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MOTION CONTROLLER DESIGN FOR A WALKING SUPPORT ROBOT UTILIZING ACCELERATION AND SPEED INFORMATION OF DESIRED TRAJECTORY

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Abstract. *A walking support robot has been developed for the people with walking disabilities. However, motion performance of this robot is seriously affected by center of gravity shifts and load changes due to users. The aim of this study is to design an asymptotically stable controller that can ensure the motion accuracy for the robot. The dynamic model of the robot is derived that the center of gravity shifts and load changes are taken into consideration. A controller that can satisfy asymptotic stability is proposed based on a Lyapunov function by effectively utilizing the acceleration and speed information of desired trajectory. The proposed controller can compensate the constant center of gravity shifts and load changes; furthermore, the acceleration and speed information can improve the tracking accuracy. The effectiveness of the controller is valid by simulations.*
Keywords: Walking support robot, Motion control, Center of gravity shift, Lyapunov stability

1. **Introduction.** Walking is the most vital mode of transportation upon which all societal activities depend for people in daily life [1,2]. In addition, walking is the most useful and convenient fitness exercise for people to keep their health [3]. However, because the problem of aging population goes worsen in addition with a low birthrate [4], an increasing number of people suffer from walking impairment due to aging, illness or accidents, and there is no enough young to assist them. Therefore, to enable daily life convenient and keep healthy, a walking support robot is essential for the people with walking disabilities to be able to continue walking every day.

In such a social background, many kinds of walking support robots have been developed, such as the power-assist exoskeletons studied in [5,6]. However, this kind of robot is difficult to be put on and taken off and has the danger of falling down. Another kind of walking assist device with bodyweight support has been developed in [7]. Although easy to put on and take off, it is dangerous and not convenient for elderly and the people with walking disabilities.

In our previous study, a walking support robot (WSR) has been developed [8]. While a user walking with the assist of WSR, he/she places his/her arm on the armrest leading the pressure and thrust to the WSR. These forces may seriously affect the motion accuracy of the WSR. Therefore, to address these issues, in this paper, we firstly simplify the pressure as a load which leads load changes and center of gravity (COG) shifts problems to the WSR. A controller that can compensate the constant load changes and COG shifts is proposed based on a Lyapunov function. This controller effectively utilizes the acceleration and speed information of desired trajectory which improves the tracking accuracy of the WSR. The stability of this controller is proved by Lyapunov theorem and its effectiveness is valid by simulations.

This paper is organized as follows. Section 2 describes the structure of the WSR and derives its dynamic model. In Section 3, a controller that can satisfy asymptotic stability is proposed based on a Lyapunov function by effectively utilizing the acceleration and speed information of desired trajectory. Section 4 shows simulations of the proposed method, and the results show that the proposed control method is feasible and effective. A conclusion is given in Section 5.

2. **Walking Support Robot and Modeling.** A photo of the WSR is shown in Figure 1. Four Mecanum wheels enable the robot to realize the omnidirectional movement function. With this function, the robot could freely move in a narrow space.

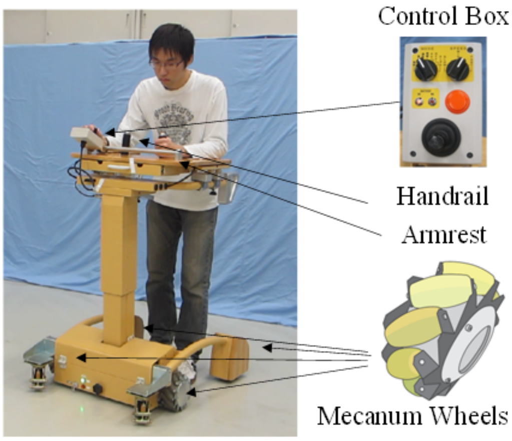


FIGURE 1. Walking support robot

In this paper, to address the effects from users on the motion accuracy, a right motion controller is required that a dynamic model of this robot is firstly to be derived. We simplify the robot on a 2-dimensional (2-D) plane considering these forces as the equivalent load on the same plane with four drive forces as shown in Figure 2. The parameters and coordinate systems are defined as follows:

