ICIC Express Letters
Part B: Applications
Volume **7**, Number **12**, December **2016**

ICIC International ©2016 ISSN 2185-2766
pp. **2603–2609**

AN EFFECTIVE METHOD FOR TRAFFIC ASSIGNMENT PROBLEM UNDER UNCERTAIN ENVIRONMENT

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Received December 2015; accepted March 2016

Abstract. *Traffic assignment is a fundamental problem in the field of transportation science. In the past decades, many algorithms have been proposed to find the optimal solution. However, the travel demand is not deterministic. Instead, it is varying day by day. In this paper, we investigate the traffic assignment problem with the fuzzy travel demand. By representing the travel demand using the triangular fuzzy numbers, we propose a generalized* Physarum*-based approach to approach the optimal solution to the traffic assignment problem. A numerical example is used to demonstrate the performance of the proposed algorithm.*

Keywords: *Physarum*, Traffic assignment, Uncertainty, Optimization

1. **Introduction.** Traffic networks are playing more and more significant roles in our daily life in the sense that they ship massive products from the manufacturers to the demand markets and conveys millions of persons between home and work place as well. In the past decades, traffic networks have received increasing attention. Seminal work has been done by various researchers, for example, Wardrop built two individual principles to depict the two equilibrium statuses of the traffic networks: user equilibrium (UE) flows, and system optimal (SO) flows [1].

In terms of user equilibrium, it is based on the fact that humans choose a route so as to minimize his/her travel time and on the assumption that such a behaviour on the individual level creates an equilibrium at the system (or network) level. In contrast, it implies that each user behaves cooperatively in choosing his own route to ensure the most efficient use of the whole system. Among them, due to the importance of user equilibrium to depict the pattern of user behavior and the flow distribution in the equilibrate traffic networks, it draws lots of attention and many algorithms have been proposed to solve this problem. For instance, Fukushima put forward an improved Frank-Wolfe (FW) algorithm for the traffic assignment problem [2]. Lately, Mitradjieva and Lindberg presented the Conjugate Frank-Wolfe algorithm and Bi-Conjugate Frank-Wolfe by improving the FW search direction [3]. In recent years, many bush-based algorithms are emerging. For instance, Dial proposed Algorithm B by shifting the traffic flow from the longest used path to the shortest path within a given bush [4].

However, the travel demand is affected by many uncertain factors, such as seasons, and holidays. For example, in China, in the national day, the traffic volume in many attractions is much larger than usual. In USA, at the holiday of Thanksgiving, millions of persons are going back to their hometowns to celebrate the holiday, which leads to a totally different traffic pattern from usual. As a result, it is not realistic to represent the traffic demand in a deterministic way. Instead, a research on the traffic assignment problem, uncertain environment, is much more meaningful. Seminal work has been presented by many researchers [5, 6, 7]. However, some of the proposed models are too complicated to be implemented in many large networks. In the past decades, fuzzy numbers are developing very fast and have been widely used to solve many practical problems, such as supplier selection [8], path finding [9], and risk analysis [10]. They show great potential to be applied into real-world applications.

Recently, *Physarum* has been proved to be capable of solving the user equilibrium problem [11, 12]. Liu et al. [13] employed the *Physarum* model to solve the traffic assignment problem in the presence of uncertainty, in which they decomposed the fuzzy numbers into three individual parts. However, this in turn increases the size of solving linear equations. Given a network with N nodes, they enlarge the problem with $3 * N$ linear equations, which might hinder its application in real-world scenarios. In this paper, we are motivated to propose another generalized *Physarum*-based approach to address the traffic assignment problem under uncertain environment, in which we transform the fuzzy numbers into crisp numbers. By doing this, on one hand, we capture the uncertainty in the traffic demand. On the other hand, we retain the same time complexity as the original *Physarum* model, which makes it different from the existing methods.

The remainder of this paper is structured as follows. In Section 2, we introduce the basic theories, including the mathematical model of *Physarum* and traffic assignment problem. In Section 3, we present the proposed methodology in detail. In Section 4, we use several numerical examples to demonstrate the efficiency. In Section 5, we summarize this paper with conclusions.

2. **Preliminaries.**

In this section, the basic theories, including the mathematical model of *Physarum* and the traffic assignment problem, are briefly introduced.

