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FUZZY DEMAND QUANTITY AND FUZZY PRODUCTION QUANTITY BASED ON STATISTICAL DATA

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Abstract. *In production inventory, when demand quantity r and production quantity d per day are unknown, how to consider production inventory is a problem. Let demand quantity and production quantity per day of population be R^* and D^* respectively. From the statistical data in the past, we can find the point estimate of R^* and D^*. However, we cannot find the probability of the error of the point estimate. We use the $100(1-\alpha)\%$ confidence intervals of R^* and D^* to decide level $(1-\alpha)$ fuzzy numbers $\widetilde{r}$ and $\widetilde{d}$. Then we get fuzzy total cost and its centroid to represent the estimate of total cost in the fuzzy sense.*

Keywords: Fuzzy numbers, Interval-valued fuzzy numbers

1. **Introduction.** In series of papers [1,3,4] the authors considered the fuzzified problems for the production inventory model. In [3], Lee and Yao fuzzified both of demand quantity per day and production quantity per day as the triangular fuzzy numbers, obtained the fuzzy total cost and investigated a computing schema for the economic production quantity (EPQ) in the fuzzy sense. In [4], Lin and Yao fuzzified the production quantity per cycle as the trapezoid fuzzy numbers and obtained the fuzzy total cost. They applied the extension principle to find the membership functions of the fuzzy total cost, and applied the centroid method to estimate the total cost in the fuzzy sense and obtained the optimal production quantity per cycle. Lee and Chiang [1] fuzzified the quantity produced per cycle, the holding cost, production cost, production quantity per day, the total demand quantity and the demand quantity per day as the triangular fuzzy numbers, obtained the fuzzy total cost. They applied the signed distance method instead of the extension principle and centroid method to solve the estimated total cost in the fuzzy sense to obtain the optimal quantity produced per cycle. For the total cost of the inventory without backorder model, Lee and Lin [2] fuzzified the order quantity, the total demand, the cost of storing and the cost of placing an order as fuzzy numbers then obtained the fuzzy total cost, and then applied the signed distance method to defuzzifying the fuzzy total cost to solve the optimal order quantity. Both seasonal demand r and total demand R in the production inventory model are difficult to estimate precisely to actual r_0 and R_0 values, respectively [5]. Shih et al. [5] set the membership grade at r_0 and R_0 in an interval $[\lambda, 1]$, $0 < \lambda < 1$, and then, obtained the interval-valued fuzzy sets $\widetilde{r}$ and $\widetilde{R}$, respectively;

for each production quantity period q, we can obtain the fuzzy total cost $G_q\left(\widetilde{r}, \widetilde{R}\right)$ and determine the defuzzification of $G_q\left(\widetilde{r}, \widetilde{R}\right)$ as the estimate of the total cost.

The statistical data were not used in the fuzzy inventories stated above. In practical situations, it is more appropriate to use statistical data in the past to solve the problem. In this article, we use statistical data to consider fuzzy production inventory. If we have n groups of data (d_j, r_j), $j = 1, 2, \ldots, n$, where d_j and r_j are production quantity and demand per day, then we have confidence intervals $100(1-\alpha)\%$ for production quantity and demand. Corresponding to these intervals, we set level $1-\alpha$ fuzzy numbers $\widetilde{d}$ and $\widetilde{r}$. Using $\widetilde{d}$ and $\widetilde{r}$, we can fuzzify crisp total cost to fuzzy total cost. Through extension principle and centroid, we get total cost in the fuzzy sense. We give an example in Section 4 and Section 5 is the conclusion.

