ and $\mathcal{H}_2^{\perp}$

$$\mathcal{H}_2 := \left\{ f : f(s) \in \mathbb{C}_+ \, \|f\|_2^2 := \sup_{\sigma > 0} (1/2\pi) \int_{-\infty}^{+\infty} \|f(\sigma + jw)\|_F^2 dw < 0 \right\},$$

$$\mathcal{H}_2^{\perp} := \left\{ f : f(s) \in \mathbb{C}_- \, \|f\|_2^2 := \sup_{\sigma > 0} (1/2\pi) \int_{-\infty}^{+\infty} \|f(\sigma + jw)\|_F^2 dw < 0 \right\}.$$

Finally, $\mathcal{RH}_\infty$ defines all stable, proper rational function.

We discuss the stability problem of SISO networked control systems with network-induced delay such as Figure 1.

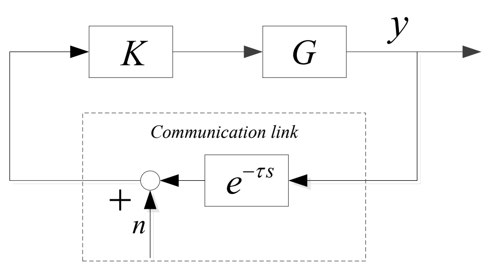

FIGURE 1. The NCSs with network-induced delay

In Figure 1, G represents the given plant and K is denoted as one-parameter compensator, whose transfer function matrices are $G(s)$ and $K(s)$, respectively. $y(t)$ and $n(t)$ are denoted as the system output signal and channel noise, respectively. $\hat{y}(s)$ and $\hat{n}(s)$ represent their Laplace transform, $\hat{n}(s)$ is assumed to be a zero mean white noise sequence and variance Φ. τ represents the network-induced delay.

Furthermore, in the practical application, the energy of the communication channel having an upper bound, is defined as

$$E\left\{\|\hat{y}\|^2\right\} < \Gamma.$$

where Γ is the upper bound of energy of the communication channel.

According to Figure 1, we can get

$$\hat{y} = GK\left(e^{-\tau s}\hat{y} + \hat{n}\right). \tag{1}$$

From [15], we have

$$S_{\hat{y}}(j\omega) = \frac{G(j\omega)K(j\omega)}{1 - e^{-\tau s}G(j\omega)K(j\omega)} S_{\hat{n}\hat{y}}(j\omega).$$

Furthermore,

$$E\left\{\|\hat{y}\|^2\right\} = P = \left\|\frac{GK}{1 - e^{-\tau s}GK}\right\|_2^2 \Phi \tag{2}$$

where P is represented as input energy of communication channel. Denote the SNR such as $\gamma = \frac{P}{\Phi}$; therefore, if the system is stable, $\gamma = \frac{P}{\Phi}$ should meet

$$\left\|\frac{GK}{1 - e^{-\tau s}GK}\right\|_2^2 < \frac{P}{\Phi}. \tag{3}$$

3. **Stability Analysis of Networked Control Systems.** The main goal of this paper is to get the minimum SNR of the NCSs, which makes the system stable. We denote $\frac{P}{\Phi} \geq J^*$, then

$$J^* = \inf_{K \in \mathcal{K}} \left\|\frac{GK}{1 - e^{-\tau s}GK}\right\|_2^2. \tag{4}$$

According to [16], every stabilizing compensator K can be expressed as Youla parameterization

$$\mathcal{K} := \left\{K : K = \frac{(Y - MQ)}{X - NQ},\ Q \in \mathcal{RH}_\infty\right\} \tag{5}$$

and satisfy the Bezout equation such as

$$MX - NY = 1 \tag{6}$$

where $N, M \in \mathcal{RH}_\infty$.

For the transfer function $e^{-\tau s}G$, we consider a coprime factorization of $e^{-\tau s}G$ as

$$e^{-\tau s}G = NM^{-1}. \tag{7}$$

For the nonminimum phase transfer function, it could be factorized as the product of the minimum phase part and all pass factor, and thus we have

$$N = e^{-\tau s}L_z N_n, \quad M = B_p M_m \tag{8}$$

where L_z and B_p are represented as all pass factors and N_n, M_m are the minimum phase parts; furthermore, L_z includes all the right half plane zeros of the plant $z_i \in \mathbb{C}_+$, $i = 1, \cdots, N_z$, B_p includes all the right half plane poles of plant $p_j \in \mathbb{C}_+$, $j = 1, \cdots, N_p$, and L_z and B_p can be decomposed respectively

$$L_z(s) = \prod_{i=1}^{N_z} \frac{s - z_i}{s + \bar{z}_i}, \quad B_p(s) = \prod_{j=1}^{N_p} \frac{s - p_j}{s + \bar{p}_j} \tag{9}$$