$\Sigma(x, y, O)$: World coordinate system;
$\Sigma(x', y', C)$: Body coordinate system fixed on the WSR;
$G(x_g, y_g)$: COG of WSR considering the effects from users;
$C(x_c, y_c)$: Geometric center of WSR;
d: Distance between the G and C;
θ: WSR orientation defined as the angle between Cy' and x-axis;
α: Angle between Cx' and CG;
$2L$: Length of WSR;
$2W$: Width of WSR;
f_i: Force of each Macanum wheel ($i = 1, 2, 3, 4$);
l_i': Distance between COG to each force ($i = 1, 2, 3, 4$);
D: Distance between geometric center to each force.

Based on the 2-D dynamics analysis in Figure 2, the dynamic model of the robot is derived. Firstly, the robot is considered as a rigid body, in which deformation is neglected. Therefore, the dynamic model at COG is derived based on Newton's motion equation and Euler's momentum equation.

$$M_G \ddot{X}_G = F_G \tag{1}$$

where $M_G = \mathrm{diag}(M + m, M + m, I_G)$, $\ddot{X}_G = \begin{bmatrix} \ddot{x}_g & \ddot{y}_g & \ddot{\theta} \end{bmatrix}^T$, $F_G = [f_{Gx} \quad f_{Gy} \quad \tau_G]^T$. M is the mass of WSR, m is the user's equivalent mass, and I_G is the inertia of mass of the WSR. f_{Gx} is the x-axis component of drive resultant force at COG, f_{Gy} is the y-axis component of drive resultant force at COG, and τ_G is the drive torque around COG.

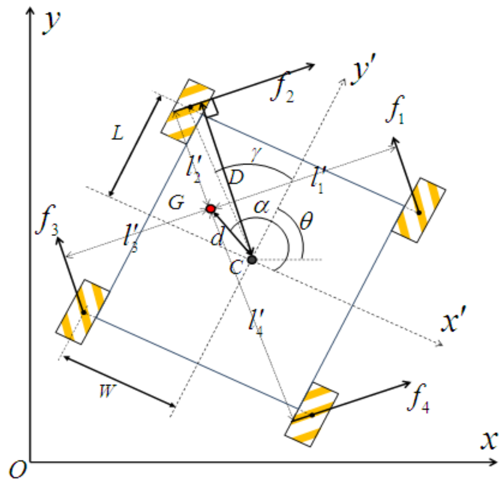


FIGURE 2. Dynamics analysis of the robot on a 2-D plane

However, in practice, the motion of geometric center is more important than COG; therefore, based on the motion relationship between COG and geometric center shown as follows:

$$\dot{X}_G = A\dot{X}_C = \begin{bmatrix} 1 & 0 & p \\ 0 & 1 & q \\ 0 & 0 & 1 \end{bmatrix} \dot{X}_C \tag{2}$$

where $p = d \cdot \cos(\alpha + \theta)$, $q = d \cdot \sin(\alpha + \theta)$, $\dot{X}_C = \begin{bmatrix} \dot{x}_c & \dot{y}_c & \dot{\theta} \end{bmatrix}^T$ is the x-axis component, y-axis component of velocity, and angular speed at the geometric center of the WSR, we can derive the acceleration relationship between COG and geometric center as follows:

$$\ddot{X}_G = A\ddot{X}_C + \dot{A}\dot{X}_C \tag{3}$$

furthermore, we can obtain the dynamic model in geometric center shown as Equation (4).

$$M_G A\ddot{X}_C + M_G\dot{A}\dot{X}_C = F_G \tag{4}$$

According to Figure 2, using the relationship between resultant F_G at COG and drive forces of four wheels, the dynamic model is further developed as follows:

$$M_G A\ddot{X}_C + M_G\dot{A}\dot{X}_C = K_G^T(\theta)F \tag{5}$$

where

$$F = [f_1 \quad f_2 \quad f_3 \quad f_4]^T$$

$$K_G^T(\theta) = \begin{bmatrix} -\sin(\theta - \pi/4) & \cos(\theta - \pi/4) & -\sin(\theta - \pi/4) & \cos(\theta - \pi/4) \\ \cos(\theta - \pi/4) & \sin(\theta - \pi/4) & \cos(\theta - \pi/4) & \sin(\theta - \pi/4) \\ l_1' & -l_2' & -l_3' & l_4' \end{bmatrix}$$

