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COMPUTING TECHNIQUES IN NONLINEAR FINITE ELEMENT ANALYSIS FOR EARTH RETAINING STRUCTURES

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Abstract. *Finite element method has been used to study the behavior of earth retaining structures with construction sequences. The solutions can be obtained much more easily assuming linear elastic soil behavior. However, this paper treats the soil as an elastic-plastic medium. An excavation simulation technique that can formulate the excavation loads accurately in the finite element analysis is presented first. Also, simulation techniques for other types of construction sequences are also presented. Furthermore, since the solutions for the incremental nonlinear finite element analysis for excavation problems could not be obtained or obtained easily using the traditional iteration techniques such as the tangent stiffness method or the constant stiffness method, an iteration solution technique called the mixed stiffness method is used. The above computing techniques are applied to a field problem where the soil behavior is simulated as elastic-plastic by employing the Drucker-Prager model. The techniques presented can be applied to future nonlinear soil-structure interaction problems with construction sequences if good nonlinear soil models were used.*
Keywords: Finite element analysis, Excavation, Mixed stiffness method, Plasticity

1. **Introduction.** Finite element method has been used to study the behavior of earth retaining structures with construction sequences. For the excavation simulation, it is very important to obtain the nodal forces of the excavated body on the excavated surface. Most available procedures for simulating excavation are based on the procedure originally proposed by Goodman and Brown [1]. In this procedure, the insitu stresses are first determined. Excavation is simulated in a number of increments. As shown in Figure 1(a), the excavation surface is considered to be a stress free surface. This condition is satisfied by applying equivalent nodal forces which are equal but opposite in direction to the existing stresses along the surface at a given increment of excavation. The displacements and stresses are then calculated and added to the values for the previous step.

Two methods have been used to formulate the equivalent nodal forces. The first one involves using the stresses computed at the nodal points directly in computing the equivalent nodal forces. The stresses are usually evaluated at internal points inside an element first, and then, through an interpolation function, they are extrapolated to the nodal points. After the stresses at the nodal points are established, the equivalent nodal forces are obtained from equilibrium considerations. It has long been recognized that there are difficulties in evaluating accurately the stresses along the excavation surface by using the first method. The second method makes use of overall equilibrium to compute the equivalent nodal forces. In this method, the stresses are usually evaluated at Gauss points inside an element first, and then, instead of computing the nodal stresses, the equivalent nodal forces due to the stresses at Gauss point are calculated. An evaluation of the equivalent nodal forces can be accomplished by considering the equilibrium of the excavated body.

The second method described above has been proved to be a better one by Mana [2], Sargand [3] and Yang [4] and is used in this study.

The finite element method solutions can be obtained much more easily assuming linear elastic soil behavior. However, this paper treats the soil as an elastic-plastic medium. Due to the mathematical complexities in deriving the tangent stiffness matrix for the nonlinear material models, such as the elastic-plastic models, the constant stiffness method is usually adopted in many finite element codes to solve nonlinear problems. The drawback of the constant stiffness method is that a large number of iterations within each loading increment are required for a converged or accurate solution and tolerance errors may be cumulative. The tangent stiffness method usually is the preferred method to simulate and solve nonlinear finite element problems. However, Tillerson et al. [5] reported that tangent stiffness method has been widely used for geometrically nonlinear problems but difficulties might arise for problems involving material nonlinearities with unloading. Excavation is associated with unloading. Lightner [6] also discussed the difficulties of using tangent stiffness method with plasticity model in soil structure interaction problems. Since the solutions for the incremental nonlinear finite element analysis for excavation problems could not be obtained or obtained easily using the traditional iteration techniques such as the tangent stiffness method or the constant stiffness method, a new iteration technique, termed as the mixed stiffness method, has been proposed by Yang [7] to solve unloading problems and is used in this study.

This paper tries to present the computing techniques used in the simulation of construction sequences, apply them to a field problem and discuss the importance of having numerically sound material models in nonlinear finite element analysis.

