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A BALANCED DEMATEL THEORY WITH NORMALIZED INDIRECT RELATION MATRIX

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Abstract. *The generalized DEMATEL theory pointed out that the results of the traditional DEMATEL theory are always unbalanced and unfair, since their indirect relations are always far greater than their responding direct relations, respectively, it can be improved by using an external shrinkage coefficient of the indirect relation matrix of a DEMATEL. However, on other contrary, the indirect relations sometimes may be smaller than their corresponding direct relations respectively, it is still unbalanced, and it can be not improved by using the above-mentioned method. In this paper, Liu's balanced coefficient was proposed, which can be used to detect any DEMATEL whether it is balanced; furthermore, we proved that if the internal shrinkage coefficient is smaller than 0.5, then its indirect relation is smaller than its direct relation. For overcoming this drawback, we propose a new method, called the balanced DEMATEL theory, by normalizing the indirect relation matrix as the direct relation matrix has been done. For any cases, according to Liu's validity index, we can find that the performance of the new method is better than that of the generalized DEMATEL theory. Some important properties of this new theory were discussed, and a simple data was also provided in this paper to illustrate the advantages of the proposed theory.*
Keywords: Liu's balanced coefficient, Normalized indirect relation matrix, Liu's validity index

1. **Introduction.** Decision Making Trial and Evaluation Laboratory (DEMATEL) was developed between 1972 to 1979 by Science and Human Affairs Program of the Battelle Memorial Institute of Geneva [1]. It can be used to resolve complex and difficult problems in the world, and it has been widely used as one of the best tools to solve the cause and effect relationship among the evaluation factors [1,2]. However, the indirect relation of a DEMATEL is always far greater than its direct relation, which is unbalanced and unfair [5]. Our previous paper proposed an external shrinkage coefficient to improve it

[5]. However, if the indirect relation of a DEMATEL is less than its direct relation, the previous method can do nothing to it. In this paper, a better method is considered by normalizing the indirect relation matrix as follows.

2. The Traditional DEMATEL. The procedure of the traditional DEMATEL method is briefly introduced below [1-4].

Step 1: Calculate the initial direct relation matrix Q

N experts are asked to evaluate the degree of direct influence between two factors based on pair-wise comparison. The degree to which the expert e perceived factor i effects on factor j is denoted as

$$q_{ij}^{(e)}, \quad e = 1, 2, \ldots, N. \quad q_{ij}^{(e)} \in \{0, 1, 2, 3, 4\}, \quad i, j = 1, 2, \ldots, n \tag{1}$$

For each expert e, an individual direct relation matrix is constructed as

$$Q_e = \left[q_{ij}^{(e)}\right]_{n\times n}, \quad e = 1, 2, \ldots, N, \quad q_{ii}^{(e)} = 0, \quad i = 1, 2, \ldots, n \tag{2}$$

We can obtain their average direct relation matrix, called the initial direct relation matrix Q as follows:

$$Q = [q_{ij}]_{n\times n} = \frac{1}{N}\sum_{e=1}^{N} Q_e, \quad q_{ij} = \frac{1}{N}\sum_{e=1}^{N} q_{ij}^{(e)}, \quad i, j = 1, 2, \ldots, n \tag{3}$$

Step 2: Calculate the direct relation matrix A

$$A = [a_{ij}]_{n\times n} = \lambda^{-1}Q, \quad \lambda = \max_{1\le i,j\le n}\left\{\sum_{j=1}^{n} q_{ij}, \sum_{i=1}^{n} q_{ij}\right\} \tag{4}$$

$$a_{ii} = 0, \quad i = 1, 2, \ldots, n, \quad 0 \le a_{ij} \le 1, \quad i \ne j, \quad i, j = 1, 2, \ldots, n$$

$$0 \le \sum_{i=1}^{n} a_{ij}, \sum_{j=1}^{n} a_{ij} \le 1, \quad i, j = 1, 2, \ldots, n \tag{5}$$

