ICIC Express Letters
Part B: Applications
Volume **7**, Number **2**, February **2016**

ICIC International ©2016 ISSN 2185-2766
pp. 357–362

A CLASS OF TIME-DELAY DISTURBANCE DISCRETE SYSTEM FOR ITERATIVE LEARNING CONTROL

Yinjun Zhang[1,2,*], Yinghui Li[1] and Jianhuan Su[2]

[1]Aeronautics and Astronautics Engineering Institute
Air Force Engineering University
No. 1, East Changle Road, Xi'an 710038, P. R. China
*Corresponding author: txeb@163.com

[2]School of Physics and Electrical Engineering
Hechi University
No. 42, Longjiang Road, Yizhou 546300, P. R. China

Received August 2015; accepted November 2015

Abstract. *The work is connected with the development of stability theory methods for a class of linear discrete time-delay system for iterative learning control with multi-state multi-input and measurement noise. This research works out the specific control law for the system, and proves its robustness and convergence via 2-dimensional linear inequalities and mathematical induction. It allows us to obtain new interesting results both for theory and for applications, and also for knowledge theory as a whole. The new algorithm can also bring a new idea to apply the adaptive algorithm.*

Keywords: Discrete system, Time delay, Iterative learning control, Disturbance

1. **Introduction.** During the last two decades, the study on stability analysis for time-delay systems has been widely investigated. Time delay occurs in various physical, industrial and engineering systems such as biological systems, neural networks, networked control systems, and multi-agent systems. It is well known that the existence of time delay is a source of poor performance and instability of dynamic systems. For more details, see [7,8,11,13,14] and references therein.

On the other hand, these days, most systems use digital computers. Therefore, discrete-time modeling with time delay plays an important role in many fields of science and engineering applications. In this regard, various approaches to stability and stabilization for discrete-time systems with time delay have been investigated in the literature, and parameter identification of continuous-time systems using iterative learning control is presented [3-5,9,10,12].

Iterative learning control (ILC) is an approach for improving the transient performance of systems that operate repetitively over a fixed time interval [1,2]. Owing to its simplicity and effectiveness, ILC has been found to be a good alternative in many areas and applications (see, for instance, [6] and the references therein). Iterative learning control was first proposed by Arimoto et al. in 1984 in Reference [1]. Since then, ILC has become a very important issue of control field. A lot of achievements have been published as References [2-4]. Many of the systems discussed before are described by ordinary differential equations. However, there are so many systems that can be modeled by partial differential equations but relevant papers are rarely seen. On the other hand, discrete system cannot be approached by ordinary differential equations. So ILC of discrete system is a very important research field.

The paper studied a class of time-delay time-varying discrete system for iterative learning control. Sufficient conditions are given for the convergence of system by employing special norm. Numerical simulation is presented for a discrete system solved using ILC based on Eular difference format. The numerical example is provided to illustrate the effectiveness of the proposed method.

2. **Problem Formulation.**
Consider a class of time-delay time-varying discrete system as follows:

$$x(t+1,k) = A(t)x(t,k) + A_1(t)x(t-\tau,k) + B(t)u(t,k) + W(t,k) \tag{1a}$$

$$y(t,k) = C(t)x(t,k) + V(t,k) \tag{1b}$$

where τ is the state time-delay, which satisfies $0 \le \tau \le t_0$; $x(t,k)$, $x(t+1,k) \in R^n$ are the state vector; $A(t)$, $B(t)$, $C(t)$ are the unknown matrices representing parametric uncertainties in the state matrices; $W(t,k) \in R^n$, $V(t,k) \in R^r$ are the state disturbance and measurement noise; $y_d(t) \in R^r$ is expected output trajectory; $u(t,k) \in R^m$ is input vector of the system. For system (1), P-type learning law is being applied.

$$u(t,k+1) = u(t,k) + P(t+1)e(t+1,k) \tag{2}$$

where $e(t,k)$ is output error and $e(t,k) = y_d(t) - y(t,k)$.

For the time-delay varying discrete system (1), the following assumptions can be made.

