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ROBUST SPECTRUM SENSING WITH NON-IDENTICAL NOISE LEVELS AMONG ANTENNAS

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Abstract. *This work focuses on the spectrum sensing for the case of non-identical noise levels among antennas in cognitive radio systems. In this scenario, conventional eigenvalue-based detectors (EBDs), including the maximum-minimum eigenvalue detection (MMED) and the maximum eigenvalue detection (MED), have difficulty in setting the thresholds for a constant false alarm rate (CFAR), and their performances may degrade seriously. This work proposes a robust eigenvalue-based detector (REBD) to deal with the non-identical noise levels. It is shown that the proposed detector is robust to the non-identity of noise levels and moreover, its threshold is tractable for a CFAR. Finally, the numerical simulations validate the proposed detector.*
Keywords: Cognitive radio, Spectrum sensing, Non-identical noise levels, Eigenvalue detection

1. **Introduction.** Reliable spectrum sensing is a crucial issue for cognitive radio (CR) not causing unacceptable interference to primary users (PUs) [1]. Here, the reliability is twofold, i.e., the decision threshold of a spectrum sensing detector for any false alarm probability (P_f) is tractable, and the detector has high detection probability (P_d).

Several spectrum sensing detectors, such as energy detection (ED) [2, 3, 4, 5], matched filter detection [6], cyclostationary detection [7] and eigenvalue-based detection [8, 9, 10], have been proposed. Each spectrum sensing detector has its advantages and disadvantages. For instance, matched filter detection requires the prior information of transmitted waveforms of primary users. The cyclostationary detection requires the knowledge of cyclic frequencies of primary signals. ED has been extensively studied due to its simplicity operation and no requirement of prior knowledge of primary signals. However, it exhibits signal-to-noise radio (SNR) wall phenomenon due to noise uncertainty, which degrades its performance significantly [11, 12]. To overcome the noise uncertainty, eigenvalue-based detectors, such as the maximum-minimum eigenvalue detection (MMED) and the maximum eigenvalue detection (MED), have been proposed. However, these detectors work only under the case of identical noise levels among antennas. In practice, the noise levels among antennas may not be identical after some array calibrations [13]. In this scenario, these eigenvalue-based detectors have difficulty in setting the decision thresholds for a constant false alarm rate (CFAR). Thus, their performances may degrade seriously.

This work proposes a robust eigenvalue-based detection (REBD) which exploits all the eigenvalues and the elements of covariance matrices. The ratio between the product of the diagonal elements and the eigenvalues is considered as part of the test-statistic. Under the noise only case, the statistical covariance matrix of received signal is a diagonal matrix which means the eigenvalues are equal to the diagonal elements. This implies that the non-identity of noise levels has no effect on the threshold for the test-statistic, i.e., the threshold is tractable. With the presence of primary signals, the statistical covariance matrix is no longer a diagonal matrix. Hence, the product of diagonal elements is bigger than that of eigenvalues. This property is used to distinguish whether primary users exist or not.

The rest of the paper is organized as follows. Section 2 states the signal model at a secondary user. In Section 3 proposes a robust eigenvalue-based detection. The numerical simulations in Section 4 validate the proposed detector. Finally, some conclusions are drawn in Section 5.

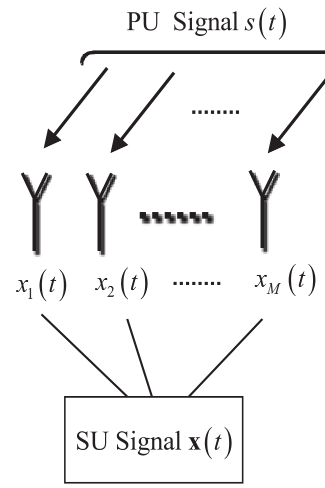

FIGURE 1. Schematic diagram of spectrum sensing in CR

2. **System Model.** Consider a secondary user equipped with M antennas. The schematic diagram of spectrum sensing in CR is given in Figure 1. Let $x_i(k)$ denote the discrete-time signal received from the ith antenna, and the received signal vector can be described as $\mathbf{x}(k) = [x_1(k), \cdots, x_M(k)]^T$, $(k = 1, \cdots, K)$, where the superscript $(\cdot)^T$ represents the transpose operator. With the binary hypotheses, the received signal $\mathbf{x}(k)$ is given by