2.1. **Mathematical model of *Physarum*.**

According to the mathematical model built by Tero et al. [14], it can be described as follows. Each segment in the diagram represents a section of tube. Two special nodes, which are also called food source nodes, are named N_1 and N_2, and the other nodes are denoted as N_3, N_4, N_5, and so on. The section of tube between N_i and N_j is denoted as M_{ij}. If several tubes connect the same pair of nodes, intermediate nodes will be placed in the center of the tubes to guarantee the uniqueness of the connecting segments. The variable Q_{ij} is used to express the flux through tube M_{ij} from N_i to N_j. Assume the flow along the tube as an approximately poiseuille flow, the flux Q_{ij} can be expressed as:

$$Q_{ij} = \frac{D_{ij}}{L_{ij}}(p_i - p_j) \tag{1}$$

where p_i is the pressure at the node N_i, p_j is the pressure at the node N_j, D_{ij} is the conductivity of the edge M_{ij}, and L_{ij} is the length of the edge M_{ij}.

Assume zero capacity at each node; hence by considering the conservation law of sol the following equation can be obtained:

$$\sum Q_{ij} = 0 \quad (j \neq 1, 2) \tag{2}$$

For the source node N_1 and the sink node N_2, the following two equations hold

$$\sum_i Q_{i1} + I_0 = 0 \tag{3}$$

$$\sum_i Q_{i2} - I_0 = 0 \tag{4}$$

where I_0 is the flux flowing from the source node. It can be seen that I_0 is a constant value in this model.

In order to describe such an adaptation of tubular thickness, we assume that the conductivity D_{ij} changes over time according to the flux Q_{ij}. The following equation for the evolution of $D_{ij}(t)$ can be used

$$\frac{d}{dt}D_{ij} = f(|Q_{ij}|) - rD_{ij} \tag{5}$$

where r is a decay rate of the tube. It can be obtained that the equation implies that the conductivity ends to vanish if there is no flux along the edge, while it is enhanced by the flux. The f is monotonically increasing continuous function satisfying $f(0) = 0$.

Then the network of Poisson equation for the pressure can be obtained from Equations (1)-(4) as follows:

$$\sum_i \frac{D_{ij}}{L_{ij}}(p_i - p_j) = \begin{cases} -1 & \text{for } j = 1, \\ +1 & \text{for } j = 2, \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

By setting $p_2 = 0$ as a basic pressure level, all p_i can be determined by solving Equation (6) and Q_{ij} can also be obtained.

In this paper, $f(Q) = |Q|$ is used. With the flux calculated, the conductivity can be derived, where Equation (7) is used instead of Equation (5), adopting the functional form $f(Q) = |Q|$.

$$\frac{D_{ij}^{n+1} - D_{ij}^{n}}{\delta t} = |Q| - D_{ij}^{n+1} \tag{7}$$

where δt is a time mesh size and the upper index n indicates a time step.

2.2. **Traffic assignment problem.** Consider a network $G(N, A)$, where N and A are sets of nodes and links, respectively. Let O and D be subsets of N, for which travel demand q^{st} is generated from origin $s \in O$ to destination $t \in D$. Suppose f_p^{st} represents the path flow originated at node s and destined to node t, then we have:

$$\sum_{p \in P^{st}} f_p^{st} = q^{st}, \quad \forall s \in O, \quad t \in D \tag{8}$$

where P^{st} is a set of cycle free paths connecting s to t. All path flows must be non-negative to guarantee a meaningful solution:

$$f_r^{st} \geq 0, \quad \forall r \in P^{st}, \quad s \in O, \quad t \in D \tag{9}$$

Let f_{ij} denote the flow along the link (i, j). Then the total flow on the link (i, j) is the sum of all paths that includes the link:

$$f_{ij} = \sum_{s \in O} \sum_{t \in D} \sum_{r \in P^{st}} f_r^{st} \delta_{rij}^{st}, \quad \forall (i, j) \in A \tag{10}$$

where $\delta_{rij}^{st} = 1$ if the link (i, j) is a segment of path r connecting s to t. Otherwise, $\delta_{rij}^{st} = 0$.