(1) When there are not initial value and disturbance, namely $x(t_0,k) = x_d(t_0)$, $W(t,k) = 0$, $V(t,k) = 0$, the expected trajectory can achieve.

$$\begin{cases} x_d(t+1,k) = A(t)x_d(t,k) + A_1(t)x_d(t-\tau,k) + B(t)u_d(t,k) \\ y(t,k) = C(t)x_d(t,k) \end{cases} \tag{3}$$

where $x_d(t)$, $y_d(t)$, $u_d(t)$ are expected state trajectory, expected output trajectory and ideal control respectively.

(2) The following conditions should be satisfied $\|x(t_0,k) - x_d(t_0)\| \le b_{x0}$, $k = 0,1,2,\cdots$; $\|W(t,k)\| \le b_w$, $k = 0,1,2,\cdots$; $\|V(t,k)\| \le b_v$, $k = 0,1,2,\cdots$;

(3) $B(t)$, $C(t)$ are row full rank.

For the error result, we consider two conditions as follows.

(1) When $k \to \infty$, if $\lim_{k\to\infty} \|e(t,k)\| \le \delta$, δ is a smaller positive integers, then we say the system has robustness.

(2) When $k \to \infty$, if $\lim_{k\to\infty} \|e(t,k)\| \le 0$, then the system is convergent.

Convergence is the most important problem for iterative learning control algorithm.

Theorem 2.1. *If system (1) meets these assumptions (1-3), we use the iterative learning control law (2) in the interval* $[t_0, t_0+T]$*, if it satisfies the following assumption, then it is being proved.*

(1) $\|I - C(t)B(t-1)P(t)\| \le \rho < 1$*, then the output error has boundary. Namely* $\lim_{k\to\infty} \|e(t,k)\| \le \delta$.

(2) $\lim_{k\to\infty} x(t,k) = x_d(t)$, $t \le t_0$, $\lim_{k\to\infty} W(t,k) = W^*(t)$, $\lim_{k\to\infty} V(t,k) = V^*(t)$, *then when* $e(t,k) \to 0$, $\lim_{k\to\infty} \|e_k(t,k)\| = 0$ *for* $k \to \infty$.

3. **The Proofs of Robustness and Convergence.**
Next we will prove Theorem 2.1.

Proof: Let

$$\eta(t,k) = x(t-1,k+1) - x(t-1,k) \tag{4}$$

From Equation (1a), we know

$$\begin{aligned} \eta(t+1,k) &= x(t,k+1) - x(t,k) \\ &= A(t-1)\eta(t,k) + A_1(t-1)\eta(t-\tau,k) + B(t-1)P(t)e_k(t,k) \\ &\quad + W(t-1,k+1) - W(t-1,k) \end{aligned} \tag{5}$$

Hence

$$\begin{aligned} &e(t,k+1) \\ =\ &y_d(t) - y(t,k+1) \end{aligned}$$

$$= e_k(t,k) + y(t,k) - y(t,k+1)$$
$$= \|I - C(t)B(t-1)P(t)\|e(t,k) - C(t)[A_1(t-1)\eta(t-\tau,k) + A(t-1)\eta(t,k)]$$
$$-C(t)[W(t-1,k+1) - W(t-1,k)] - [V(t,k+1-V(t,k)] \quad (6)$$

For Equation (5) and Equation (6), we use matrices to express. Let

$$\overline{A(t)} = \begin{bmatrix} A(t-1) & 0 & \cdots & A_1(t-1) & 0 & \cdots & A_{I-1}(t-1) & \cdots & 0 & A_I(t-1) \\ I & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & I & 0 & \cdots & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & I & 0 \end{bmatrix}$$

$$\theta(t,k) = [\eta(t,k)\ \ \eta(t-1,k)\ \ \cdots\ \ \eta(t-\tau_1,k)\ \ \cdots\ \ \eta(t-\tau_2,k)\ \ \cdots\ \ \eta(t-\tau_{I-1},k)\ \ \cdots\ \ \eta(t-\tau_I,k)]^T$$

$$\overline{B(t)} = [B(t-1)P(t)\ \ 0\ \ \cdots\ \ 0\ \ \cdots\ \ 0\ \ \cdots\ \ 0\ \ \cdots\ \ 0]^T$$