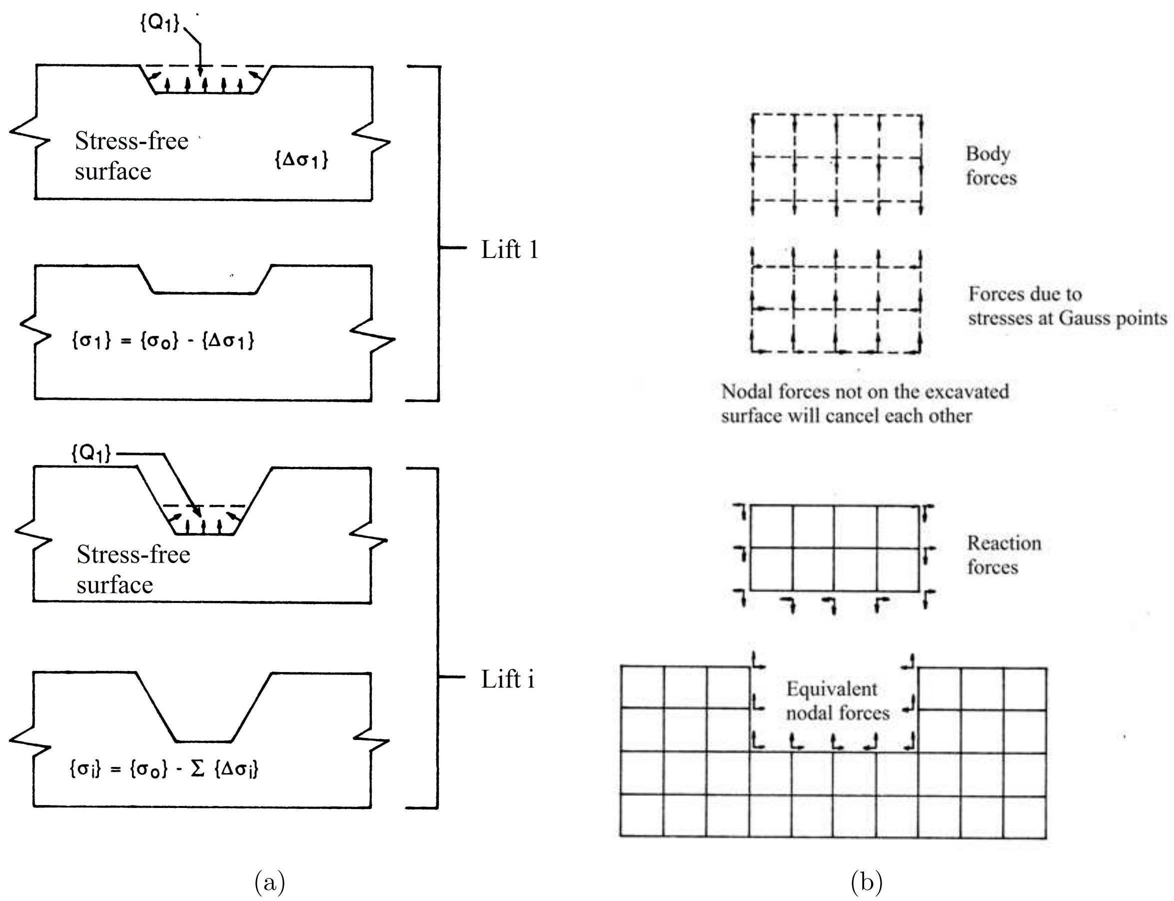

Figure 1. (a) Simulation of excavation; (b) equivalent nodal forces

2. **Excavation Simulation Technique.** As shown in Figure 1(b), the excavated area is treated as a free body. If the free body is in equilibrium, reaction forces are required to add at nodal points on the excavated surface to counteract the acting forces. The reaction forces, which are equal to the equivalent nodal forces due to excavation, can be found by forming equilibrium residuals of the free body. In the finite element analysis, the procedure to obtain equilibrium residuals is described as follows.

1. Compute the equivalent nodal loads due to the stresses at the integration points in the excavated body for all elements by
$$\{F\} = \sum \int_v [B]^{(m)T} \{\sigma\}^{(m)} dV \tag{1}$$
where $[B]^{(m)}$ is the strain-displacement transformation matrix and $\{\sigma\}^{(m)}$ is the equivalent stress vector at Gauss points for element m.
2. Compute the equivalent nodal loads due to the unit weight of the excavated body for all elements by
$$\{Q\} = \sum \int_v [N]^{(m)T} \{\gamma\}^{(m)} dV \tag{2}$$
where $[N]^{(m)}$ is the displacement interpolation matrix and $\{\gamma\}^{(m)}$ is the unit weight of soil for element m.
3. Compute the equivalent nodal loading vector, $\{E\}$, due to the effect of excavation by
$$\{E\} = \{F\} - \{Q\} \tag{3}$$

It can be verified that the excavation simulation technique employed can provide unique solutions independent of the number of excavation steps using linear elastic material.

