ICIC Express Letters
Part B: Applications
Volume **7**, Number **12**, December **2016**

ICIC International ©2016 ISSN 2185-2766
pp. 2611–2617

AN INVESTIGATION ON FUZZY Γ-HYPERFILTERS IN ORDERED Γ-SEMIHYPERGROUPS

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Received January 2016; accepted April 2016

Abstract. *In this paper, we introduce the concept of fuzzy* Γ*-hyperfilters of an ordered* Γ*-semihypergroup, and discuss its related properties. Furthermore, we introduce the concept of completely prime fuzzy* Γ*-hyperideals of an ordered* Γ*-semihypergroup* S*, and establish the relation of fuzzy* Γ*-hyperfilters and completely prime fuzzy* Γ*-hyperideals of* S*. In particular, we prove that a fuzzy subset* f *of* S *is a fuzzy* Γ*-hyperfilter of* S *if and only if the complement* f' *of* f *is a completely prime fuzzy* Γ*-hyperideal of* S*.*
Keywords: Ordered Γ-semihypergroup, Γ-hyperfilter, Fuzzy Γ-hyperfilter, Completely prime fuzzy Γ-hyperideal

1. **Introduction.** The theory of algebraic hyperstructures which is a generalization of the concept of algebraic structures was first introduced by Marty [1]. Later on, hyperstructures are widely studied from the theoretical point of view and for their applications to many subjects of pure and applied mathematics (see [2]). Recently, many researchers have worked on algebraic hyperstructures and generalized various classical algebraic structures. Especially, semihypergroups are the simplest algebraic hyperstructures which possess the properties of closure and associativity. Nowadays many authors have studied different aspects of semihypergroups, for example, see [3-5]. It is worth pointing out that Heidari and Davvaz [6] applied the theory of hyperstructures to ordered semigroups and introduced the concept of ordered semihypergroups. On the other hand, Sen and Seth [7] introduced the concept of po-Γ-semigroups (some authors called them ordered Γ-semigroups), which is a generalization of ordered semigroups. Later on, Tang and Xie [8] defined and studied the (λ, μ)-fuzzy ideals and (λ, μ)-fuzzy interior ideals of ordered Γ-semigroups.

The theory of fuzzy sets, which was initially introduced in 1965 by Zadeh, has been applied to many mathematical branches. The study of fuzzy hyperstructures is an interesting research topic of fuzzy set theory. We noticed that the relationships between the fuzzy sets and algebraic hyperstructures have been already considered by Corsini, Davvaz, Leoreanu, Zhan, Hila, Tang and others, for example, the reader can refer to [9-12]. Recently, Davvaz and Leoreanu-Fotea [13] studied the structure of fuzzy Γ-hyperideals in Γ-semihypergroups, and provided some interesting results. In [14], Tang et al. introduced the concept of ordered Γ-semihypergroups, and studied the properties of fuzzy quasi-Γ-hyperideals in ordered Γ-semihypergroups. As a further study of ordered Γ-semihypergroups, in this paper we attempt to introduce and give a detailed investigation of fuzzy Γ-hyperfilters of an ordered Γ-semihypergroup. Furthermore, we introduce the concept of completely prime fuzzy Γ-hyperideals of an ordered Γ-semihypergroup, and discuss the relationship between fuzzy Γ-hyperfilters and completely prime fuzzy Γ-hyperideals of ordered Γ-semihypergroups.

2. **Preliminaries and Some Notations.** Recall first the basic terms and definitions from the hyperstructure theory.

As we know, a *hypergroupoid* $(S, \circ)$ is a nonempty set S together with a hyperoperation, that is a map $\circ : S \times S \to P^*(S)$, where $P^*(S)$ denotes the set of all nonempty subsets of S. The image of the pair (x, y) is denoted by $x \circ y$. If $x \in S$ and A, B are nonempty subsets of S, then $A \circ B$ is defined by $A \circ B = \bigcup\limits_{a \in A, b \in B} a \circ b$. The notations $A \circ x$ and $x \circ A$ are used for $A \circ \{x\}$ and $\{x\} \circ A$, respectively. Generally, the singleton $\{x\}$ is identified by its element x.