$$\overline{C(t)} = [-C(t-1)A(t-1)\ \ 0\ \ \cdots\ \ -C(t)A_1(t-1)\ \ \cdots\ \ -C(t)A_2(t-1)\ \ \cdots\ \ -C(t)A_{I-1}(t-1)\ \ \cdots\ \ -C(t)A_I(t-1)]^T$$

$$\Delta\overline{W(t)} = [W(t-1,k+1) - W(t-1,k)\ \ 0\ \ \cdots\ \ 0\ \ \cdots\ \ 0\ \ \cdots\ \ 0\ \ \cdots\ \ 0]^T$$

Equations (5) and (6) can be transformed Equation (7).

$$\theta(t+1,k) = \overline{A(t)}\theta(t,k) + \overline{B(t)}e(t,k) + \Delta\overline{W(t)} \quad (7)$$
$$e(t,k+1) = \overline{C(t)}\theta(t,k) + [I - C(t)B(t-1)P(t)]e(t,k)$$
$$-C(t)[W(t-1,k+1) - W(t-1,k)] - [V(t,k+1) - V(t,k)] \quad (8)$$

According to Equation (5) and the definition of $\theta(t,k)$, we know $\|\theta(t_0,k)\|$ is bounded. From $e(t,0) = y_d(t) - y(t,0)$, we know $e(t,0)$ is bounded. As there exists disturbance and measurement noise, we use induction method to prove it.

1) When $t = t_0 + 1$, for Equation (8), we have and take the $e(t_0+1,k)$ norm

$$\|e(t_0+1,k)\| \leq \left\|\overline{C(t_0+1)}\theta(t_0+1,k)\right\|$$
$$+\|[I - C(t_0+1)B(t_0)P(t_0+1)]\|\|e(t_0+1,k)\|$$
$$+\|C(t_0+1)[W(t_0,k+1) - W(t_0,k)]\|$$
$$+\|V(t_0+1,k+1) - V(t_0,k)]\| \quad (9)$$

Let

$$d(t_0+1) = sup_k\left(\left\|\overline{C(t_0+1)}\theta(t_0+1,k)\right\| + \|C(t_0+1)\left[W(t_0,k+1) - W(t_0,k)\right]\|\right)$$
$$+sup_k\|[V(t_0+1,k+1) - V(t_0,k)]\| \quad (10)$$

As $\left\|\overline{C(t_0+1)}\theta(t_0+1,k)\right\|$ and $\|C(t_0+1)\left[W(t_0,k+1) - W(t_0,k)\right]\|$ are bounded, $d(t_0+1)$ is bounded.

So inequality (9) can be rewritten

$$\|e(t_0+1,k)\| \leq \|e(t_0+1,0)\| + \frac{d(t_0+1)}{1-\rho} \quad (11)$$

As $sup_k\|e(t_0+1,k)\|$ is bounded and because $\|\theta(t_0+1,k)\|$ is bounded, $sup_k\|\theta(t_0+1,k)\|$ is bounded.

2) When $t = l$, then $sup_k\left\{\begin{array}{c}\|\theta(l,k)\| \\ \|e(l,k)\|\end{array}\right\}$ is bounded.

3) When $t = l+1$, set

$$d(l+1) = sup_k\left(\left\|\overline{C(l+1)}\theta(l+1,k)\right\| + \|C(l+1)[W(l,k+1) - W(l,k)]\|\right)$$
$$+sup_k\|[V(l+1,k+1) - V(l+1,k)]\| \quad (12)$$

Similarly

$$\|e(l+1,k)\| \leq \rho^k \|e(l+1,0)\| + \sum_{j=0}^{k-1} \rho^{k-j-1} d(l+1) \leq \|e(l+1,0)\| + \frac{d(l+1)}{1-\rho} \tag{13}$$

When $t = l+1$, we can get that $\|\theta(l+1,k)\|$ is bounded, so $sup_k \|e(l+1,k+1)\|$ has bounded. Thus, we prove the robustness of system.