Step 3: Calculate the indirect relation matrix B and the total relation matrix T

Based on Markov chain theory, we have

$$\lim_{k\to\infty} A^k = 0_{n\times n} \tag{6}$$

$$B = [b_{ij}]_{n\times n} = \lim_{k\to\infty}\left[A^2 + A^3 + \ldots + A^k\right] = A^2(I - A)^{-1} \tag{7}$$

$$T = [t_{ij}]_{n\times n} = A + B = [(a_{ij} + b_{ij})]_{n\times n} \tag{8}$$

Step 4: Calculate the relation degree and prominence degree of each factor

$$r_i = \sum_{j=1}^{n} t_{ij}, \quad c_i = \sum_{k=1}^{n} t_{ki}, \quad i = 1, 2, \ldots, n \tag{9}$$

The value of r_i indicates the total dispatch both directly and indirectly effects, that factor i has on the other factors, and the value of c_i indicates the total receive both directly and indirectly effects, that factor i has on the other factors.

The relation degree of factor i is denoted as

$$x_i = r_i - c_i, \quad i = 1, 2, \ldots, n \tag{10}$$

The prominence degree of factor i is denoted as

$$y_i = r_i + c_i, \quad i = 1, 2, \ldots, n \tag{11}$$

Relation prominence matrix is denoted as

$$(x_i, y_i)_{i=1}^{n} \tag{12}$$

Step 5: Set the threshold value (α)

For eliminating some minor effects elements in matrix T to find the impact-relations map, Yang et al. [3] suggest their threshold value below

$$\alpha_Y = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} t_{ij} \tag{13}$$

Lia and Tzeng [4] suggested a more information threshold value, α_M, based on their maximum mean de-entropy (MMDE) algorithm.

Step 6: Build a cause and effect relationship diagram

If $t_{ij} > \alpha_Y$, or $t_{ij} > \alpha_M$, then factor i is a net dispatch node of factor j, and factor j is a net receive node of factor i, and denoted as

$$(x_i, y_i) \to (x_j, y_j), \quad \text{or} \quad (x_i, y_i) \leftarrow (x_j, y_j) \tag{14}$$

The graph of $(x_i, y_i)_{i=1}^{n}$ including the net direct edges can present a cause and effect relationship diagram.

3. **The Generalized DEMATEL.** Our previous paper [5] pointed out that the indirect relation of a traditional DEMATEL is always far greater than its direct relation, which is unbalanced and unfair, since it overemphasizes the influence of the indirect relation. For overcoming this drawback, an external shrinkage coefficient of the indirect relation matrix, d, was provided to construct a better indirect relation matrix, and a generalized DEMATEL theory is obtained below

$$B_d = \left[b_{ij}^{(d)}\right]_{n\times n} = dA^2(I - dA)^{-1}, \quad d \in \left[\frac{1}{2}, 1\right] \tag{15}$$

$$T_d = \left[t_{ij}^{(d)}\right]_{n\times n} = A + B_d = \left[\left(a_{ij} + b_{ij}^{(d)}\right)\right], \quad d \in \left[\frac{1}{2}, 1\right] \tag{16}$$

The Relation-Prominence of the DEMATEL (A, B_d) is defined as

$$R(A, B_d) = \left(x_i^{(d)}, y_i^{(d)}\right)_{i=1}^{n} = \left(r_i^{(d)} + c_i^{(d)}, r_i^{(d)} - c_i^{(d)}\right)_{i=1}^{n} \tag{17}$$

$$r_i^{(d)} = \sum_{j=1}^{n} t_{ij}^{(d)}, \quad c_i^{(d)} = \sum_{k=1}^{n} t_{ij}^{(d)}, \quad i = 1, 2, \ldots, n \tag{18}$$

$$\alpha_Y = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} t_{ij}^{(d)} \tag{19}$$

If $t_{ij}^{(d)} > \alpha_Y$, or $t_{ij}^{(d)} > \alpha_M$, then factor i is a net dispatch node of factor j, and factor j is a net receive node of factor i, and denoted as

$$\left(x_i^{(d)}, y_i^{(d)}\right) \to \left(x_j^{(d)}, y_j^{(d)}\right) \tag{20}$$

The graph of $(x_i, y_i)_{i=1}^{n}$ including the net direct edges can present a cause and effect relationship diagram.