We say that a hypergroupoid $(S, \circ)$ is a *semihypergroup* if the hyperoperation "$\circ$" is associative, that is, $(x \circ y) \circ z = x \circ (y \circ z)$ for all $x, y, z \in S$ (see [2]).

An algebraic hyperstructure $(S, \circ, \leq)$ is called an *ordered semihypergroup* (also called *po-semihypergroup* in [6]) if $(S, \circ)$ is a semihypergroup and $(S, \leq)$ is a partially ordered set such that the monotone condition holds as follows:

$$x \leq y \Rightarrow a \circ x \preceq a \circ y \text{ and } x \circ a \preceq y \circ a$$

for all $x, y, a \in S$, where, if A and B are nonempty subsets of S, then we say that $A \preceq B$ if for every $a \in A$ there exists $b \in B$ such that $a \leq b$.

Let $S = \{x, y, z, \cdots\}$ and $\Gamma = \{\alpha, \beta, \gamma, \cdots\}$ be two nonempty sets. Then, (S, Γ) is called Γ-*semihypergroup* if every $\gamma \in \Gamma$ is a hyperoperation on S, that is, $x\gamma y \subseteq S$ for every $x, y \in S$, and for every $\alpha, \beta \in \Gamma$ and $x, y, z \in S$, $(x\alpha y)\beta z = x\alpha(y\beta z)$ (see [7]).

We now recall the notion of ordered semihypergroups from [14].

Definition 2.1. *Let $S = \{x, y, z, \cdots\}$ and $\Gamma = \{\alpha, \beta, \gamma, \cdots\}$ be two nonempty sets. An algebraic hyperstructure $(S, \Gamma, \leq)$ is called an ordered Γ-semihypergroup if (S, Γ) is a Γ-semihypergroup and $(S, \leq)$ is a partially ordered set such that: for any $x, y, z \in S$ and $\gamma \in \Gamma$, $x \leq y$ implies $x\gamma z \preceq y\gamma z$ and $z\gamma x \preceq z\gamma y$. Here, if $A, B \in P^*(S)$, then we say that $A \preceq B$ if for every $a \in A$ there exists $b \in B$ such that $a \leq b$. In particular, if $A = \{a\}$, then we write $a \preceq B$ instead of $\{a\} \preceq B$.*

Let $(S, \circ, \leq)$ be an ordered semihypergroup and $\Gamma = \{\circ\}$. Then, $(S, \Gamma, \leq)$ is an ordered Γ-semihypergroup. Thus every ordered semihypergroup is an ordered Γ-semihypergroup. Throughout this paper, unless stated otherwise, S stands for an ordered Γ-semihypergroup.

Let A, B be nonempty subsets of an ordered Γ-semihypergroup S. We define

$$A\Gamma B := \bigcup_{\gamma \in \Gamma} A\gamma B = \bigcup_{\gamma \in \Gamma} \{a\gamma b \mid a \in A, b \in B\}.$$

Let S be an ordered Γ-semihypergroup. For $\emptyset \neq H \subseteq S$, we define

$$(H] := \{t \in S \mid t \leq h \text{ for some } h \in H\}.$$

For $H = \{a\}$, we write $(a]$ instead of $(\{a\}]$. For any $A, B \in P^*(S)$, we have: (1) $A \subseteq (A]$; (2) If $A \subseteq B$, then $(A] \subseteq (B]$; (3) $(A]\Gamma(B] \subseteq (A\Gamma B]$.

By a *sub* Γ-*semihypergroup* of an ordered Γ-semihypergroup S we mean a nonempty subset A of S such that $A\Gamma A \subseteq A$. A nonempty subset I of an ordered Γ-semihypergroup S is called a *left* (resp. *right*) Γ-*hyperideal* of S if (1) $S\Gamma I \subseteq I$ (resp. $I\Gamma S \subseteq I$) and (2) $a \in I$, and $S \ni b \leq a$ implies $b \in I$. If I is both a left and a right Γ-hyperideal of S, then it is called a Γ-*hyperideal* of S (see [14]). A Γ-hyperideal I of an ordered Γ-semihypergroup S is called *completely prime* if for any two elements a, b of S such that $a \circ b \cap I \neq \emptyset$, then $a \in I$ or $b \in I$.