$l_1' = D - d \cdot \cos(\alpha - \pi/4)$, $l_2' = D - d \cdot \sin(\alpha - \pi/4)$, $l_3' = D + d \cdot \cos(\alpha - \pi/4)$, $l_4' = D + d \cdot \sin(\alpha - \pi/4)$, $D = \cos(\gamma - \pi/4)\sqrt{L^2 + W^2}$, $\gamma = \arctan(W/L)$. To simplify the COG shift problem, multiplying the matrix A^T to the left of dynamic model (5), the dynamic model is becoming as Equation (6), in which the force F directly acts at the geometric center.

$$M_1\ddot{X}_C + N_1\dot{X}_C = K_C^T(\theta)F \tag{6}$$

where,

$$\begin{aligned} M_1 &= A^T M_G A \\ &= \begin{bmatrix} M+m & 0 & (M+m)p \\ 0 & M+m & (M+m)q \\ (M+m)p & (M+m)q & I_G+(M+m)d^2 \end{bmatrix} \\ N_1 &= A^T M_G \dot{A} \\ &= \begin{bmatrix} 0 & 0 & -(M+m)q\dot{\theta} \\ 0 & 0 & (M+m)p\dot{\theta} \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

$$K_C^T(\theta) = \begin{bmatrix} -\sin(\theta-\pi/4) & \cos(\theta-\pi/4) & -\sin(\theta-\pi/4) & \cos(\theta-\pi/4) \\ \cos(\theta-\pi/4) & \sin(\theta-\pi/4) & \cos(\theta-\pi/4) & \sin(\theta-\pi/4) \\ D & -D & -D & D \end{bmatrix}$$

3. **Controller Design.** In this section, a controller that can satisfy asymptotic stability is designed based on a Lyapunov function.

Definition 3.1. *The error state equation of the WSR system is defined as:*

$$M_1\ddot{E}(t) + N_1\dot{E}(t) = M_1\ddot{X}_{Cd} + N_1\dot{X}_{Cd} - u(t) \tag{7}$$

where $u(t) = K_C^T(\theta)F$ is the control input for the error state Equation (7), $E(t) = X_{Cd} - X_C$, $\dot{E}(t) = \dot{X}_{Cd} - \dot{X}_C$, $\ddot{E}(t) = \ddot{X}_{Cd} - \ddot{X}_C$ are the position, velocity and acceleration tracking errors. $X_{Cd} = [x_{cd}\ y_{cd}\ \theta_d]$ is the desired trajectory and $X_C = [x_c\ y_c\ \theta]$ is the actual motion trajectory.

Theorem 3.1. *Consider the error state Equation (7) with the control law.*

$$u(t) = M_1\ddot{X}_{Cd} + N_1\dot{X}_{Cd} - N_1\dot{E}(t) + Q\dot{E}(t) + PE(t) + \frac{N_1+N_1^T}{2}\dot{E}(t) \tag{8}$$

where P and Q are the positive scalar, symmetric matrices and let $M+m$ be the constant, and then the error state system is table.

Proof: Define the Lyapunov function as

$$V(t) = \frac{1}{2}\dot{E}^T(t)M_1\dot{E}(t) + \frac{1}{2}E^T(t)PE(t) \geq 0 \tag{9}$$

for M_1 and P are the constant positive symmetric matrices.