2. **Membership Function of the Fuzzy Total Cost Based on Statistical Data.** For the inventory control and the production process model, we introduce the following variables:

T: the whole period for the plan (days)
q: quantity produced per cycle
a: holding cost per unit per day
b: production cost per cycle
d: production quantity per day
R: the total demand quantity of whole plan period
r: demand quantity per day
s: the maximal stock quantity
t_s: time for production in each cycle
t_q: time for each cycle

The following formula of the total cost for the whole plan period T can be derived with the knowledge of inventory model [6,7]:

$$F(q) = \left(\frac{1}{2}at_q s + b\right)\frac{R}{q} = \frac{1}{2}a\left(1 - \frac{r}{d}\right)Tq + \frac{b}{R},\ q > 0 \tag{1}$$

The crisp optimal solutions are as follows:

$$\text{the optimal production quantity per cycle, } q_* = \sqrt{\frac{2bR}{cT}} \tag{2}$$

$$\text{the minimal total cost, } F(q_*) = \sqrt{2bcRT},\ \text{ where } c = a\left(1 - \frac{r}{d}\right). \tag{3}$$

In practical situations, r and d are estimates. We do not know the exact values of them. Suppose the production inventory has run through n times in the past. Let the production quantity per day be $d_1, d_2, \ldots, d_n$ and demand quantity per day be $r_1, r_2, \ldots, r_n$, where $0 < r_j < d_j$, $j = 1, 2, \ldots, n$. If the populations of the production quantity D^* and demand quantity R^* per day are unknown, we can get the point estimate $\overline{d}$ of D^* and the point estimate $\overline{r}$ of R^*, where $\overline{d} = \dfrac{1}{n}\sum\limits_{j=1}^{n} d_j$ and $\overline{r} = \dfrac{1}{n}\sum\limits_{j=1}^{n} r_j$. After substituting into (1), we have the following result.

Property 2.1. *Using statistical data d_j and r_j, $j = 1, 2, \ldots, n$, we get total cost $F_0(q) = \frac{1}{2}a\left(1 - \frac{\overline{r}}{\overline{d}}\right)Tq + \frac{bR}{q}$, $0 < q$. The optimal solution occurs when the production quantity per cycle is $q_0 = \sqrt{\frac{2bR}{c_0 T}}$ and the minimal total cost is*

$$F_0(q_0) = \sqrt{2bc_0RT},\ \textit{where } c_0 = a\left(1 - \frac{\overline{r}}{\overline{d}}\right) \tag{4}$$

Since we do not know the probability of the error of the point estimate, we consider the following confidence interval. The $100(1-\alpha)\%$ confidence intervals of D^* and R^* are

$$\left[\overline{d}-z(\alpha),\overline{d}+z(\alpha)\right] \tag{5}$$

and

$$\left[\overline{r}-w(\alpha),\overline{r}+w(\alpha)\right] \tag{6}$$

respectively, where $z(\alpha)=t_{n-1}(\alpha)\dfrac{u_1}{\sqrt{n}}$, $w(\alpha)=t_{n-1}(\alpha)\dfrac{u_2}{\sqrt{n}}$, $u_1^2=\dfrac{1}{n-1}\sum\limits_{j=1}^{n}\left(d_j-\overline{d}\right)^2$ and $u_2^2=\dfrac{1}{n-1}\sum\limits_{j=1}^{n}\left(r_j-\overline{r}\right)^2$.

If T^* is the probability density function ($p.d.f.$) of t distribution with degree $n-1$, then $t_{n-1}(d)$ satisfies $P\left(|T^*|>t_{n-1}(\alpha)\right)=\alpha$. Then

$$P\left(\overline{d}-z(\alpha)\le D^*\le\overline{d}+z(\alpha)\right)=1-\alpha \tag{7}$$

and

$$P\left(\overline{r}-w(\alpha)\le R^*\le\overline{r}+w(\alpha)\right)=1-\alpha \tag{8}$$

Let $0<\alpha_k<1$, $0<\beta_k<1$, $k=1,2$, $0<\alpha<1$, $0<\beta<1$ and $\alpha_1+\alpha_2=2\alpha$, $\beta_1+\beta_2=2\alpha$. From (7), (8) and the fact that the graph of T^* distribution is symmetric with respect to the y axis, we have

$$\begin{aligned}1-\alpha_k&=2P\left(0\le D^*\le\overline{d}+z(\alpha_k)\right),\ k=1,2\\ 1-\beta_k&=2P\left(0\le R^*\le\overline{r}+w(\beta_k)\right),\ k=1,2\end{aligned}$$