We next state some fuzzy logic concepts.

Let S be an ordered Γ-semihypergroup. By a *fuzzy subset* of S, we mean a function from S into the real closed interval $[0, 1]$, that is, $f : S \to [0, 1]$. We denote by $F(S)$

the set of all fuzzy subsets of S. Let $f, g \in F(S)$. Then, the inclusion relation $f \subseteq g$ is defined by $f(x) \le g(x)$ for all $x \in S$, and $f \cup g$ is defined by

$$(f \cup g)(x) = \max\{f(x), g(x)\} = f(x) \vee g(x)$$

for all $x \in S$.

Definition 2.2. ([14]) *Let S be an ordered Γ-semihypergroup and $f \in F(S)$. The set*

$$f_t := \{x \in S \mid f(x) \ge t\}, \text{ where } t \in (0, 1]$$

is called a level subset of f.

Definition 2.3. *Let S be an ordered semihypergroup and $f \in F(S)$. Then the function defined by*

$$f' : S \to [0, 1] \mid x \mapsto f'(x) := 1 - f(x),$$

is a fuzzy subset of S, called the complement of f in S.

Similar to Lemmas 2.6 and 2.7 in [11], we have the following two lemmas.

Lemma 2.1. *Let S be an ordered Γ-semihypergroup and $f \in F(S)$. Then the following statements are equivalent.*

(1) $\bigwedge_{z \in x\Gamma y} f(z) \ge f(x) \wedge f(y), \forall x, y \in S$.

(2) $\bigvee_{z \in x\Gamma y} f'(z) \le f'(x) \vee f'(y), \forall x, y \in S$.

Lemma 2.2. *Let S be an ordered Γ-semihypergroup and $f \in F(S)$. Then the following statements are equivalent.*

(1) $\bigvee_{z \in x\Gamma y} f(z) \le f(x) \wedge f(y), \forall x, y \in S$.

(2) $\bigwedge_{z \in x\Gamma y} f'(z) \ge f'(x) \vee f'(y), \forall x, y \in S$.

The reader is referred to [2,14] for notation and terminology not defined in this paper.

3. **Fuzzy Γ-Hyperfilters of Ordered Γ-Semihypergroups.**

In this section, we introduce the concepts of Γ-hyperfilters and fuzzy Γ-hyperfilters of ordered Γ-semihypergroups. Moreover, we discuss the properties of fuzzy Γ-hyperfilters of ordered Γ-semihypergroups.

Definition 3.1. *Let S be an ordered Γ-semihypergroup. A sub Γ-semihypergroup F of S is called a Γ-hyperfilter of S if*

(1) $(\forall\, a, b \in S)\ a\Gamma b \cap F \neq \emptyset \Rightarrow a, b \in F$.

(2) $a \in F, b \in S, a \le b \Rightarrow b \in F$.

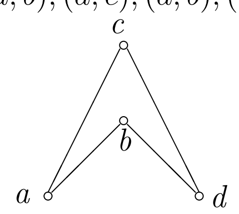
Example 3.1. *Let $S := \{a, b, c, d\}$ and $\Gamma = \{\gamma\}$ with the hyperoperation and the relation "$\le$" on S defined by*

γ	a	b	c	d
a	$\{a, d\}$	$\{a, d\}$	$\{a, d\}$	$\{a\}$
b	$\{a, d\}$	$\{b\}$	$\{a, d\}$	$\{a, d\}$
c	$\{a, d\}$	$\{a, d\}$	$\{c\}$	$\{a, d\}$
d	$\{a\}$	$\{a, d\}$	$\{a, d\}$	$\{d\}$

$$\le := \{(a, a), (a, b), (a, c), (b, b), (c, c), (d, b), (d, c), (d, d)\}.$$

We give the covering relation "$\prec$" and the figure of S as follows:

$$\prec = \{(a, b), (a, c), (d, b), (d, c)\}.$$



Then $(S, \Gamma, \leq)$ is an ordered Γ-semihypergroup. With a small amount of effort one can verify that the sets $\{b\}$, $\{c\}$ and S are all Γ-hyperfilters of S.