In addition, when $\lim_{k\to\infty} W(t,k) = W^*(t)$, $\lim_{k\to\infty} V(t,k) = V^*(t)$, set $v_1 = C(t)$ $[W(t-1,k+1) - W(t-1,k)] + [V(t,k+1) - V(t,k)]$, taking the limit, we have

$$\lim_{k\to\infty} v_1 = \lim_{k\to\infty} C(t)[W(t-1,k+1) - W(t-1,k)] + \lim_{k\to\infty} [V(t,k+1) - V(t,k)] = 0 \tag{14}$$

When $\lim_{k\to\infty} x(t,k) = x_d(t)$, $t \leq t_0$, $\lim_{k\to\infty} \eta(t+1,k) = \lim_{k\to\infty}(x(t,k+1) - x(t,k)) = 0$.

We take the norm of Equation (6),

$$\begin{aligned} \|e(t+1,k)\| \leq & \left\|\overline{C(t)}\theta(t,k)\right\| + \|I - C(t)B(t-1)P(t)\|\|e(t,k)\| \\ & + \|-C(t)[W(t-1,k+1) - W(t-1,k)] - [V(t,k+1) - V(t,k)]\| \end{aligned} \tag{15}$$

Then take the limit of Equation (15)

$$\lim_{k\to\infty} \|e(t+1,k)\| \leq \lim_{k\to\infty} \|I - C(t)B(t-1)P(t)\|\|e(t,k)\| \tag{16}$$

Because $\|I - C(t)B(t-1)P(t)\| < \rho < 1$, we have $\lim_{k\to\infty} \|e(t,k)\| \leq \lim_{k\to\infty} \rho^k \|e(t,0)\| = 0$. Thus, the convergence can be proved.

4. **Simulation Analysis.** In this section, we will use some examples to illustrate that the system is robust and convergent. We construct the system as follows:

$$\begin{aligned} x(t+1,k) = & \begin{bmatrix} -0.5\cos t & 0.1\sin t \\ 0.2 & -0.4\sin t \end{bmatrix} x(t,k) + \begin{bmatrix} -0.01 & 0.02t \\ 0.5\cos t & 0.21 \end{bmatrix} x(t-4,k) \\ & + \begin{bmatrix} 0.011t \\ 0.53t \end{bmatrix} u(t,k) + W(t,k) \\ y(t,k) = & [0.09 \;\; 0.0033t] x(t,k) + V(t,k) \end{aligned} \tag{17}$$

The expected trajectory is $y_d(t) = 0.005t^2 + 0.008$. The learning gain is $K(k) = 0.5(C(t)B(t-1))^T \left[C(t)B(t-1)(C(t)B(t-1))^T\right]^{-1}$, and $C(t)B(t-1)$ are row full matrices.

Part A: Let the interference and initial value offset be limited stochastic error. The initial value offset, state interference and output interference satisfy $\|x(t_0,k) - x_d(t_0)\| \leq 0.02$, $k = 0,1,2,\cdots$, $\|W(t,k)\| \leq 0.02$, $k = 0,1,2,\cdots$, $\|V(t,k)\| \leq 0.06$, $k = 0,1,2,\cdots$ respectively. Let the interference be bounded and verify robustness of the system.

Part B: Let the interference and initial value offset be convergent.

The initial value offset, state interference and output interference satisfy $\lim_{k\to\infty} x(t_0,k) = x^*(t_0)$, $\lim_{k\to\infty} W(t,k) = W^*(t)$, $\lim_{k\to\infty} V(t,k) = V^*(t)$ respectively.

For part A, we assume $\|x(t_0,k) - x_d(t_0)\| = 0.02(2rand(2,1) - ones(2,1))$, $W(t,k) = 0.2(2rand(2,1) - ones(2,1))$, $V(t,k) = 0.06(2rand(2,1) - ones(2,1))$.

For system (1), the simulation results are shown in Figures 1 and 2.

For part B, we assume: $[x_d(0) - x(0,k)] = \begin{bmatrix} 0.1^k \\ 0.1 + 0.5^k \end{bmatrix}$, $\|W(t,k)\| = \left(0.5^k + 1\right) \begin{bmatrix} \sin t \\ \sin t \end{bmatrix}$, $\|V(t,k)\| = [(k+1)^{-4} + 0.5]\cos t$. For system (1), the simulation results are shown in Figures 3 and 4.