Then, the time derivative of $V(t)$ along the trajectory of the system (7) is given as

$$\begin{aligned} \dot{V}(t) &= \dot{E}^T(t)M_1\ddot{E}(t) + \frac{1}{2}\dot{E}^T(t)\dot{M}_1E(t) + \dot{E}^T(t)PE(t) \\ &= \dot{E}^T(t)\left[M_1\ddot{X}_{Cd}(t) + N_1\dot{X}_{Cd} - N_1\dot{E}(t) - u(t)\right] + \dot{E}^T(t)PE(t) + \frac{1}{2}\dot{E}^T(t)\dot{M}_1E(t) \\ &= \dot{E}^T(t)\left[M_1\ddot{X}_{Cd}(t) + N_1\dot{X}_{Cd} - N_1\dot{E}(t) - u(t) + \frac{N_1+N_1^T}{2}\dot{E}(t) + pE(t)\right] \\ &\quad - \dot{E}^T(t)\frac{N_1+N_1^T}{2}\dot{E}(t) + \frac{1}{2}\dot{E}^T(t)\dot{M}_1E(t) \end{aligned} \tag{10}$$

Let $M_1\ddot{X}_{Cd}(t) + N_1\dot{X}_{Cd} - N_1\dot{E}(t) - u(t) + \frac{N_1+N_1^T}{2}\dot{E}(t) + pE(t) = -Q\dot{E}(t)$, Q is the positive scalar, symmetric matrix; $M+m$ is the constant, $\dot{M}_1 - \left(N_1+N_1^T\right) = 0$, and therefore

$$\dot{V}(t) = -\dot{E}^T(t)Q\dot{E}(t) + \frac{1}{2}\dot{E}^T(t)\left[\dot{M}_1 - \left(N_1+N_1^T\right)\right]\dot{E}(t) = -\dot{E}^T(t)Q\dot{E}(t) < 0 \tag{11}$$

The error state Equation (7) is asymptotically stable.

Then, we can obtain the control input F for the WSR system (6) as

$$F = K_C(\theta)\left[K_C^T(\theta)K_C(\theta)\right]^{-1}\left[M_1\ddot{X}_{Cd} + N_1\dot{X}_{Cd} - N_1\dot{E}(t) + Q\dot{E}(t) + PE(t) + \frac{N_1+N_1^T}{2}\dot{E}(t)\right] \tag{12}$$

4. **Simulation.** In this section, we validate the proposed control method through simulations. The WSR is set to track a predefined lemniscate path since this path can examine all the possible postures of the WSR. Dynamic Equation (6) is used to simulate the practical motion and dynamic characteristics of the WSR. The physical parameters of the WSR used in the simulation are given as: $W = 0.3$m, $L = 0.225$m, $M = 80$kg, $I_G = 1.3$kg·m^2, the parameters of COG shift and load changes are set as $d = 0.2$m, $\alpha = 2\pi/3$rad, $m = 40$kg. The required path to track is shown in (13)

$$36[x_{cd}(t) - x_0]^4 + 625[y_{cd}(t) - y_0]^2 - 900[x_{cd}(t) - x_0]^2 = 0 \tag{13}$$

where $(x_0, y_0) = (5\text{m}, 5\text{m})$ specifies the center of the lemniscate. The desired trajectory of x position, y position and orientation angle to be tracked is described by

$$\begin{aligned} x_{cd}(t) &= x_0 + 5\cos\sigma(t) \\ y_{cd}(t) &= y_0 + 3\sin 2\sigma(t), \\ \theta_d(t) &= \arctan\left[\dot{y}_{cd}(t)/\dot{x}_{cd}(t)\right] \end{aligned} \qquad \sigma(t) = \begin{cases} \dfrac{4\pi}{t_0^2}t^2 & 0 \le t \le \dfrac{t_0}{2} \\ 2\pi - \dfrac{4\pi}{t_0^2}(t-t_0)^2 & \dfrac{t_0}{2} < t \le t_0 \end{cases}$$