Then

$$\begin{aligned}&P\left(\overline{d}-z(\alpha_1)\le D^*\le\overline{d}+z(\alpha_2)\right)\\ =\ &P\left(\overline{d}-z(\alpha_1)\le D^*\le 0\right)+P\left(0\le D^*\le\overline{d}+z(\alpha_2)\right)\\ =\ &\frac{1}{2}(1-\alpha_1)+\frac{1}{2}(1-\alpha_2)\\ =\ &1-\alpha\end{aligned}$$

It follows that $\left[\overline{d}-z(\alpha_1),\overline{d}+z(\alpha_2)\right]$ is the $100(1-\alpha)\%$ confidence interval. Corresponding to the confidence interval, we set level $(1-\alpha)$ fuzzy number

$$\widetilde{d}=\left(\overline{d}-z(\alpha_1),\overline{d},\overline{d}+z(\alpha_2);1-\alpha\right) \tag{9}$$

Similarly, we consider the $100(1-\alpha)\%$ confidence interval $[\overline{r}-w(\beta_1),\overline{r}+w(\beta_2)]$ of R^*. We set level $(1-\alpha)$ fuzzy number

$$\widetilde{r}=\left(\overline{r}-w(\beta_1),\overline{r},\overline{r}+w(\beta_2);1-\alpha\right) \tag{10}$$

For any $0<q$, let

$$G_q(d,r)=F(q)=\frac{1}{2}a\left(1-\frac{r}{d}\right)Tq+\frac{bR}{q}$$

We fuzzify d in (9) to a level $(1-\alpha)$ fuzzy number $\widetilde{d}$ and fuzzify r in (10) to a level $(1-\alpha)$ fuzzy number $\widetilde{r}$. Then we get fuzzy total cost $G_q\left(\widetilde{d},\widetilde{r}\right)$. Through extension principle, we can find its membership function. Let

$$S_1 = \frac{1}{2q\left(\overline{d} - z(\alpha_1)\right)}\left[aTq^2\left(\overline{d} - z(\alpha_1) - \overline{r} - w(\beta_2)\right) + 2bR\left(\overline{d} - z(\alpha_1)\right)\right]$$

$$S_2 = \frac{aTq^2\left(\overline{d} - \overline{r}\right) + 2bR\overline{d}}{2q\overline{d}} > 0 \quad \left(\because\ 0 < r_j < d_j,\ j = 1, 2, \ldots, n \Rightarrow \overline{r} < \overline{d}\right)$$

and

$$S_3 = \frac{1}{2q\left[\overline{d} + z(\alpha_2)\right]}\left[aTq^2\left(\overline{d} + z(\alpha_2) - \overline{r} + w(\beta_1)\right) + 2bR\left(\overline{d} + w(\alpha_2)\right)\right].$$

We have the following property.

Property 2.2. *For any $q > 0$,*
(1) if $\overline{d} - z(\alpha_1) > 0$, $\overline{d} - z(\alpha_1) - \overline{r} - w(\beta_2) > 0$, $S_1 < S_2 < S_3$ and satisfies
(1-1) $-\overline{r} + w(\beta_1) > 0$, then

$$\mu_{G_q(\tilde{d},\tilde{r})}(z) = \begin{cases} \dfrac{(1-\alpha)K_1(z)}{H_1(z)}, & S_1 \le z \le S_2 \\ \dfrac{(1-\alpha)K_2(z)}{H_2(z)}, & S_2 \le z < \dfrac{aTq^2 + 2bR}{2q} \\ 0, & \textit{otherwise} \end{cases} \tag{11}$$

(1-2) $-\overline{r} + w(\beta_1) < 0$, then

$$\mu_{G_q(\tilde{d},\tilde{r})}(z) = \begin{cases} \dfrac{(1-\alpha)K_1(z)}{H_1(z)}, & S_1 \le z \le S_2 \\ \dfrac{(1-\alpha)K_2(z)}{H_2(z)}, & S_2 \le z < S_3 \\ 0, & \textit{otherwise} \end{cases} \tag{12}$$