3. **Other Simulation Techniques.** In addition to excavation, the other types of construction sequences to be dealt with in the excavation problems are examined and loading vector corresponding to the simulation of each sequence is presented below.

3.1. **Insitu stresses.** The method chosen was to do a cycle of finite element analysis to compute the insitu stresses from applied loads due to the unit weight of the specified region. The applied loads can be computed by using Equation (2). In the case with water in soil, the Buoyant unit weight should be used. The proper lateral stresses, which are equal to K_0 times effective vertical stresses, are obtained by choosing the value of Poisson's ratio according to the following equation:
$$\upsilon = K_0/(1 + K_0) \tag{4}$$

3.2. **Dewatering.** The change in the state of stress caused by dewatering can be included in the finite element analysis as an equivalent load that modifies the unit weight in the soil. Such an equivalent load can be computed as
$$\{Q\} = \sum \int_v [N]^{(m)T} \{\gamma_w\}^{(m)} dV \tag{5}$$
where $\{\gamma_w\}^{(m)}$ is the added apparent unit weight for element m in the dewatered region. The added weight occurs due to the loss of the Buoyant force of water in the region.

3.3. **Embankment.** Embankment is the process of adding material to the domain. The equivalent nodal forces due to the added material are computed by using Equation (2).

3.4. **Reestablishment of the water table.** Raising the water table is the process of adding water to the domain. The simulation is to use Equation (5) to compute the equivalent nodal forces due to the removed apparent weight of the region in which water is added.

4. **Iteration Solution Technique.** Experimental observations of several investigators (e.g., Duncan and Chang [8] and Holubec [9]) have indicated that the unloading and reloading behavior of many soils is nearly linear and elastic in nature, and is independent of the stress levels at which unloading starts, particularly when unloading occurs at stress levels not close to failure. Based on these observations, the unloading and reloading to the maximum previous stress level may be approximated by an isotropic linear elastic model. The constant stiffness method can be used to solve nonlinear problems with unloading since it always uses the linear elastic model to formulate the stiffness matrix and unloading is a linear elastic behavior. However, when the tangent stiffness method is applied to nonlinear problems, the loading stress state of a Gauss (integration) point after unloading may not show linear-elastic behavior. This is due to the fact that the tangent stiffness matrix is used at the Gauss points when unloading occurs. Thus, the linear elastic unloading behavior cannot be detected and the correct global stiffness matrix cannot be reassembled. Furthermore, a loading case even without unloading, due to its boundary conditions, the stress state of a Gauss point may exhibit unloading behavior.

In order that the linear elastic matrix at a stress point is used when unloading occurs, a new method was proposed by Yang [7]. This method takes the advantages of both the constant stiffness method and the tangent stiffness method. Within each load increment, the constant stiffness method is used for the first iteration and after that, the tangent stiffness method is used. The first iteration uses the constant stiffness method to formulate the linear stiffness matrix at each Gauss point to insure the linear stiffness matrix is used when unloading does occur and uses the nonlinear material model to define the stress state for all Gauss points. If the stress state is found linear-elastic for a particular Gauss point then the rest of the iterations for that particular point are performed using linear stiffness matrix. On the other hand, if unloading does not occur then the subsequent iterations in that particular Gauss point are performed using tangent stiffness matrix. In other words, for the first iteration in each load increment, the global linear matrix is formed to take care of the occurrence of unloading behavior at some Gauss point and thereafter the global tangent stiffness matrices are formed to accelerate the convergence of the incremental solution. The incremental form of the discretized nonlinear equation can be written as

$$
\begin{aligned}
{}^{0}[K]\{dU\}^{(i)} &= {}^{n+1}\{Q\} - {}^{n+1}\{F\}^{(i-1)} \quad \text{for } i = 1 \\
{}^{n+1}[K]^{(i-1)}\{dU\}^{(i)} &= {}^{n+1}\{Q\} - {}^{n+1}\{F\}^{(i-1)} \quad \text{for } i > 1 \qquad (6) \\
{}^{n+1}\{U\}^{(i)} &= {}^{n+1}\{U\}^{(i-1)} + \{dU\}^{(i)} \qquad (7)
\end{aligned}
$$