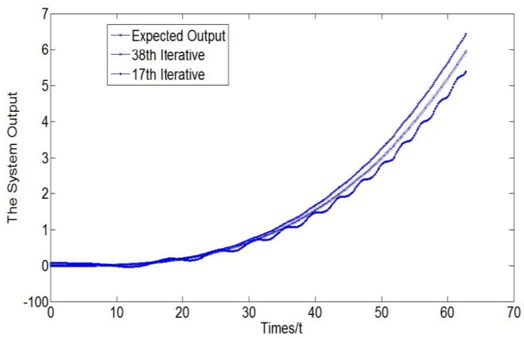

Figure 1. Comparison with 17^{th}, 38^{th} iterative output trajectory and expected output trajectory

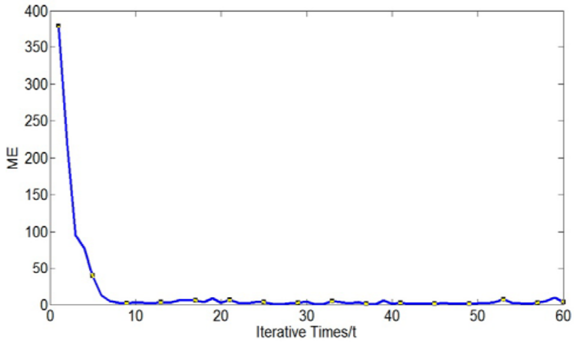

Figure 2. The relationship between iterative times and ME

$$\text{ME (Maximum Error)} = \begin{array}{cccccc} 379.0883 & 216.3278 & 94.9492 & 77.0205 & 38.8554 & 12.9564 \\ 5.1270 & 2.6077 & 1.5576 & 3.3849 & 2.8157 & 2.2167 \\ 3.1503 & 2.3755 & 6.0973 & 5.8784 & 5.8200 & 3.4241 \\ 8.6467 & 2.8612 & 6.5419 & 3.0400 & 1.6388 & 3.5837 \\ 3.2528 & 1.1944 & 0.7497 & 2.0157 & 2.9416 & 3.9901 \\ 0.6243 & 1.0232 & 5.0303 & 3.5680 & 1.6472 & 3.4707 \\ 1.0529 & 0.4023 & 5.5902 & 1.2127 & 2.5782 & 1.4675 \\ 1.5536 & 1.0690 & 1.3159 & 2.4366 & 1.0192 & 1.0692 \\ 1.3709 & 2.2331 & 1.6842 & 3.2071 & 7.3576 & 2.6422 \\ 1.5728 & 1.3386 & 2.960l & 5.2212 & 9.3661 & 3.1395 \end{array}$$

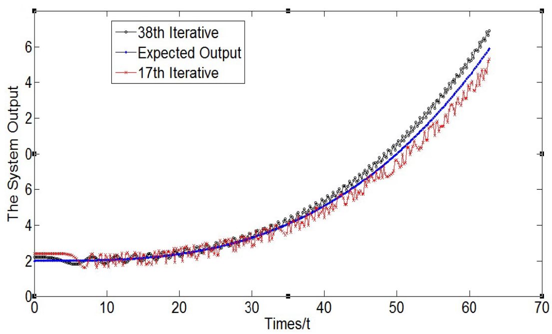

Figure 3. Comparison with 17^{th}, 38^{th} iterative output trajectory and expected output trajectory