(2) if $\overline{d} - z(\alpha_1) > 0$, $\overline{d} - z(\alpha_1) - \overline{r} - w(\beta_2) < 0$, $S_2 < S_3$ and satisfies
(2-1) $-\overline{r} + w(\beta_1) > 0$, then

$$\mu_{G_q(\tilde{d},\tilde{r})}(z) = \begin{cases} \dfrac{(1-\alpha)K_1(z)}{H_1(z)}, & \dfrac{bR}{q} \le z \le S_2 \\ \dfrac{(1-\alpha)K_2(z)}{H_2(z)}, & S_2 \le z \le \dfrac{aTq^2 + 2bR}{2q} \\ 0, & \textit{otherwise} \end{cases} \tag{13}$$

(2-2) $-\overline{r} + w(\beta_1) < 0$, then

$$\mu_{G_q(\tilde{d},\tilde{r})}(z) = \begin{cases} \dfrac{(1-\alpha)K_1(z)}{H_1(z)}, & \dfrac{bR}{q} < z \le S_2 \\ \dfrac{(1-\alpha)K_2(z)}{H_2(z)}, & S_2 \le z < S_3 \\ 0, & \textit{otherwise} \end{cases} \tag{14}$$

(3) if $\overline{d} - z(\alpha_1) < 0$, $S_2 < S_3$ and satisfies
(3-1) $-\overline{r} + w(\beta_1) > 0$, then

$$\mu_{G_q(\tilde{d},\tilde{r})}(z) = \begin{cases} \dfrac{(1-\alpha)K_1(z)}{H_1(z)}, & \dfrac{bR}{q} < z \le S_2 \\ \dfrac{(1-\alpha)K_2(z)}{H_2(z)}, & S_2 \le z < \dfrac{aTq^2 + 2bR}{2q} \\ 0, & \textit{otherwise} \end{cases} \tag{15}$$

(3-2) $-\overline{r}+w(\beta_1)<0$, *then*

$$\mu_{G_q(\tilde{d},\tilde{r})}(z)=\begin{cases}\dfrac{(1-\alpha)K_1(z)}{H_1(z)}, & \dfrac{bR}{q}<z\le S_2\\ \dfrac{(1-\alpha)K_2(z)}{H_2(z)}, & S_2\le z\le S_3\\ 0, & otherwise\end{cases}\tag{16}$$

where

$$\begin{aligned}H_1(z)&=z(\alpha_1)\left[aTq^2\left(z(\alpha_1)+w(\beta_2)\right)-2z(\alpha_1)qz+2z(\alpha_1)bR\right],\\ K_1(z)&=aTq^2\left[\overline{r}z(\alpha_1)+\overline{d}w(\beta_2)\right]-\left(\overline{d}-z(\alpha_1)\right)\left[aTq^2\left(z(\alpha_1)+w(\beta_2)\right)\right.\\ &\quad\left.-2z(\alpha_1)qz+2z(\alpha_1)bR\right],\\ H_2(z)&=z(\alpha_2)\left[aTq^2\left(z(\alpha_2)+w(\beta_1)\right)-2z(\alpha_2)qz+2z(\alpha_2)bR\right],\\ K_2(z)&=\left(\overline{d}+z(\alpha_2)\right)\left[aTq^2\left(z(\alpha_2)+w(\beta_1)\right)-2z(\alpha_2)qz+2z(\alpha_2)bR\right]\\ &\quad-aTq^2\left[\overline{r}z(\alpha_2)+\overline{d}w(\beta_1)\right]\end{aligned}$$

3. **The Centroid of Fuzzy Total Cost.** Defuzzify the fuzzy total cost by the centroid method, and we have the following property.

Let

$$\begin{aligned}A_1&=z(\alpha_1)\left[aTq^2\left(z(\alpha_1)+w(\beta_2)\right)+2z(\alpha_1)bR\right]\\ B_1&=aTq^2\left[\overline{r}z(\alpha_1)+\overline{d}w(\beta_2)\right]-\left(\overline{d}-z(\alpha_1)\right)\left[aTq^2\left(z(\alpha_1)+w(\beta_2)\right)+2z(\alpha_1)bR\right]\\ A_2&=z(\alpha_2)\left[aTq^2\left(z(\alpha_2)+w(\beta_1)\right)+2z(\alpha_2)bR\right]\end{aligned}$$

and

$$B_2=\left(\overline{d}+z(\alpha_2)\right)\left[aTq^2\left(z(\alpha_2)+w(\beta_1)\right)+2z(\alpha_2)bR\right]-aTq^2\left(\overline{r}z(\alpha_1)+\overline{d}w(\beta_2)\right)$$