$$\mathbf{x}(k) = \begin{cases} \mathbf{n}(k), & \mathcal{H}_0 \\ \mathbf{s}(k) + \mathbf{n}(k), & \mathcal{H}_1 \end{cases} \tag{1}$$

where $\mathcal{H}_0$ denotes the absence of PU, and $\mathcal{H}_1$ stands for the presence of PU; $\mathbf{n}(k) = [n_1(k), \cdots, n_M(k)]^T$ denotes an independent and identically distributed (i.i.d.) additive white circularly symmetric complex Gaussian noise vector with mean zero and covariance $\mathbf{R_n} = \text{diag}\left(\sigma_1^2, \cdots, \sigma_M^2\right)$. Here $\text{diag}\left(\sigma_1^2, \cdots, \sigma_M^2\right)$ denotes a diagonal matrix with $\sigma_1^2, \cdots, \sigma_M^2$ being its diagonal elements. $\mathbf{s}(k) = [s_1(k), \cdots, s_M(k)]^T$ with $s_i(k)$ representing the primary signal at the ith antenna. It is assumed that the primary signal vector has mean zero and covariance $\mathbf{R_s} = \text{E}\left[\mathbf{s}(k)\mathbf{s}^H(k)\right]$, where $\text{E}[\cdot]$ denotes expectation operator, and $\mathbf{R_s}$ is not a diagonal matrix because of antennas correlation. Assuming that the PU signal $\mathbf{s}(k)$ and noise $\mathbf{n}(k)$ are independent, we have the statistical covariance matrix of the received signal as

$$\begin{aligned} \mathbf{R}_x|\mathcal{H}_0 &= \text{E}\left[\mathbf{x}(k)\mathbf{x}^H(k)|\mathcal{H}_0\right] \\ &= \text{E}\left[\mathbf{n}(k)\mathbf{n}^H(k)\right] \\ &= \mathbf{R_n} \end{aligned} \tag{2}$$

and

$$\begin{aligned} \mathbf{R}_x|\mathcal{H}_1 &= \text{E}\left[\mathbf{x}(k)\mathbf{x}^H(k)|\mathcal{H}_1\right] \\ &= \text{E}\left[\mathbf{s}(k)\mathbf{s}^H(k)\right] + \text{E}\left[\mathbf{n}(k)\mathbf{n}^H(k)\right] \\ &= \mathbf{R_s} + \mathbf{R_n} \end{aligned} \tag{3}$$

under $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively.

The test-statistic of MMED and MED are given by

$$T_{\text{MMED}} = \frac{\lambda_{\max}}{\lambda_{\min}} \tag{4}$$

and

$$T_{\text{MED}} = \frac{\lambda_{\max}}{\frac{1}{M}\sum_{i=1}^{M} \mathbf{R}_{\mathbf{x}}(i,i)} \tag{5}$$

where $\lambda_{\max}$ and $\lambda_{\min}$ are the maximum and minimum eigenvalues of $\mathbf{R}_x$, respectively. If the noise levels from antennas are equal to each other, it can be easily verified that $T_{\text{MMED}} = T_{\text{MED}} = 1$ under $\mathcal{H}_0$ hypothesis. Hence, the threshold level for any P_f is traceable with finite samples. However, when the noise levels are not equal to each other, both T_{MMED} and T_{MED} are untraceable for having no prior knowledge of the noise levels. Therefore, the threshold become untraceable with finite samples.

3. **Robust Eigenvalue-Based Detection.** In order to overcome the previously mentioned problem, all the eigenvalues and the diagonal elements of $\mathbf{R}_{\mathbf{x}}$ are exploited in the proposed REBD. The test-statistic of the REBD is given by

$$T_{\text{REBD}} = \frac{\sum_{i\neq j;\, i,j=1}^{M} \mathbf{R}_{\mathbf{x}}(i,j)}{\sum_{i=1}^{M} \mathbf{R}_{\mathbf{x}}(i,i)} + \frac{\prod_{i=1}^{M} \mathbf{R}_{\mathbf{x}}(i,i)}{\prod_{i=1}^{M} \lambda_i} \tag{6}$$

where λ_i is the eigenvalue of $\mathbf{R}_{\mathbf{x}}$. Further, the test-statistic can be rewritten as

$$T_{\text{REBD}} = \frac{\sum_{i\neq j;\, i,j=1}^{M} \mathbf{R}_{\mathbf{x}}(i,j)}{\sum_{i=1}^{M} \mathbf{R}_{\mathbf{x}}(i,i)} + \frac{\prod_{i=1}^{M} \mathbf{R}_{\mathbf{x}}(i,i)}{\det(\mathbf{R}_{\mathbf{x}})} \tag{7}$$

where $\det(\mathbf{R}_{\mathbf{x}})$ denotes the determinant of $\mathbf{R}_{\mathbf{x}}$.