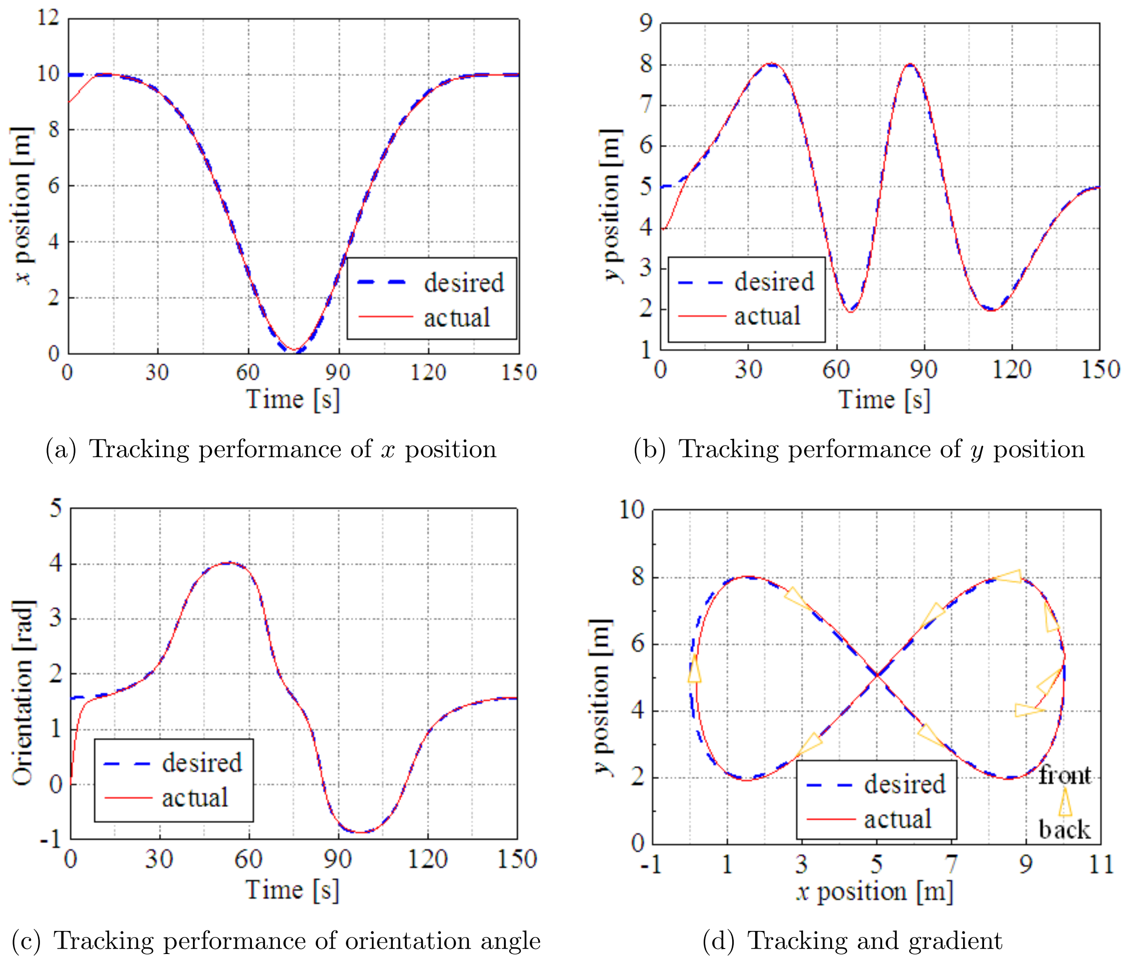
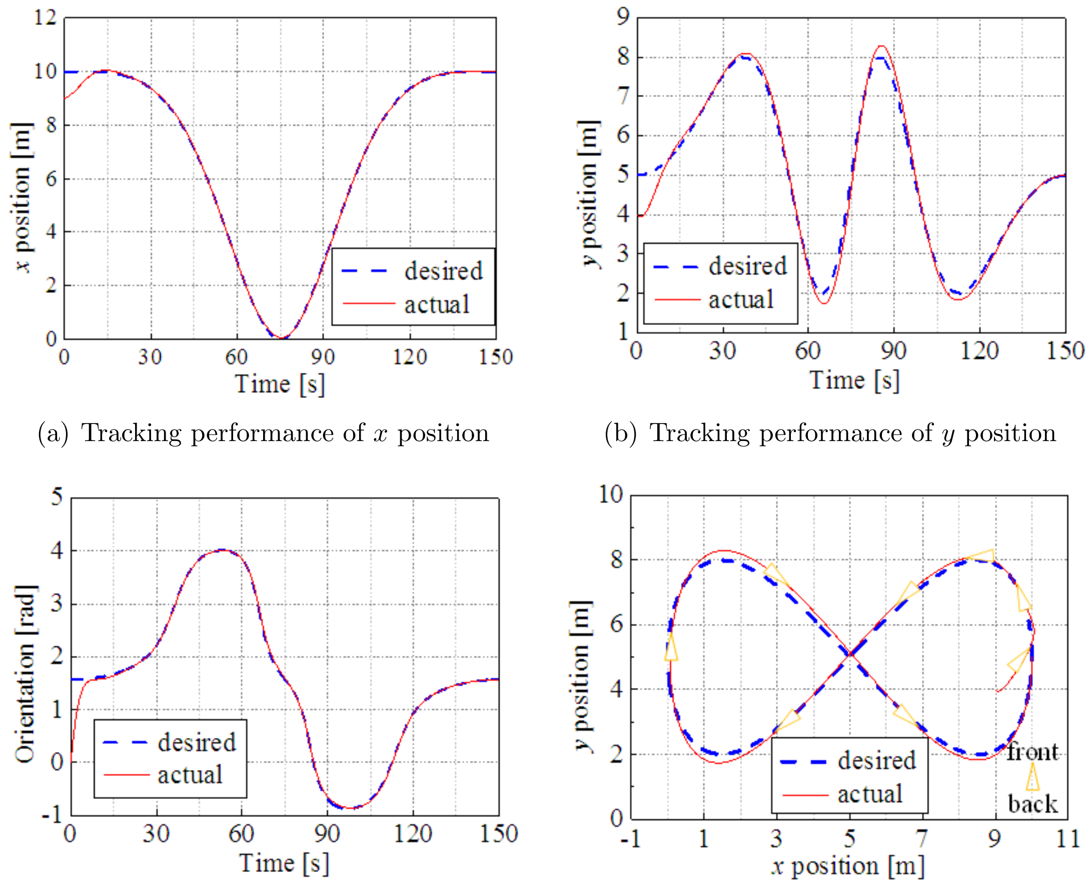