Property 3.1. *Centroid of* $\mu_{G_q(\tilde{d},\tilde{r})}(z)$

(1) If $\overline{d}-z(\alpha_1)>0$, $\overline{d}-z(\alpha_1)-\overline{r}-w(\beta_2)>0$, $S_1<S_2<S_3$ *and satisfies*

(1-1) $-\overline{r}+w(\beta_1)>0$, *then*

$$c_{11}(q)=\frac{T_{12}(S_1,S_2)+T_{22}\left(S_2,\left(\frac{aTq^2+2bR}{2q}\right)\right)}{T_1(S_1,S_2)+T_2\left(S_2,\left(\frac{aTq^2+2bR}{2q}\right)\right)}\tag{17}$$

(1-2) $-\overline{r}+w(\beta_1)<0$, *then*

$$c_{12}(q)=\frac{T_{12}(S_1,S_2)+T_{22}(S_2,S_3)}{T_1(S_1,S_2)+T_2(S_2,S_3)}\tag{18}$$

(2) If $\overline{d}-z(\alpha_1)>0$, $\overline{d}-z(\alpha_1)-\overline{r}-w(\beta_2)<0$, $S_2<S_3$ *and satisfies*

(2-1) $-\overline{r}+w(\beta_1)>0$, *then*

$$c_{21}(q)=\frac{T_{12}\left(\frac{bR}{q},S_2\right)+T_{22}\left(S_2,\left(\frac{aTq^2+2bR}{2q}\right)\right)}{T_1\left(\frac{bR}{q},S_2\right)+T_2\left(S_2,\left(\frac{aTq^2+2bR}{2q}\right)\right)}\tag{19}$$

(2-2) $-\overline{r}+w(\beta_1)<0$, *then*

$$c_{22}(q)=\frac{T_{12}\left(\frac{bR}{q},S_2\right)+T_{22}(S_2,S_3)}{T_1\left(\frac{bR}{q},S_2\right)+T_2(S_2,S_3)}\tag{20}$$

(3) If $\overline{d}-z(\alpha_1)<0$, $S_2<S_3$ *and satisfies*

(3-1) $-\overline{r}+w(\beta_1)>0$, *then*

$$c_{31}(q)=\frac{T_{12}\left(\frac{bR}{q},S_2\right)+T_{22}\left(S_2,\left(\frac{aTq^2+2bR}{2q}\right)\right)}{T_1\left(\frac{bR}{q},S_2\right)+T_2\left(S_2,\left(\frac{aTq^2+2bR}{2q}\right)\right)} \tag{21}$$

(3-2) $-\overline{r}+w(\beta_1)<0$, *then*

$$c_{32}(q)=\frac{T_{12}\left(\frac{bR}{q},S_2\right)+T_{22}(S_2,S_3)}{T_1\left(\frac{bR}{q},S_2\right)+T_2(S_2,S_3)} \tag{22}$$

where

$$\begin{aligned}
T_1(a_1,a_2)&=\frac{\overline{d}-z(\alpha_1)}{z(\alpha_1)}(a_1-a_2)+\frac{1}{2z(\alpha_1)^2q}\left[B_1+\frac{\overline{d}-z(\alpha_1)}{z(\alpha_1)}A_1\right][\ln|A_1-2z(\alpha_1)^2a_1q|\\
&\quad-\ln|A_1-2z(\alpha_1)^2a_2q|]\\
T_{12}(a_1,a_2)&=\frac{\overline{d}-z(\alpha_1)}{2z(\alpha_1)}\left(a_1^2-a_2^2\right)+\frac{1}{2z(\alpha_1)^2q}\left(B_1+\frac{\overline{d}-z(\alpha_1)}{z(\alpha_1)}A_1\right)(a_1-a_2)\\
T_2(a_1,a_2)&=\frac{\overline{d}+z(\alpha_2)}{z(\alpha_2)}(a_2-a_1)+\frac{1}{2z(\alpha_2)^2q}\left(B_2-\frac{\overline{d}+z(\alpha_2)}{z(\alpha_2)}A_2\right)[\ln|A_2-2z(\alpha_2)^2a_1q|\\
&\quad-\ln|A_2-2z(\alpha_2)^2a_2q|]\\
T_{22}(a_1,a_2)&=\frac{\overline{d}+z(\alpha_2)}{2z(\alpha_2)}\left(a_2^2-a_1^2\right)+\frac{1}{2z(\alpha_2)^2q}\left(B_2-\frac{\overline{d}+z(\alpha_2)}{z(\alpha_2)}A_2\right)(a_1-a_2)\\
&\quad+\frac{A_2}{4z(\alpha_2)^4q^2}\left(B_2-\frac{\overline{d}+z(\alpha_2)}{z(\alpha_2)}A_2\right)[\ln|A_2-2z(\alpha_2)^2a_1q|\\
&\quad-\ln|A_2-2z(\alpha_2)^2a_2q|]
\end{aligned}$$