Theorem 3.1. *Considering the network control systems such as Figure 1, in order to make the network control systems stable, the SNR must be satisfied*

$$\frac{P}{\Phi} > \sum_{i,j=1}^{N_p} \frac{4Re(p_j)Re(p_i)}{(\bar{p}_j + p_i)\,\bar{b}_j b_i} \left(e^{\tau s} L_z^{-1}(p_i)\right)^2$$

where $b_j = \prod\limits_{\substack{i\in N \\ i\neq j}}^{N_p} \frac{p_i - p_j}{p_j + \bar{p}_i}$.

Proof: According to (4), (5), (6), (7) and (8), we can get

$$J^* = \inf_{Q\in\mathcal{Q}} \|L_z N_n (Y - MQ)\|_2^2.$$

According to (8) and B_P is represented as all pass factors, we can write

$$J^* = \inf_{Q\in\mathcal{Q}} \left\| \frac{N_n Y}{B_p} - N_n M_m Q \right\|_2^2.$$

According to partial fraction expansion

$$\frac{N_n Y}{B_p} = \sum_{j=1}^{N_p} \frac{\bar{p}_j + s}{s - p_j} \frac{N_n(p_j) Y(p_j)}{b_j} + R_1,$$

where $R_1 \in \mathcal{RH}_\infty$, $b_j = \prod\limits_{\substack{i\in N \\ i\neq j}}^{N_p} \frac{p_i - p_j}{p_j + \bar{p}_i}$, therefore

$$J^* = \inf_{Q\in\mathcal{Q}} \left\| \sum_{j=1}^{N_p} \left(\frac{\bar{p}_j + s}{s - p_j} - 1 \right) \frac{N_n(p_j) Y(p_j)}{b_j} + R_1 + \sum_{j=1}^{N_p} \frac{N_n(p_j) Y(p_j)}{b_j} - N_n M_m Q \right\|_2^2.$$

Because of $\sum_{j=1}^{N_p} \left(\frac{\bar{p}_j + s}{s - p_j} - 1 \right) \frac{N_n(p_j) Y(p_j)}{b_j} \in \mathcal{H}_2^\perp$ and $\left(R_1 + \sum_{j=1}^{N_p} \frac{N_n(p_j) Y(p_j)}{b_j} - N_n M_m Q \right)$ $\in \mathcal{H}_2$. Hence, we can get

$$J^* = \left\| \sum_{j=1}^{N_p} \left(\frac{\bar{p}_j + s}{s - p_j} - 1 \right) \frac{N_n(p_j) Y(p_j)}{b_j} \right\|_2^2 + \inf_{Q\in\mathcal{Q}} \left\| R_1 + \sum_{j=1}^{N_p} \frac{N_n(p_j) Y(p_j)}{b_j} - N_n M_m Q \right\|_2^2.$$

From (6) and $M(p_j) = 0$, after the calculation, we have

$$N_n(p_j) Y(p_j) = -e^{\tau p_j} L_z^{-1}(p_j).$$

Thus, J^* can be obtained

$$J^* = \left\| \sum_{j=1}^{N_p} \left(\frac{\bar{p}_j + s}{s - p_j} - 1 \right) \frac{-e^{\tau p_j} L_z^{-1}(p_j)}{b_j} \right\|_2^2 + \inf_{Q\in\mathcal{Q}} \left\| R_1 + \sum_{j=1}^{N_p} \frac{-e^{\tau p_j} L_z^{-1}(p_j)}{b_j} - N_n M_m Q \right\|_2^2.$$

Because N_n and M_m are the minimum phase parts and $R_1 \in \mathcal{RH}_\infty$, $Q \in \mathcal{RH}_\infty$, we can choose an appropriate Q, making

$$\inf_{Q\in\mathcal{Q}} \left\| R_1 + \sum_{j=1}^{N_p} \frac{-e^{\tau p_j} L_z^{-1}(p_j)}{b_j} - N_n M_m Q \right\|_2^2 = 0.$$

The proof is completed.

The obtained theorem shows that the SNR of NCSs is related to the nonminimum phase zero, the unstable pole of given plant and the network-induced delay in the communication channel.

4. **Illustrative Example.** In this section, typical examples are given to illustrate the efficiency of the result.