Definition 3.2. *Let S be an ordered Γ-semihypergroup. A fuzzy subset f of S is called a fuzzy sub Γ-semihypergroup of S if $\bigwedge_{z \in x\Gamma y} f(z) \geq f(x) \wedge f(y)$ for all $x, y \in S$.*

Lemma 3.1. *Let S be an ordered Γ-semihypergroup and $f \in F(S)$. Then f is a fuzzy sub Γ-semihypergroup of S if and only if the level subset f_t $(t \in (0,1])$ of f is a sub Γ-semihypergroup of S whenever $f_t \neq \emptyset$.*

Proof: Assume that f is a fuzzy sub Γ-semihypergroup of S. Let $x, y \in f_t$. Then $f(x) \geq t$, $f(y) \geq t$. By hypothesis, we have $\bigwedge_{z \in x\Gamma y} f(z) \geq f(x) \wedge f(y)$. Thus, for any $z \in x\Gamma y$, we have $f(z) \geq f(x) \wedge f(y) \geq t$, i.e., $z \in f_t$. It follows that $x\Gamma y \subseteq f_t$. Consequently, f_t is a sub Γ-semihypergroup of S for $f_t \neq \emptyset$.

Conversely, suppose that f_t $(\neq \emptyset)$ is a sub Γ-semihypergroup of S. If $\bigwedge_{z \in x\Gamma y} f(z) < f(x) \wedge f(y)$ for some $x, y \in S$, then there exists $t \in (0,1)$ such that

$$\bigwedge_{z \in x\Gamma y} f(z) < t < f(x) \wedge f(y),$$

which implies that $x, y \in f_t$ and $x\Gamma y \nsubseteq f_t$. It contradicts the fact that f_t is a sub Γ-semihypergroup of S. Therefore, f is a fuzzy sub Γ-semihypergroup of S.

Definition 3.3. *Let S be an ordered Γ-semihypergroup. A fuzzy sub Γ-semihypergroup f of S is called a fuzzy Γ-hyperfilter of S if*

(1) $\bigvee_{z \in x\Gamma y} f(z) \leq f(x) \wedge f(y)$ *for all* $x, y \in S$.

(2) $x \leq y$ *implies* $f(x) \leq f(y)$.

Theorem 3.1. *Let S be an ordered Γ-semihypergroup and $f \in F(S)$. Then f is a fuzzy Γ-hyperfilter of S if and only if the level subset f_t $(t \in (0,1])$ of f is a Γ-hyperfilter of S whenever $f_t \neq \emptyset$.*

Proof: Suppose that f is a fuzzy Γ-hyperfilter of S. Then f is a fuzzy sub Γ-semihypergroup of S. By Lemma 3.1, f_t is a sub Γ-semihypergroup of S. Let $x, y \in S$ be such that $x\Gamma y \cap f_t \neq \emptyset$. Then there exists $z \in x\Gamma y$ such that $z \in f_t$, and $f(z) \geq t$. By hypothesis, we have

$$t \leq \bigvee_{z \in x\Gamma y} f(z) \leq f(x) \wedge f(y),$$

which implies that $f(x) \geq t$ and $f(y) \geq t$. It thus follows that $x, y \in f_t$. Furthermore, let $x \in f_t, x \leq y \in S$. Then $y \in f_t$. Indeed, since $x \in f_t, f(x) \geq t$ and f is a fuzzy Γ-hyperfilter of S, we have $f(y) \geq f(x) \geq t$, which means that $y \in f_t$. Consequently, f_t is a Γ-hyperfilter of S.