$c_{ij}(q)$, $i=1,2,3$, $j=1,2$ *are the total cost in the fuzzy sense under different cases.*

4. **Numerical Example.** Given $a=10$, $b=500$, $T=40$ and $R=80$. Suppose we get $n=5$ data points. The demand quantities per day are $r_1=2.4$, $r_2=3$, $r_3=2.8$, $r_4=3.2$ and $r_5=2.6$. The production quantities per day are $d_1=8.6$, $d_2=9$, $d_3=9.4$, $d_4=9.3$ and $d_5=8.7$. We find $\overline{r}=\frac{1}{5}\sum_{j=1}^{5}r_j=2.8$ and $\overline{d}=\frac{1}{5}\sum_{j=1}^{5}d_j=9$, $\mu_1^2=\frac{1}{4}\sum_{j=1}^{5}\left(d_j-\overline{d}\right)^2=0.1$ and $\mu_2^2=\frac{1}{4}\sum_{j=1}^{5}\left(r_j-\overline{r}\right)^2=0.125$. From Property 2.1, the optimal solution is $F(q_0)=4695.15$ when $q_0=17.04$.

Let $\alpha=0.05$, $\alpha_1=0.06$, $\alpha_2=0.04$. Then $\alpha_1+\alpha_2=2\alpha$, $t_4(\alpha_1)=2.647$, $t_4(\alpha_2)=3.1$. Let $\beta_1=0.08$ and $\beta_2=0.02$. Then $\beta_1+\beta_2=2\alpha$, $t_4(\beta_1)=2.390$, $t_4(\beta_2)=3.747$

$$z(\alpha_1)=t_4(\alpha_1)\frac{\mu_1}{\sqrt{5}}=0.3743,\quad z(\alpha_2)=t_4(\alpha_2)\frac{\mu_1}{\sqrt{5}}=0.4384$$
$$w(\beta_1)=t_4(\beta_1)\frac{\mu_2}{\sqrt{5}}=0.3779,\quad w(\beta_2)=t_4(\beta_2)\frac{\mu_2}{\sqrt{5}}=0.5925$$

Then we have level 0.95 fuzzy numbers

$$\widetilde{d}=(9-0.3743,9,9+0.4384;0.95)=(8.6257,9,9.4384;0.95).$$
$$\widetilde{r}=(2.8-0.3779,2.8,2.8+0.5928;0.95)=(2.4221,2.8,3.3928;0.95).$$

Implementing the Properties 2.2 and 3.1 by computer, we have the following results:

a) The minimum total cost 4810.345 in the fuzzy sense occurs at $q=16.6$,

b) Then, $\dfrac{16.6-17.04}{17.04}\times 100\%=-2.5\%$ is the relative error of quantity produced per cycle, and

c) $\dfrac{4810.345 - 4695.15}{4695.15} \times 100\% = 2.4\%$ is the relative error of fuzzy total cost, respectively.

5. **Conclusions.** This literature uses the statistical data in the past and fuzzy concept to treat the inventory problem. It is more practical and meaningful in applications.

For the further research, we will use signed distance method instead of centroid, since the formulas for centroid are too complex. And then we can compare these two methods to see which is more practical in real situations.

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