If $d = 1$, new DEMATEL (A, B_d) is just the traditional DEMATEL (A, B). If $d = 0.5$, then $\max\limits_{1\le i,j\le n} \left\{\sum_{j=1}^{n} b_{ij}^{(d)}, \sum_{i=1}^{n} b_{ij}^{(d)}\right\} \le 1$, and the new DEMATEL (A, B_d) is feasible, since its indirect relation influence is no longer greater than its direct relation influence.

For evaluating the performance of any DEMATEL, the Liu's validity index [5] was defined below,

$$V_L(A, B_d) = 1 - \frac{1}{1 + 5\sqrt{\sum_{i=1}^{n} \sqrt{\left(x_i^{(d)} - \bar{x}_d\right)^2 + \left(y_i^{(d)} - \bar{y}_d\right)^2}}} \tag{21}$$

$$\bar{x}_d = \frac{1}{n}\sum\nolimits_{j=1}^{n} x_i^{(d)}, \quad \bar{y}_d = \frac{1}{n}\sum\nolimits_{j=1}^{n} y_i^{(d)} \tag{22}$$

Example 3.1. [5]

$$\begin{gathered} If \ \ A = [a_{ij}]_{4\times 4} = \begin{bmatrix} 0 & 0.36 & 0.32 & 0.32 \\ 0.32 & 0 & 0.34 & 0.30 \\ 0.34 & 0.30 & 0 & 0.30 \\ 0.28 & 0.28 & 0.30 & 0 \end{bmatrix}, \ then \\ B = [b_{ij}]_{4\times 4} = A^2(I-A)^{-1} = \begin{bmatrix} 3.9448 & 3.8483 & 3.9290 & 3.7995 \\ 3.7483 & 3.8237 & 3.7991 & 3.6907 \\ 3.6807 & 3.6963 & 3.8253 & 3.6331 \\ 3.4499 & 3.4478 & 3.4963 & 3.4508 \end{bmatrix}, \end{gathered} \tag{23}$$

$$\mu = \max_{1\le i,j\le n} \left\{\sum\nolimits_{j=1}^{n} b_{ij}, \sum\nolimits_{i=1}^{n} b_{ij}\right\} = 15.5215 \gg \max_{1\le i,j\le n} \left\{\sum\nolimits_{j=1}^{n} a_{ij}, \sum\nolimits_{i=1}^{n} a_{ij}\right\} = 1 \tag{24}$$

$$\begin{gathered} T = [t_{ij}]_{4\times 4} = \begin{bmatrix} 3.9448 & 4.2083 & 4.2490 & 4.1195 \\ 4.0683 & 3.8237 & 4.1391 & 3.9907 \\ 4.0207 & 3.9963 & 3.8253 & 3.9331 \\ 3.7299 & 3.7278 & 3.7963 & 3.4508 \end{bmatrix}, \\ R(A,B) = (x_i, y_i)_{i=1}^{n} = \begin{pmatrix} 0.7579 & 32.2851 \\ 0.2658 & 31.7780 \\ -0.76344 & 31.7850 \\ -0.77893 & 30.1989 \end{pmatrix} \end{gathered} \tag{25}$$

$$\begin{gathered} B_{0.5} = \begin{bmatrix} 0.2538 & 0.1993 & 0.2144 & 0.2047 \\ 0.2012 & 0.2450 & 0.2002 & 0.2012 \\ 0.1916 & 0.2020 & 0.2450 & 0.1971 \\ 0.1882 & 0.1879 & 0.1872 & 0.2182 \end{bmatrix}, \\ T_{0.5} = \begin{bmatrix} 0.2538 & 0.5593 & 0.5344 & 0.5247 \\ 0.5212 & 0.2450 & 0.5402 & 0.5012 \\ 0.5316 & 0.5020 & 0.2450 & 0.4971 \\ 0.4682 & 0.4679 & 0.4872 & 0.2182 \end{bmatrix} \end{gathered} \tag{26}$$

$$R(A, B_{0.5}) = \left(x_i^{(0.5)}, y_i^{(0.5)}\right)_{i=1}^{n} = \begin{pmatrix} 0.0973 & 3.6469 \\ 0.0334 & 3.5718 \\ -0.0312 & 3.5824 \\ -0.0996 & 3.3526 \end{pmatrix} \tag{27}$$

$$V_L(A, B_{0.5}) = 0.7044 > V_L(A, B) = 0.6952 \tag{28}$$

4. **The Balanced DEMATEL with Normalized Indirect Relation Matrix.** In this paper, a useful balanced index, Liu's balanced coefficient, is provided to test the balanced degree of any DEMATEL.