Example 4.1. *Consider the given plant such as* $G(s) = \frac{s-0.2}{(s-0.5)(s+3)}$. *In this plant, it is clear to see that the unstable pole is located at* $p = 0.5$ *and nonminimum zero is* $z = 0.2$. *According to the theorem, we can get the minimal SNR such as* $J^* = 2.333e^{\tau}$.

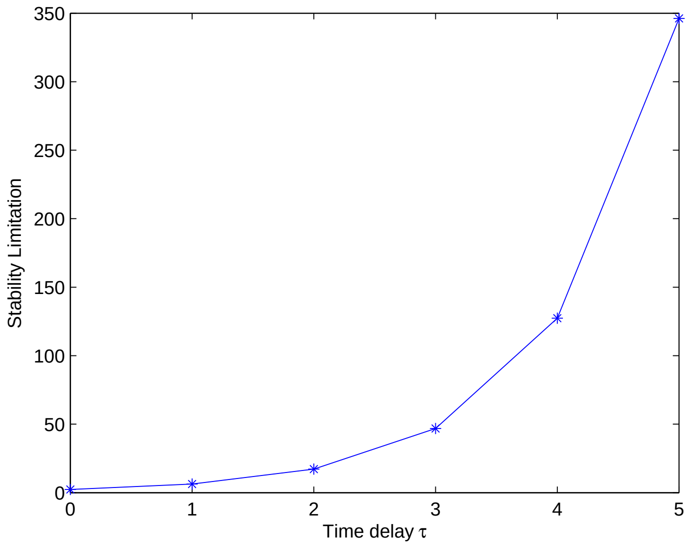

FIGURE 2. The minimal SNR under different network-induced delay

From Figure 2, we can see that the minimum SNR is affected by the network-induced delay of the communication channel, and it can also be seen that the larger the network induced delay is, the greater the SNR of NCSs will be.

Example 4.2. *Consider the unstable plant model described by*

$$G(s) = \frac{s-k}{s(s-2)(s+1)}.$$

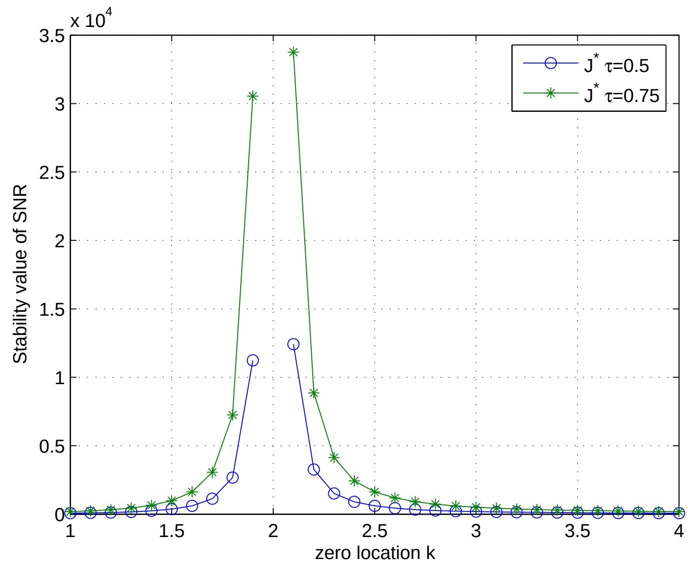

FIGURE 3. The minimal SNR under different NMP zero

This plant is a nonminimum phase. For any $k > 0$, the nonminimum phase zero is located at $z_1 = k$, and it has an unstable pole at $p_1 = 2$.

τ^1 and τ^2 represent different network induced delay, respectively.

$$\tau^1 = 0.5, \quad \tau^2 = 0.75$$

From Theorem, the stability value of SNR is obtained

$$J^* = 4\left(e^{2\tau}\frac{2+k}{2-k}\right)^2.$$

The stability value of SNR with different NMP zero or networked induced-delay is shown in Figure 3. It can be seen from Figure 3 that the stability value of SNR has been degraded because of the networked induced-delay.

5. **Conclusion.** This paper studies the stability issue of networked control systems with network-induced delay and channel noise constraints in communication channel. The minimal value of SNR to stabilize the networked system is obtained by co-prime factorization, partial factorization and spectral decomposition. The obtained result shows that the minimal SNR is dependent on two aspects: one factor is the communication channel parameter such as network-induced delay and channel noise, the other aspect is the internal structure of the given plant such as nonminimum phase zero and unstable poles.

Possible future extensions to this work include studying more general plants such as multiple-input multiple-output nonlinear networked control systems, more complex channel model, e.g., the fading channel case, and more parameters of communication channel constraints such as quantization effect, and the bandwidth effect.

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