Conversely, assume that f_t $(\neq \emptyset)$ is a Γ-hyperfilter of S. By Definition 3.3 and Lemma 3.1, f is a fuzzy sub Γ-semihypergroup of S. Let x, y be arbitrary elements of S. We claim that $\bigvee_{z \in x\Gamma y} f(z) \leq f(x) \wedge f(y)$. If $\bigvee_{z \in x\Gamma y} f(z) > f(x) \wedge f(y)$ for some $x, y \in S$, then there exists $t \in (0,1]$ such that

$$\bigvee_{z \in x\Gamma y} f(z) \geq t > f(x) \wedge f(y).$$

It thus follows that $f(z) > t$ for some $z \in x\Gamma y$. Then $z \in f_t$ and $x\Gamma y \cap f_t \neq \emptyset$. Since f_t is a Γ-hyperfilter of S, we have $x \in f_t$ and $y \in f_t$. Then $f(x) \wedge f(y) \geq t \wedge t = t$, which is a contradiction. Hence $\bigvee_{z \in x\Gamma y} f(z) \leq f(x) \wedge f(y)$ for all $x, y \in S$. Moreover, let $x, y \in S$ be such that $x \leq y$. Then $f(x) \leq f(y)$. In fact, let $t = f(x)$. Then $x \in f_t$. Since f_t is a Γ-hyperfilter of S, we have $y \in f_t$. Then $f(y) \geq t = f(x)$. Therefore, f is a fuzzy Γ-hyperfilter of S.

Example 3.2. *Consider the ordered* Γ*-semihypergroup* $(S, \Gamma, \leq)$ *given in Example 3.1. We have shown that the sets* $\{b\}$, $\{c\}$ *and* S *are all* Γ*-hyperfilters of* S*. Now let* f *be a fuzzy subset of* S *such that*

$$f(a) = f(c) = f(d) = 0, \ f(b) = 0.5.$$

Then

$$f_t = \begin{cases} \{b\}, & \text{if } t \in (0, 0.5], \\ \emptyset, & \text{if } t \in (0.5, 1]. \end{cases}$$

Thus all nonempty level subsets f_t $(t \in (0, 1])$ *of* f *are* Γ*-hyperfilters of* S *and by Theorem 3.1,* f *is a fuzzy* Γ*-hyperfilter of* S*.*

In order to characterize the fuzzy Γ-hyperfilters of ordered Γ-semihypergroups, we need introduce the following concept.

Definition 3.4. *Let* S *be an ordered* Γ*-semihypergroup. A fuzzy subset* f *of* S *is called completely prime if* $\bigvee_{z \in x\Gamma y} f(z) \leq f(x) \vee f(y)$ *for any* $x, y \in S$*.*

Definition 3.5. *Let* S *be an ordered* Γ*-semihypergroup. A fuzzy subset* f *of* S *is called a fuzzy left (resp. right)* Γ*-hyperideal of* S *if*

(1) $x \leq y$ *implies* $f(x) \geq f(y)$ *for all* $x, y \in S$*, and*

(2) $\bigwedge_{z \in x\Gamma y} f(z) \geq f(y)$ *(resp.* $\bigwedge_{z \in x\Gamma y} f(z) \geq f(x)$*) for all* $x, y \in S$*.*

A fuzzy Γ*-hyperideal of* S *is a fuzzy subset of* S *which is both a fuzzy left and a fuzzy right* Γ*-hyperideal of* S*.*

Lemma 3.2. *Let* S *be an ordered* Γ*-semihypergroup and* $f \in F(S)$*. Then* f *is a fuzzy* Γ*-hyperideal of* S *if and only if the level subset* f_t $(t \in (0, 1])$ *of* f *is a* Γ*-hyperideal of* S *for* $f_t \neq \emptyset$*.*

Proof: Assume that f is a fuzzy Γ-hyperideal of S. Let $t \in (0, 1]$ be such that $f_t \neq \emptyset$. Let now $x \in S, y \in f_t$. Then $f(y) \geq t$. By hypothesis, we have

$$\bigwedge_{z \in x\Gamma y} f(z) \geq f(y) \geq t.$$

Thus for any $z \in x\Gamma y$, we have $f(z) \geq t$, i.e., $z \in f_t$. It follows that $x\Gamma y \subseteq f_t$. Furthermore, let $x \in f_t$, $S \ni y \leq x$. Then $y \in f_t$. Indeed, since $x \in f_t$, $f(x) \geq t$ and f is a fuzzy Γ-hyperideal of S, we have $f(y) \geq f(x) \geq t$, so $y \in f_t$. Hence f_t is a left Γ-hyperideal of S. In the same way, we can show that f_t is also a right Γ-hyperideal of S. Consequently, f_t is a Γ-hyperideal of S for $f_t \neq \emptyset$.