All nodes, except the supply nodes and the demand nodes, must satisfy the flow conservation law:

$$\sum_{(i,j) \in A} f_{ij} = \sum_{(j,k) \in A} f_{jk}, \quad \forall j \in N \backslash \{O, D\} \tag{11}$$

In a transportation network, each user non-cooperatively seeks to minimize his own cost by taking the path of least perceived cost from his origin to his destination. The network is said to be at equilibrium if nobody can reduce his cost by shifting to other alternative routes. Suppose the cost of shipping f_{ij} units along link (i, j) is $g_{ij}(f_{ij})$. The cost function $g_{ij}(f_{ij})$ is a monotonically increasing function of the total flow passing through the link

(i, j). The traffic equilibrium assignment problem can be formulated as:

$$\begin{aligned}
Min \quad & \sum_{(i,j)\in A} \int_0^{f_{ij}} g_{ij}\,(f_{ij})df \\
\text{s.t.} \quad & f_{ij} = \sum_{s\in O}\sum_{t\in D}\sum_{r\in P^{st}} f_r^{st}\delta_{pij}^{st}, \ \ \forall(i,j)\in A \\
& \sum_{(i,j)\in A} f_{ij} = \sum_{(j,k)\in A} f_{jk}, \ \ \forall j \in N\backslash\{O,D\} \\
& \sum_{p\in P^{st}} f_p^{st} = q^{st}, \ \ \forall s\in O, \ \ t\in D \\
& f_p^{st} \geq 0, \ \ \forall p\in P^{st}, \ \ s\in O, \ \ t\in D \\
& f_{ij} \geq 0, \ \ \forall\,(i,j)\in A
\end{aligned} \tag{12}$$

3. **Development of the Proposed Algorithm.** The proposed method consists of three steps. First of all, we use the triangular fuzzy numbers (TFN) to represent the fuzzy travel demand. Secondly, the fuzzy travel demand is converted into a crisp one. At last, with the help of *Physarum*, we are able to find the solution to the traffic assignment problem.

Step 1: Expressing the uncertain travel demand using triangular fuzzy numbers. In this paper, the nondeterministic travel demand between the OD pair (i, j) is expressed by triangular fuzzy numbers in the form of $R_{ij}(r_{ijl}, r_{ijm}, r_{ijr})$.

Step 2: Transforming TFN into crisp number. There are various ways to transform TFN into a crisp number. In this paper, we perform the transformation according to the following method:

$$d\,(i,j) = \frac{r_{ijl} + 4r_{ijm} + r_{ijr}}{6} \tag{13}$$

In this way, we transform the fuzzy travel demand in a canonical representation.

Step 3: Apply *Physarum*-based solution to the fuzzy traffic assignment problem. Since *Physarum* is able to adapt to the update of the link cost, we can make full use of this mechanism to approach the solution to the traffic assignment problem. The specific algorithm is illustrated in Algorithm 1.

There are several alternate conditions to determine the termination of the *Physarum* algorithm, for example, arriving the maximum iterations, and the difference between two consecutive iterations is less than a predefined threshold. In our paper, when the sum of the absolute difference in two consecutive iterations is less than 0.01, the program ends.

4. **Applications.** The network shown in Figure 1 is derived from [15]. In this network, it has 13 nodes, 19 links, and 4 OD pairs. The origin-destination demands, in vehicles per hour, are $q^{1,2} = [520, 660, 740]$, $q^{1,3} = [400, 480, 500]$, $q^{4,2} = [300, 400, 500]$, and $q^{4,3} = [500, 520, 630]$. The link characteristics are shown in Table 1.

The following function developed by the US Bureau of Public Roads (BPR) is used to represent the cost on each link [16]:

$$g_{ij} = \alpha_{ij}\left(1 + 0.15\left(\frac{v_{ij}}{c_{ij}}\right)^4\right) \tag{14}$$

where g_{ij}, α_{ij}, v_{ij}, c_{ij} denote the travel time (cost), free-flow travel time, flow and capacity on link (i, j), respectively.

With the proposed method, we need to convert the fuzzy travel demand into a crisp number. For example, with respect to $q^{1,2}$, we have $\frac{520+4*660+740}{6} = 650$. In a similar way, we can apply the same operation to the other fuzzy travel demand. Afterwards, based on Algorithm 1, we can find the optimal solution to the network shown in Figure 1. In

Algorithm 1 *Physarum* Solver in Traffic Network Equilibrium Assignment Problem (L, n, O, D)

// n is the size of the network;
// O, D are the set of origin nodes and destination nodes;
// L_{ij} is the length of the link connecting node i with node j;
// Transform the fuzzy travel demand into a crisp one
$C_{ij} \leftarrow (0, 1]$ $(\forall i, j = 1, 2, \ldots, n)$;
$Q_{ij} \leftarrow 0$ $(\forall i, j = 1, 2, \ldots, n)$;
$p_i \leftarrow 0$ $(\forall i = 1, 2, \ldots, n)$;
repeat
  Calculate the pressure associated with each node according to Equation (6)

$$\sum_i \frac{D_{ij}}{L_{ij}}(p_i - p_j) = \begin{cases} -1 & \text{for } j = 1, \\ +1 & \text{for } j = 2, \\ 0 & \text{otherwise} \end{cases}$$

