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THE CALCULATION OF FALL USING A RIGID BODY MODEL IN WHICH THE LOWER END MOVES FREELY

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Abstract. *Aging involves various diseases and impairments. This is a very important problem in Japan as well as other developed countries. Elderly people often fall in their residence or the hospital. Falling sometimes brings about serious damage such as fracture or subdural hematoma. In some cases, patients become bedridden. It is a risk factor of dementia. Therefore, we have conducted studies about falling for some years, treating cases with constraint forces such as friction. In this paper, we calculated the behavior of a rod just after falling without friction. We used the model of the rigid rod whose lower end moves freely. We also explain the features of the solution.*
Keywords: Fall, Rigid rod, Normal force, The velocity of the upper end

1. **Introduction.** Recently, Japan has problems with declining birth rate and aging population. In general, elderly people easily fall down and those accidents sometimes lead to patients being bedridden. As a result, medical care is getting to require large amounts of money. Therefore, research, on how to prevent falling is very important for modern Japanese society. We have studied this problem for the past several years [1-3]. At first we used a rigid rod with a fixed bottom end as a mode, in which the upper end was regarded as a human head. We investigated the speed just before colliding with a floor and how constraint forces such as normal force and friction force behave during falling. We also considered a model with a bottom end that can move freely without friction force as a primary study.

In this study, we investigated the latter case more precisely. We calculated the behavior of the upper end of the rod just after beginning to fall. We found that the velocity of the upper end will increase in non-linear way more rapidly than we had expected.

2. **The Rigid Model.** We will consider a case where the lower end of the rod moves freely. That is to say, it is assumed that the lower end of the rod is not fixed and the surface of the floor is smooth and has no friction force. In this case, two parameters are needed to express the motion of the rod, as shown in Figure 1.

The x in the axis of abscissas is the position of the center of mass of the rod in the horizontal direction. The angle θ is the degree of leaning from the horizontal plane and l is the length of the rigid body.

3. **The Equation of Motion and the Approximation.** We assume that the mass of this rigid body is m and its mass density is constant. The total kinetic energy is the sum of the total kinetic energy in the center of the mass frame and the kinetic energy of the total mass in the inertia frame. Let T and V be the total kinetic energy and potential

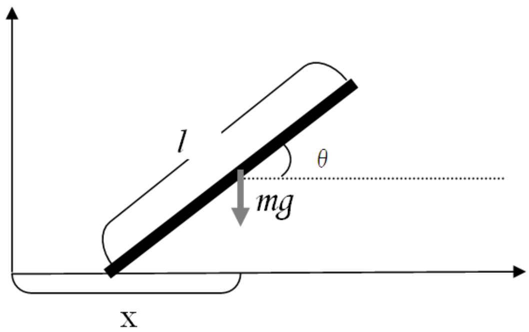

FIGURE 1. The rigid model whose lower end can move freely

energy, respectively. They can be expressed as below, respectively [4]:

$$\begin{aligned} T &= \frac{I}{2}\dot{\theta}^2 + \frac{1}{2}m\dot{x}^2 + \frac{1}{8}ml^2\dot{\theta}^2\cos^2\theta \\ V &= \frac{l}{2}mg\sin\theta \end{aligned} \tag{1}$$

where I is the moment of inertia around the axis perpendicular to the plain containing the rod in the position of the center of mass and is given by $I = \frac{1}{12}ml^2$.

Let L be the Lagrangian of the rod in this coordinate system. It can be given by

$$L = T - V = \frac{ml^2}{24}\dot{\theta}^2 + \frac{1}{2}m\dot{x}^2 + \frac{1}{8}ml^2\dot{\theta}^2\cos^2\theta - \frac{l}{2}mg\sin\theta \tag{2}$$

We can derive the equation of motion from this Lagrangian in a familiar way. Consequently, the set of the equations of motion is described as

$$l\ddot{\theta} - 3l\dot{\theta}^2\sin\vartheta\cos\theta + 3l\ddot{\theta}\cos^2\theta + 6g\cos\theta = 0 \tag{3}$$

$$m\ddot{x} = 0 \tag{4}$$

Equation (4) tells us the well-known fact that the center of mass moves with constant velocity under the condition that there is no force in the horizontal direction. On the contrary, since Equation (3) is non-linear and very complicated differential equation of θ, it is very difficult to solve it analytically in general. However, in this study we are interested in the motion of the rod just after beginning to fall. In this case, θ can be supposed to be nearly equal to 90 degrees. Thus, we adopt the approximation as below.