with the initial conditions ${}^{n+1}[K]^{0} = {}^{0}[K]$, ${}^{n+1}\{U\}^{0} = {}^{n}\{U\}$, and ${}^{n+1}\{F\}^{0} = {}^{n}\{F\}$, where ${}^{0}[K]$ is the initial linear stiffness matrix for iteration $i = 1$ at increment $n+1$, ${}^{n+1}[K]^{(i-1)}$ is tangent stiffness matrix corresponding to the geometric and material condition for iteration $i > 1$ at increment $n + 1$, $\{U\}$ is a vector of nodal point displacements, $\{dU\}$ is a vector of incremental nodal point displacements, $\{Q\}$ is a vector of externally applied nodal point forces and $\{F\}$ is a vector of nodal point forces that correspond to the finite element stresses.

5. **Numerical Simulation of a Field Problem.** The above simulation and iteration techniques have been applied to the construction sequences of Port Allen Lock.

5.1. **Problem description.** The interaction of Port Allen Lock with its surrounding soil was analyzed by Clough and Duncan [10] using the hyperbolic soil model. It was reanalyzed using Drucker-Prager model [11] and the mixed stiffness method. A cross section of Port Allen Lock is shown in Figure 2. The initial soil profile is shown in Figure 3. The material properties used for the analysis are shown in Table 1. The concrete and interface element materials were assumed to be linear elastic. The finite element mesh, shown in Figure 4, was used throughout the incremental finite element analyses of various construction sequences. The mesh has 277 nodes and 242 4-node quadrilateral elements. Depending on the sequences represented, the elements in the mesh were assigned properties representative of natural soils, air (after excavation), concrete, backfill soil (after construction), or interface element. The analyses were done using the finite element program, FEMCON by Yang.