Conversely, suppose that f_t $(\neq \emptyset)$ is a Γ-hyperideal of S. If $\bigwedge_{z \in x\Gamma y} f(z) < f(y)$ for some $x, y \in S$, then there exists $t \in (0, 1]$ such that

$$\bigwedge_{z \in x\Gamma y} f(z) < t \leq f(y),$$

which implies that $y \in f_t$ and $x\Gamma y \not\subseteq f_t$. It contradicts the fact that f_t is a Γ-hyperideal of S. Consequently, $\bigwedge_{z \in x\Gamma y} f(z) \geq f(y)$ for all $x, y \in S$. In a similar way, we can show that $\bigwedge_{z \in x\Gamma y} f(z) \geq f(x)$ for all $x, y \in S$. Moreover, let $x, y \in S$ and $x \leq y$. Then $f(x) \geq f(y)$. In fact, let $t = f(y)$. Then $y \in f_t$. Since f_t is a Γ-hyperideal of S, we have $x \in f_t$. Then $f(x) \geq t = f(y)$. Therefore, f is a fuzzy Γ-hyperideal of S.

Lemma 3.3. *Let* S *be an ordered* Γ*-semihypergroup. If* f *is a completely prime fuzzy* Γ*-hyperideal of* S*, then* $\bigwedge_{z \in x\Gamma y} f(z) = f(x) \vee f(y)$ *for any* $x, y \in S$*.*

Proof: Assume that f is a completely prime fuzzy Γ-hyperideal of S. Let $x, y \in S$. Then, since f is a fuzzy Γ-hyperideal of S, $\bigwedge_{z \in x\Gamma y} f(z) \geq f(x) \vee f(y)$. On the other hand, since f is completely prime, by Definition 3.4 we have

$$\bigwedge_{z\in x\Gamma y} f(z) \leq \bigvee_{z\in x\Gamma y} f(z) \leq f(x)\vee f(y).$$

Therefore, $\bigwedge_{z\in x\Gamma y} f(z) = f(x)\vee f(y)$ for any $x, y \in S$.

Theorem 3.2. *Let S be an ordered Γ-semihypergroup and $f \in F(S)$. Then f is a completely prime fuzzy Γ-hyperideal of S if and only if the level subset f_t $(t \in (0,1])$ of f is a completely prime Γ-hyperideal of S for $f_t \neq \emptyset$.*

Proof: Suppose that f is a completely prime fuzzy Γ-hyperideal of S. By Lemma 3.2, f_t $(t \in (0,1])$ is a Γ-hyperideal of S for $f_t \neq \emptyset$. To prove that f_t is completely prime, let $x, y \in S$ be such that $x\Gamma y \cap f_t \neq \emptyset$. Then there exists $z \in x\Gamma y$ such that $z \in f_t$. It implies that $\bigvee_{z\in x\Gamma y} f(z) \geq t$. Then, by hypothesis we have

$$f(x)\vee f(y) \geq \bigvee_{z\in x\Gamma y} f(z) \geq t,$$

which means that $f(x) \geq t$ or $f(y) \geq t$, and thus $x \in f_t$ or $y \in f_t$. Therefore, f_t is completely prime.

Conversely, assume that f_t $(\neq \emptyset)$ is a completely prime Γ-hyperideal of S. Then, by Lemma 3.2, f is a fuzzy Γ-hyperideal of S. Now let $x, y \in S$ and $t = \bigvee_{z\in x\Gamma y} f(z)$. Then there exists $z \in x\Gamma y$ such that $f(z) \geq t$, and $z \in f_t$. It implies that $x\Gamma y \cap f_t \neq \emptyset$. Since f_t is a completely prime Γ-hyperideal of S, we have $x \in f_t$ or $y \in f_t$, and thus $f(x) \geq t$ or $f(y) \geq t$. Consequently, $f(x)\vee f(y) \geq t = \bigvee_{z\in x\Gamma y} f(z)$. Thus, f is a completely prime fuzzy Γ-hyperideal of S.