  $Q_{ij} \leftarrow C_{ij} \times (p_i - p_j)/L_{ij}$ // Using Equation (1);
  $C_{ij} \leftarrow Q_{ij} + C_{ij}$ // Using Equation (7)
  Update the cost on each link;
  for $i = 1 : n$ **do**
    for $j = 1 : n$ **do**
      $L_{ij} = \dfrac{L_{ij} + g_{ij}(Q_{ij})}{2}$
    end for
  end for
until a termination criterion is met

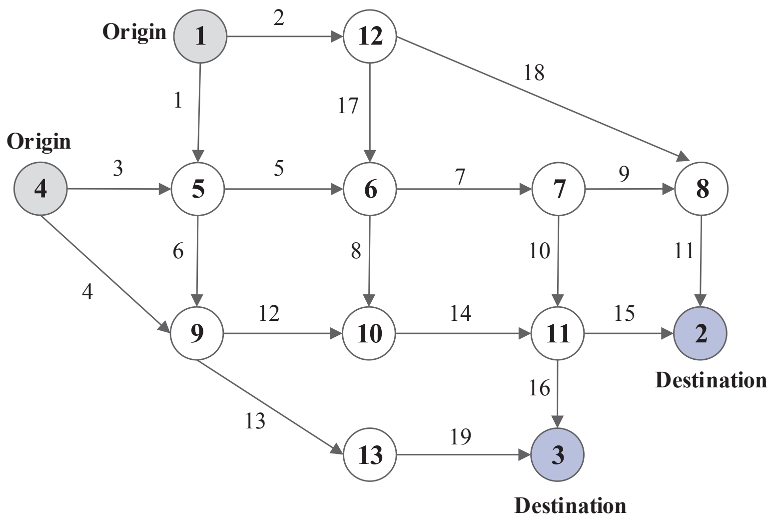

Figure 1. A transportation network with 13 nodes

the optimal solution, all the used paths have the same cost. Specifically, we can calculate the cost on each link, e.g., the cost on link 1 is $7 * (1 + 0.15 * (685.15/300)^4) = 35.5658$. Thus, the cost of the path $1 \to 12 \to 8 \to 2$ is 77.5739. Similarly, the cost of path $1 \to 5 \to 6 \to 7 \to 8 \to 2$ is also 77.5739. In addition, we also implement Frank-Wolfe algorithm to solve this problem, and the results are consistent.

As can be noted, all the used paths having the same origin and destination have the same cost. In other words, nobody can reduce his/her travel cost by switching to other paths, which is the corresponding user equilibrium state.

Table 1. Link characteristics for Nguyen-Dupuis's 13-node network shown in Figure 1

Link	Free-flow travel time (min/trip)	Capacity (veh/h)	Link	Free-flow travel time (min/trip)	Capacity (veh/h)
1	7	300	11	9	500
2	9	200	12	10	550
3	9	200	13	9	200
4	12	200	14	6	400
5	3	350	15	9	300
6	9	400	16	8	300
7	5	500	17	7	200
8	13	250	18	14	300
9	5	250	19	11	200
10	9	300			

Table 2. The optimal solution for Nguyen-Dupuis's 13-node network shown in Figure 1

Link	Optimal flow (veh/h)	Link	Optimal flow (veh/h)
1	685.15	11	753.32
2	483.84	12	545.34
3	474.08	13	365.26
4	460.92	14	545.34
5	709.55	15	296.68
6	449.68	16	639.74
7	719.50	17	9.95
8	0	18	424.89
9	328.42	19	365.26
10	391.08		

5. **Conclusions.** In this paper, we investigate the traffic assignment problem under uncertain environment. Specifically speaking, the travel demand is imprecise in our paper and we represent its uncertainty using the triangular fuzzy numbers. By converting the TFN into a crisp number, we apply *Physarum* algorithm to approach the optimal traffic distribution across the network. We validate its performance through a classical transportation network with 13 nodes. The result demonstrates that our algorithm is capable of finding the optimal solution.

Future researches can be carried out in two different directions. On one hand, we can try to apply the proposed model into the traffic assignment problem with multiple user classes. On the other hand, how to reduce the time spent on solving the linear equations shown in Equation (6) is worthwhile to investigate.

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