Figure 3. Simulation results without load and COG shifts



(a) Tracking performance of x position

(b) Tracking performance of y position

(c) Tracking performance of orientation angle

(d) Tracking and gradient

FIGURE 4. Simulation results with a load and one kind of COG shift

where t_0 is the simulation execution time, which can be changed to adjust the moving speed of the WSR. Here, t_0 is set to be 150s. The initial position $x_c(0) = 9$m, $y_c(0) = 4$m and the initial angle $\theta(0) = 0$rad. To verify the effectiveness of the proposed method in dealing with constant load changes and COG shift, simulations are conducted with and without considering load changes and COG shifts. The parameters of the proposed control method are adjusted manually in the simulation under the assumption that load is zero and that COG is the same with the geometric center ($d = 0$, $\alpha = 0$rad, $m = 0$kg). The selected proper control parameters are given as $Q = \text{diag}(38, 38, 28)$, $P = \text{diag}(10, 10, 12)$.

Figure 3 shows the tracking ability of the WSR under the condition of zero load and without COG shifts. Figure 4 shows the tracking ability of the WSR with a constant load change and one kind of COG shift with no changes to the control parameters. Figures 3(a)-3(d) and Figures 4(a)-4(d) show the x position, y position, orientation angle tracking results, and path tracking results; in each, the dotted line represents the reference response and the solid line represents the control response. Triangles in Figure 3(d) and Figure 4(d) represent the posture of the WSR. The simulation curves in Figure 3 and Figure 4 show good tracking results of the WSR. In Figure 4 the response curves are similar to those in Figure 3, which indicates that the proposed algorithm allows for good tracking ability even with load change and COG shift in the system.

5. **Conclusion.** A WSR has been developed for the people with walking disabilities in our previous study. The motion of the WSR is affected by load changes and COG shifts. In this paper, to address these issues, a controller that effectively utilizes the acceleration and velocity information of the desired trajectory is designed based on a Lyapunov function. The proposed controller can effectively compensate the constant load changes and COG

shifts, which was demonstrated by comparing simulations while taking the problems of constant load changes and COG shifts into consideration or not. In the future, the real-time changing loads and COG position issues will be addressed.

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