Example 3.3. *Consider the ordered Γ-semihypergroup $(S, \Gamma, \leq)$ given in Example 3.1. It can be easily shown that the set $\{a, b, d\}$ is a completely prime Γ-hyperideal of S. Now let f be a fuzzy subset of S such that*

$$f(a) = f(b) = f(d) = 0.5,\ f(c) = 0.$$

Then

$$f_t = \begin{cases} \{a, b, d\}, & \text{if } t \in (0, 0.5], \\ \emptyset, & \text{if } t \in (0.5, 1]. \end{cases}$$

Thus all nonempty level subsets f_t $(t \in (0,1])$ of f are completely prime Γ-hyperideals of S, and f is a completely prime fuzzy Γ-hyperideal of S by Theorem 3.2.

Now we characterize the fuzzy Γ-hyperfilters of ordered Γ-semihypergroups in terms of completely prime fuzzy Γ-hyperideals.

Theorem 3.3. *Let S be an ordered Γ-semihypergroup and $f \in F(S)$. Then f is a fuzzy Γ-hyperfilter of S if and only if the complement f' of f is a completely prime fuzzy Γ-hyperideal of S.*

Proof: $\Rightarrow$. Suppose that f is a fuzzy Γ-hyperfilter of S. Then f is a fuzzy sub Γ-semihypergroup of S. Let $x, y \in S$, $x \leq y$. Since f is a fuzzy Γ-hyperfilter of S, we have $f(x) \leq f(y)$. Then $-f(x) \geq -f(y)$ implies that $1 - f(x) \geq 1 - f(y)$. Hence $f'(x) \geq f'(y)$. Let x, y be arbitrary elements of S. Since f is a fuzzy Γ-hyperfilter of S, we have $\bigvee_{z\in x\Gamma y} f(z) \leq f(x)\wedge f(y)$. Then, by Lemma 2.2, $\bigwedge_{z\in x\Gamma y} f'(z) \geq f'(x)\vee f'(y)$. Thus f' is a fuzzy Γ-hyperideal of S. To prove that f' is completely prime, let $x, y \in S$. Then, since f is a fuzzy sub Γ-semihypergroup of S, $\bigwedge_{z\in x\Gamma y} f(z) \geq f(x)\wedge f(y)$. By Lemma 2.1, we have $\bigvee_{z\in x\Gamma y} f'(z) \leq f'(x)\vee f'(y)$. Therefore, f' is a completely prime fuzzy Γ-hyperideal of S.

$\Leftarrow$. Assume that f' is a completely prime fuzzy Γ-hyperideal of S. Let $x, y \in S$ such that $x \leq y$. Since f' is a fuzzy Γ-hyperideal of S, we have $f'(x) \geq f'(y)$. Then $1 - f(x) \geq 1 - f(y)$, and thus $f(x) \leq f(y)$. Let $x, y \in S$. Then, since f' is a fuzzy Γ-hyperideal of S, $\bigwedge_{z \in x\Gamma y} f'(z) \geq f'(x) \vee f'(y)$. By Lemma 2.2, we have $\bigvee_{z \in x\Gamma y} f(z) \leq f(x) \wedge f(y)$. In addition, since f' is completely prime, we have $\bigvee_{z \in x\Gamma y} f'(z) \leq f'(x) \vee f'(y)$. Then, by Lemma 2.1, $\bigwedge_{z \in x\Gamma y} f(z) \geq f(x) \wedge f(y)$. Hence f is a fuzzy sub Γ-semihypergroup of S. Therefore, f is a fuzzy Γ-hyperfilter of S.

4. **Conclusions.** It is well known that ordered Γ-semihypergroups generalize ordered Γ-semigroups and ordered semihypergroups. The details can be referred to [8,14]. The purpose of this paper is to introduce and study the fuzzy Γ-hyperfilters of an ordered Γ-semihypergroup. In particular, we discussed the relationship between fuzzy Γ-hyperfilters and completely prime fuzzy Γ-hyperideals of ordered Γ-semihypergroups. This paper generalized some results in [11]. We hope that this work would offer foundation for further study of the theory on ordered Γ-semihypergroups and fuzzy ordered Γ-semihypergroups.

Acknowledgment. This work was supported by the University Natural Science Project of Anhui Province (No. KJ2015A161).

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