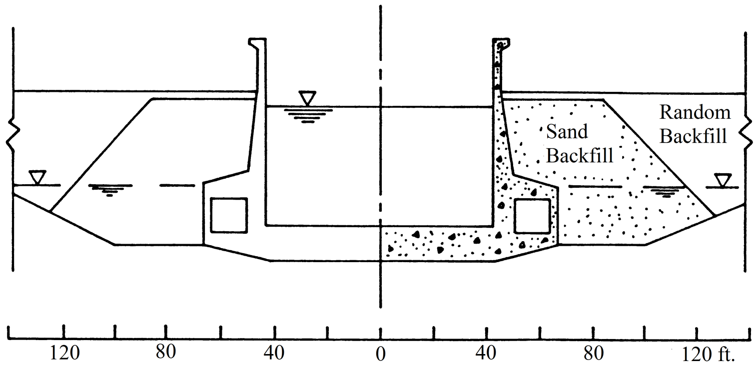

FIGURE 2. Cross section of Port Allen Lock

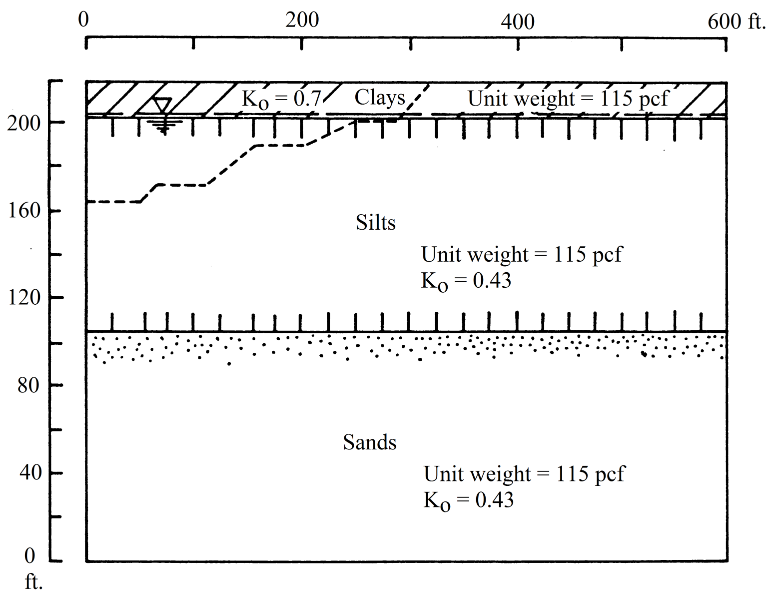

FIGURE 3. Initial soil profile for Port Allen Lock site

TABLE 1. Materials properties for finite element analyses of Port Allen Lock

Location	Soil type	E [psi]	υ	γ [pcf]	K_0	c [psf]	ϕ [degree]
Foundation	Silt	8,700	0.3	115.0	0.43	40.0	33.0
	Sand	10,600	0.3	115.0	0.43	0	40.0
Backfill	Clay	1,500	0.3	115.0	0.43	80.0	26.0
	Sand	3,000	0.3	115.0	0.43	0	40.0
Interface element properties: $E = 3000.0$ psi, $\upsilon = 0.33$, $G = 1000.0$ psi							
Concrete properties: $E = 3.0 \times 10^6$ psi, $\upsilon = 0.2$, $\gamma = 150.0$ pcf							

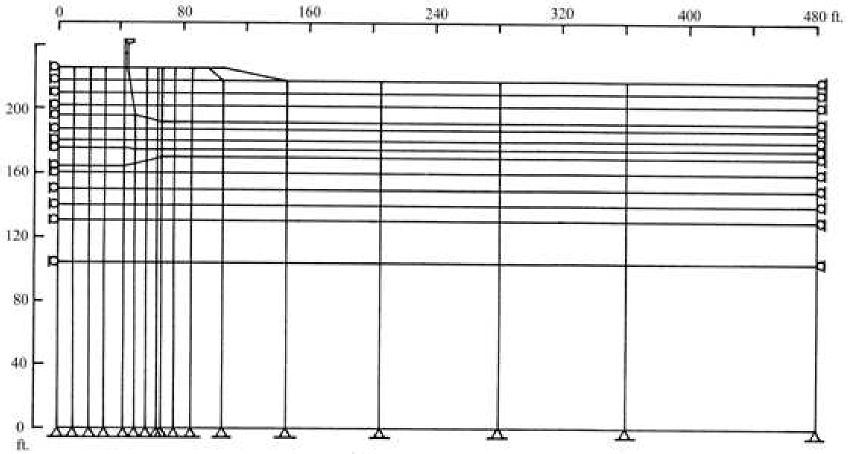

FIGURE 4. Finite element mesh for Port Allen Lock

5.2. **Incremental finite element analyses.** The incremental finite element analyses of Port Allen Lock were performed by simulating each of the actual construction operations in one or more analytical sequences involving changes in loading and mesh geometry. Beginning from the initial computation of insitu stresses, the steps progressed through excavation and dewatering, placement of concrete and backfill, reestablishment of normal ground water conditions, and filling of the lock with water. The analysis involved one linear and 16 nonlinear finite element analyses:

Stage 1: Initial insitu stress computation, linear solution.
Stage 2: 1st excavation, solution converged in 4 iterations.
Stage 3: 1st dewatering, solution converged in 6 iterations.
Stage 4: 2nd excavation, solution converged in 18 iterations.
Stage 5: 2nd dewatering, solution converged in 7 iterations.
Stage 6: 3rd excavation, solution converged in 7 iterations.
Stage 7: 1st embankment, solution converged in 6 iterations.
Stage 8: 1st water table reestablishment, solution converged in 8 iterations.
Stage 9: 2nd embankment, solution converged in 7 iterations.
Stage 10: 2nd water table reestablishment, solution converged in 7 iterations.
Stage 11: 3rd embankment, solution converged in 15 iterations.
Stage 12: 3rd water table reestablishment, solution converged in 5 iterations.
Stage 13: 4th water table reestablishment, solution converged in 6 iterations.
Stage 14: 4th embankment, solution converged in 9 iterations.
Stage 15: 5th embankment, solution converged in 16 iterations.
Stage 15: 6th embankment, solution converged in 22 iterations.
Stage 16: 7th embankment, solution converged in 23 iterations.
Stage 17: 5th water table reestablishment, solution converged in 8 iterations.

5.3. **Discussions.** The variation of the calculated rebound of the center line of the excavation at the bottom of the base slab over a period of time is shown in Figure 5. Approximately 50% of the rebound occurred during the first increment because in the second and third increments dewatering served partially to counteract the effects of excavation. The only value of observed rebound reported by Sherman and Trahan was that for the end condition. This value at the center line was 0.29 ft and at the edge of the excavation was 0.26 feet. The corresponding values calculated by the author were 0.30 feet and 0.26 feet and by Clough and Duncan were 0.22 feet and 0.21 feet. The differences between two finite element analyses are probably due to the different excavation techniques and soil models used. The technique used by the author gives results closer to the field observation.