Definition 4.1. *Liu's balanced coefficient of DEMATEL* (A,B), $\beta(A,B)$

If $A=[a_{ij}]_{n\times n}$ *is the direct relation matrix of a DEMATEL,* $B=[b_{ij}]_{n\times n}=A^2(I-A)^{-1}$, $\mu=\max\limits_{1\le i,j\le n}\left\{\sum_{j=1}^n b_{ij},\sum_{i=1}^n b_{ij}\right\}$, *then the Liu's balanced coefficient of the DEMATEL is defined below*

$$\beta(A,B)=\frac{2\sqrt{\mu}}{1+\mu},\quad 0\le\beta(A,B)\le 1 \tag{29}$$

Note that

$$\mu=1\Leftrightarrow\beta(A,B)=1,\quad \mu\ne 1\Leftrightarrow\beta(A,B)<1 \tag{30}$$

The internal shrinkage coefficient is defined below.

Definition 4.2. *Internal shrinkage coefficient of the indirect relation matrix,* γ_δ

Let $A=[a_{ij}]_{n\times n}$ *be the direct relation matrix of a DEMATEL, and* $B=A^2(I-A)^{-1}$

$$A^m=\left[a_{ij}^{(m)}\right]_{n\times n}=\left[\sum_{k=1}^n a_{ik}^{(m)}a_{kj}\right]_{n\times n},\quad \delta^{(m)}=\max_{1\le i,j\le n}\left\{\sum_{j=1}^n a_{ij}^{(m)},\sum_{i=1}^n a_{ij}^{(m)}\right\} \tag{31}$$

If $\gamma_\delta=\mathop{Sup}\limits_{m\in N}\left[\left(\delta^{(m)}\right)^{-1}\delta^{(m+1)}\right]\le 1$, *then* γ_δ *is the internal shrinkage coefficient of indirect relation matrix* B.

Some important properties of the external and internal shrinkage coefficients are provided as follows.

Theorem 4.1. *Important properties of the shrinkage coefficients of a DEMATEL*

$$(a)\quad A=[a_{ij}]_{n\times n}=\left[a_{ij}^{(1)}\right]_{n\times n},\quad A^{m+1}=\left[a_{ij}^{(m+1)}\right]_{n\times n}=\left[\sum_{k=1}^n a_{ik}^{(m)}a_{kj}\right]$$

$$a_{ii}=0,\quad i=1,2,\ldots,n,\quad a_{ij}\ge 0,\quad i\ne j,\quad i,j=1,2,\ldots,n$$

$$(b)\quad \delta^{(m)}=\max_{1\le i,j\le n}\left\{\sum_{j=1}^n a_{ij}^{(m)},\sum_{i=1}^n a_{ij}^{(m)}\right\},\quad m\in N,\quad \delta^{(1)}=1,$$

$$\gamma_\delta=\mathop{Sup}_{m\in N}\left[\left(\delta^{(m)}\right)^{-1}\delta^{(m+1)}\right]\le 1$$

$$(c)\quad B_d=\left[b_{ij}^{(d)}\right]_{n\times n}=dA^2(I-dA)^{-1},\quad d\in[\,0.5,1\,],$$

d is the external shrinkage coefficient

$$(d)\quad \mu_d=\max_{1\le i,j\le n}\left\{\sum_{j=1}^n b_{ij}^{(d)},\sum_{i=1}^n b_{ij}^{(d)}\right\},\quad \mu_1=\mu$$

$$\Rightarrow (i)\ \delta^{(m+1)}\le\delta^{(m)}\le 1,\ m\in N\ (ii)\ \mu=\lim_{t\to\infty}\sum_{m=2}^{t}\delta^{(m)}\ (iii)\ \mu<1\Rightarrow\mu_d<1$$

$$(iv)\ \gamma_\delta<0.5\Rightarrow\mu<1$$

Note that: (a) From (i) and (ii) of Theorem 4.1, we know that the traditional DEMATEL is always unbalanced.