$$\theta \cong 90 \tag{5}$$

$$\alpha = 90 - \theta \tag{6}$$

If we transform θ to α, then Equation (3) will become

$$l\ddot{\alpha} + l\dot{\alpha}^2\sin\alpha\cos\alpha + 3l\ddot{\alpha}\sin^2\alpha - 6g\sin\alpha = 0 \tag{7}$$

To make the equation simple, we will use the notation f instead of $\frac{g}{l}$. Then we get the following equation.

$$\ddot{\alpha} + \dot{\alpha}^2\sin\alpha\cos\alpha + 3\ddot{\alpha}\sin^2\alpha - 6f\sin\alpha = 0 \tag{8}$$

In this case, it can be considered that

$$\alpha \ll 1, \quad \sin\alpha \cong \alpha, \quad \cos\alpha \cong 1 \tag{9}$$

Using this approximation changes Equation (8) into

$$\ddot{\alpha} + \dot{\alpha}^2\alpha + 3\ddot{\alpha}\alpha^2 - 6f\alpha = 0 \tag{10}$$

Equation (10) can be integrated analytically as below.

$$\ddot{\alpha}\left(1 + 3\alpha^2\right) + \left(\dot{\alpha}^2 - 6f\right)\alpha = 0$$

$$\ddot{\alpha}\left(1+3\alpha^2\right)=\left(-\dot{\alpha}^2+6f\right)\alpha$$
$$\frac{\ddot{\alpha}}{(-\dot{\alpha}^2+6f)}=\frac{\alpha}{(1+3\alpha^2)}$$
$$\frac{\ddot{\alpha}\dot{\alpha}}{(-\dot{\alpha}^2+6f)}=\frac{\alpha\dot{\alpha}}{(1+3\alpha^2)}$$

We can integrate both terms at this stage, as is described below.

$$-\frac{1}{2}\log\left|-\dot{\alpha}^2+6f\right|=\frac{1}{6}\log\left|1+3\alpha^2\right|+C_1' \tag{11}$$

C_1 is the integral constant. Calculating formulation (11) leads to the solution as below.

$$\dot{\alpha}^2=C_1\left(1+3\alpha^2\right)^{-\frac{1}{3}}+6f \tag{12}$$

We must remember that the initial condition is as follows:

$$\alpha(0)=0,\quad \dot{\alpha}(0)=0 \tag{13}$$

Considering these initial conditions, it turns out that the integral constant C_1 should be $-6f$. Then we finally got the solution as follows:

$$\dot{\alpha}=\sqrt{6f\left\{1-(1+3\alpha^2)^{-\frac{1}{3}}\right\}}=\sqrt{6\frac{g}{l}\left\{1-(1+3\alpha^2)^{-\frac{1}{3}}\right\}} \tag{14}$$

4. **Result.** Using Equation (14), the velocity V_0 of the upper end of the rod just after beginning to fall will be given by

$$\begin{aligned} V_0 &= l\dot{\alpha}\sqrt{\left(\frac{5}{4}\sin^2\alpha+1\right)} \\ &= l\sqrt{6\frac{g}{l}\left\{1-(1+3\alpha^2)^{-\frac{1}{3}}\right\}}\sqrt{\left(\frac{5}{4}\alpha^2+1\right)} \\ &= \sqrt{6gl\left\{1-(1+3\alpha^2)^{-\frac{1}{3}}\right\}}\sqrt{\left(\frac{5}{4}\alpha^2+1\right)} \\ &\cong \left(\sqrt{6gl}\right)\left(\alpha+\frac{5}{8}\alpha^3\right) \end{aligned} \tag{15}$$

Equation (15) tells us that the velocity will increase in a non-linear way as the angle of the degree of leaning of the rigid rod increases. Once the person begins to fall, the velocity of the head will increase very quickly. According to our previous study [4], the velocity of the upper end just before colliding with the floor is as fast as the velocity of a bike. Therefore, there would be some people who cannot avoid falling down. It may cause serious damages, such as head injury, skull fracture or subdural hematoma. This result is consistent with clinical experiences.

5. **Conclusions.** In this paper, we calculated the velocity of the upper end of a rod just after falling. Then we could get the analytical solution under the primary approximation described above. We found that the velocity will increase in a non-linear way as the angle of the degree of leaning of the rigid rod increases. This finding tells us that once an aged person begins to fall, it is very difficult to avoid falling down on the floor. In our clinical experiences, falling sometimes results in impaired consciousness or syncope. The large angular acceleration may play some role in this event. Thus, understanding the accurate features of falling can help us to elucidate the mechanism of the clinical damages and find out how to prevent falling.

For this purpose, we will try to expand this result to the whole motion of the rod during falling.

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