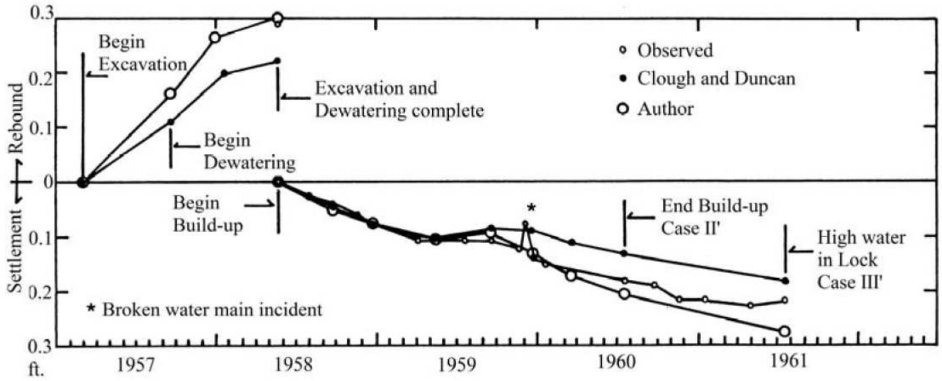

FIGURE 5. Variation of observed and calculated rebounds and settlement with time

The variation of the settlement of the center line of the lock with time is also shown in Figure 5. From May 1958 to March 1959 the three sets of settlements agreed very closely. From this time to about December 1959, the water pressures on the base of the lock increased because the backfill was maintained in a saturated condition. Also, during this period, little weight was being added to the lock as the backfill was not yet being placed over the culvert. For these reasons, the observed settlements were small and the calculated settlements decreased slightly until September 1959. After that, the backfill was placed over the culvert and the calculated settlements increased gradually. The final settlement of the lock at the end of construction before the lock filled with water (Case II') is shown in Figure 5. The calculated settlement was 0.21 feet and the observed value was 0.19 feet. Filling of the lock with water produced additional calculated settlement of 0.06 feet as opposed to an observed additional settlement of 0.03 feet as shown in Figure 5. The total calculated settlement for this condition (Case III') was 0.27 feet as compared to the observed value of 0.22 feet. It can be seen that the present analysis, in general, provides more accurate results than that by Clough and Duncan. The reason for predicting more additional settlements for Cases II' and III' may be due to that the elastic soil modulus used is a little lesser and the soils under the base slab have already yielded fully.

For Port Allen Lock, 16 nonlinear incremental finite element analyses were performed and each incremental solution can converge from 4 to 23 iterations with an average of 10.875 iterations. Those numbers indicate that how fast a solution can converge if the mixed stiffness method is used.

This paper tries to present a modified iteration technique to successively solve nonlinear finite element analysis with unloading. However, nonlinear material constitutive models also play a key role in the analysis. Verghese et al. [12] stated that for an effective analysis to be carried out, it is imperative for the user to recognize the uncertainties and limitations each model has to offer, and consequently make a judgement between the reliability of the numerical results and the uncertainty therein. Today there are still a lot of proposed soil models that can only apply to static loading and not yet to generalized stress paths found in field situations (e.g., Rahnema [13] and Joseph [14]). Without sound material models, the numerical singularity may occur and successful applications of them to solve complicated field problems are unlikely.

6. **Conclusions.** Simulation techniques for various types of construction sequences are presented. Furthermore, since the solutions for the incremental nonlinear finite element analysis for excavation problems could not be obtained or obtained easily using the traditional iteration techniques such as the tangent stiffness method or the constant stiffness method, an iteration solution technique called the mixed stiffness method is used. The

above computing techniques are applied successfully to analyzing a field problem where the soil behavior is simulated as elastic-plastic by employing the Drucker-Prager model. It is believed that the computing techniques presented here will help researchers solve the earth retaining structures with construction sequences when they try to apply nonlinear material models in the finite element analysis. However, further research directions should be emphasized on establishing numerically sound material models to be used in the finite element analysis in order that a converged solution can be obtained successfully.

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