(b) From (iii) and (iv) of Theorem 4.1, we know that if $\mu<1$, then we can obtain no balanced DEMATEL for taking any external shrinkage coefficient.

In this paper, we can always obtain the balanced DEMATEL by normalizing its indirect relation matrix as follows.

Definition 4.3. *Balanced DEMATEL with normalized indirect relation matrix*

If $A=[a_{ij}]_{n\times n}$ *is the direct relation matrix,* $B=[b_{ij}]_{n\times n}=A^2(I-A)^{-1}$ *and*

$$\mu=\max_{1\le i,j\le n}\left\{\sum_{j=1}^n b_{ij},\sum_{i=1}^n b_{ij}\right\} \tag{32}$$

The normalized indirect relation matrix, B_N is defined by

$$B_N = \left[b_{ij}^{(N)}\right]_{n\times n} = \mu^{-1}B = \left[(\mu^{-1}b_{ij})\right]_{n\times n} \tag{33}$$

Normalized total relation matrix is defined as

$$T_N = \left[t_{ij}^{(N)}\right]_{n\times n} = A + B_N = \left[(a_{ij} + \mu^{-1}b_{ij})\right]_{n\times n} \tag{34}$$

The Relation-Prominence of the DEMATEL (A, B_N) is defined as

$$R(A, B_N) = \left(x_i^{(N)}, y_i^{(N)}\right)_{i=1}^{n} = \left(r_i^{(N)} + c_i^{(N)}, r_i^{(N)} - c_i^{(N)}\right)_{i=1}^{n} \tag{35}$$

$$r_i^{(N)} = \sum\nolimits_{j=1}^{n} t_{ij}^{(N)}, \quad c_i^{(N)} = \sum\nolimits_{i=1}^{n} t_{ij}^{(N)} \tag{36}$$

Example 4.1. *Letting $A, B, \mu, \beta(A, B)$ be the same as those of Example 3.1, then we can obtain*

$$B_N = \mu^{-1}B = \begin{bmatrix} 0.2541 & 0.2479 & 0.2531 & 0.2448 \\ 0.2415 & 0.2464 & 0.2448 & 0.2378 \\ 0.2371 & 0.2381 & 0.2464 & 0.2341 \\ 0.2223 & 0.2221 & 0.2253 & 0.2223 \end{bmatrix},$$
$$T_N = A + B_N = \begin{bmatrix} 0.2541 & 0.6079 & 0.5731 & 0.5648 \\ 0.5615 & 0.2464 & 0.5848 & 0.5378 \\ 0.5771 & 0.5381 & 0.2464 & 0.5341 \\ 0.5023 & 0.5021 & 0.5253 & 0.2223 \end{bmatrix} \tag{37}$$

$$\mu_N = \max_{1\le i,j\le n}\left\{\sum\nolimits_{j=1}^{n} b_{ij}^{(N)}, \sum\nolimits_{i=1}^{n} b_{ij}^{(N)}\right\} = 1 \Rightarrow \beta(A, B_N) = 1 \tag{38}$$

$$RP_N = \left(x_i^{(N)}, y_i^{(N)}\right)_{i=1}^{n} = \begin{pmatrix} 0.0525 & 0.9475 \\ 0.0179 & 0.9125 \\ -0.0169 & 0.9127 \\ -0.0535 & 0.8055 \end{pmatrix} \tag{39}$$

$$\beta(A, B_N) = 1 > \beta(A, B_{0.5}) = 0.9977 > \beta(A, B) = 0.4769 \tag{40}$$

$$V_L(A, B_N) = 0.7051 > V_L(A, B_{0.5}) = 0.7044 > V_L(A, B) = 0.6952 \tag{41}$$

5. **Conclusion.** In this paper, we proved that if the internal shrinkage coefficient is less than 0.5, then $\mu < 1$. And the balance and validity degree of the DEMATEL cannot be improved by taking any external shrinkage coefficient. Using the new method to normalize its indirect relation matrix as the direct relation matrix has been done can obtain a perfect balanced DEMATEL, and it is more valid than before. Some important properties of this new theory were discussed, and a simple data was also provided in this paper to illustrate the advantages of the proposed theory.

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