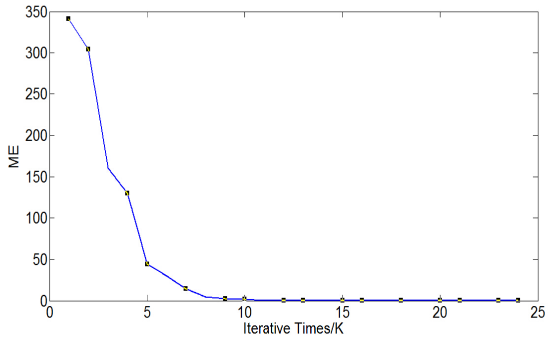

Figure 4. The relationship between iterative times and ME

$$\text{ME} = \begin{array}{ccccc} 340.9760 & 304.5506 & 160.5417 & 130.1544 & 44.0637 \\ 30.2087 & 14.2487 & 4.3268 & 2.7438 & 1.6982 \\ 0.4861 & 0.4177 & 0.3273 & 0.1830 & 0.1634 \\ 0.1161 & 0.0953 & 0.0951 & 0.0824 & 0.0747 \\ 0.0747 & 0.0729 & 0.0701 & 0.0693 & 0.0543 \end{array}$$

5. **Conclusions.** This paper presented a class of time-delay discrete system for iterative learning control algorithm. According to comprehensive analysis and summary, we design the controller of discrete system. We structure liner inequality based on 2-D theory and prove the system robustness and convergence.

According to improvement of the ordinary discrete iterative learning control algorithm, we propose and prove the improving algorithm with time-delay and disturbance in discrete system that has monotone convergence in sup norm and expend the traditional Arimoto algorithm.

REFERENCES

[1] S. Arimoto, S. Kawamura and E. Miyazaki, Bettering operation of roboties by leanring, *Robotic System*, vol.1, no.2, pp.123-140, 1984.

[2] S. Arimoto, T. Naniwa and H. Suuzki, Robusntess of P-type leaning control with a forgetting factor for robot motions, *Proc. of the Conference on Decision and Control*, Honolulu, Hawaii, pp.2640-2645, 1990.

[3] J. Haonen, D. H. Owens and K. Feng, Basis functions and parameter optimization in high-order iterative learning control, *Automatica*, vol.42, pp.287-294, 2006.

[4] X. Li, J. K. L. Ho and T. W. S. Chow, An iterative learning control method and mathematical model for robotic manipulation at undesired locations, *International Journal of Computer Integrated Manufacturing*, vol.18, no.6, 2005.

[5] K. Park, H. Z. Bien and D H. Hwang, A study on the robustness of a PID-type iterative learning controller against initial state error, *International Journal of System Science*, vol.30, no.1, pp.49-59, 1999.

[6] M. Sun and D. Wang, Closed-loop iterative learning control for nonlinear systems with initial shifts, *International Journal of Adaptive Control and Signal Processing*, vol.16, pp.515-538, 2002.

[7] D. Meng, Y. Jia, J. Du et al., Feedback approach to design fast iterative learning controller for a class of time-delay systems, *IET Control Theory and Applications*, vol.3, no.2, pp.225-238, 2009.

[8] S. Saab, On a discrete-time stochastic learning control algorithm, *IEEE Trans. Automatic Control*, vol.46, no.8, pp.1333-1336, 2001.

[9] X. H. Bu, Z. S. Hou and F. S. Yu, Stability of first and high order iterative learning control with data dropouts, *International Journal of Control, Automation, and Systems*, vol.9, no.5, pp.843-849, 2011.

[10] S. R. Oh, Z. Bien and I. H. Suh, An iterative learning control method with application for the robot manipulator, *IEEE J. Robotics Automat.*, pp.508-514, 1988.

[11] T.-Y. Doh, J. R. Ryoo and D. E. Chang, Robust iterative learning controller design using the performance weighting function of feedback control systems, *International Journal of Control, Automation, and Systems*, no.12, pp.63-70, 2014.

[12] A. Fujimori and S. Ohara, Parameter identification of continuous-time systems using iterative learning control, *International Journal of Control, Automation, and Systems*, no.9, pp.203-210, 2011.

[13] V. K. R. Kandanvli and H. Kar, Robust stability of discrete-time state delayed systems employing generalized overflow nonlinearities, *Nonlinear Anal. Theory Meth. Appl.*, vol.69, pp.2780-2787, 2008.

[14] J. H. Kim, S. J. Ahn and S. Ahn, Guaranteed cost and H_∞ filtering for discrete-time polytopic uncertain systems with time delay, *J. Franklin Inst.*, vol.342, pp.365-378, 2005.