Under $\mathcal{H}_0$ hypothesis, even when the noise levels are non-identical, it can be obtained that

$$T_{\text{REBD}} = 1 \tag{8}$$

In contrast, under $\mathcal{H}_1$ hypothesis we have

$$T_{\text{REBD}} > 1 \tag{9}$$

This property can be used to distinguish whether primary users exist or not. The proposed detector can be implemented by

$$T_{\text{REBD}} \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\lessgtr}} \lambda_{\text{REBD}} \tag{10}$$

where λ_{REBD} represents the decision threshold of the proposed REBD. In our detector, it will be verified in the next section that non-identity of noise levels has no effect on the decision threshold. Hence, the decision threshold λ_{REBD} can be obtained offline by Monte Carlo simulations using identical or any non-identical noise levels. Table 1 gives the algorithm of setting the decision threshold.

TABLE 1. Setting decision threshold through Monte Carlo simulations

Monte Carlo Simulations Procedure
Initialization:
M, K, P_f, MC where MC represents the number of Monte Carlo simulations.
Monte Carlo Trials:
for $mc = 1 : MC$
Generate noise samples with identical or any non-identical noise levels;
Compute the sample covariance matrix $\mathbf{R_x}$ using Equation (2);
Obtain the test-statistic T_{REBD} using Equation (7) from the mcth trials;
end
The decision threshold λ_{REBD} is set to be equal to the $(P_f * MC)$th value from MC T_{REBD}s of descending order.

4. **Simulation Results.** In this section, the numerical results are presented to validate the proposed spectrum sensing detector. Suppose that $M = 4$ antennas are deployed in a secondary user. Let the sample number be $K = 100$. In the first experiment, we investigate the threshold for given $P_f = 0.1$. The noise levels are assumed to be uniform distributions in the range $[-0.7, 0.7]$. For every group of the noise levels, the threshold is obtained via Monte Carlo simulations with 10^5 independent trials.

With 100 groups of randomly generated noise levels, the thresholds for the REBD, MMED and MED are shown in Figure 2. It demonstrates that the threshold for the REBD maintains almost unchanged for keeping $P_f = 0.1$. However, the thresholds for MMED and MED vary violently. The threshold of MMED varies randomly in the range from 1.9 to 2.1, and it is difficult to obtain a threshold for achieving a given P_f. It implies that the threshold is tractable for REBD, but untractable for MMED and MED under the scenario of non-identical noise levels. In addition, the blue curve of REBD in Figure 2 is very close to 1, which validated the theoretical analysis in previous section.

Two groups of noise levels are used to obtain the receiver operating characteristic (ROC) curves. Figure 3 shows the ROC curves for REBD, MMED and MED with SNR being -5 dB. In Figure 3, the terms G1 and G2 denote the first and second group of noise levels. It is observed that two red ROC curves of REBD are well matched, but the black and blue curves are not. It shows that REBD is more robust against non-identity of noise levels than MMED and MED. This is because the threshold of the proposed detector is

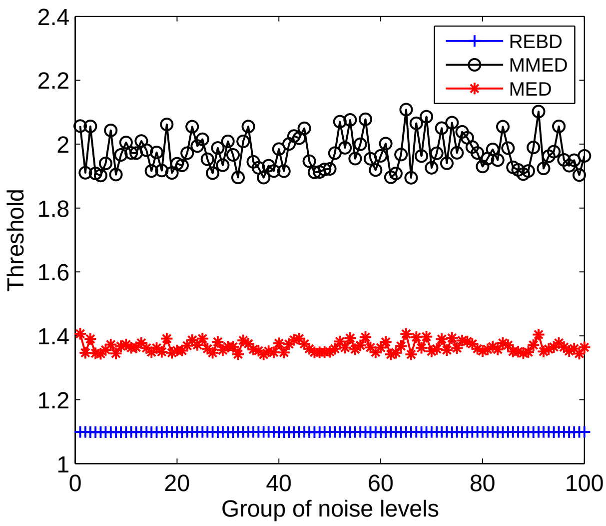

FIGURE 2. Thresholds for $P_f = 0.1$ with 100 groups of randomly generated noise levels

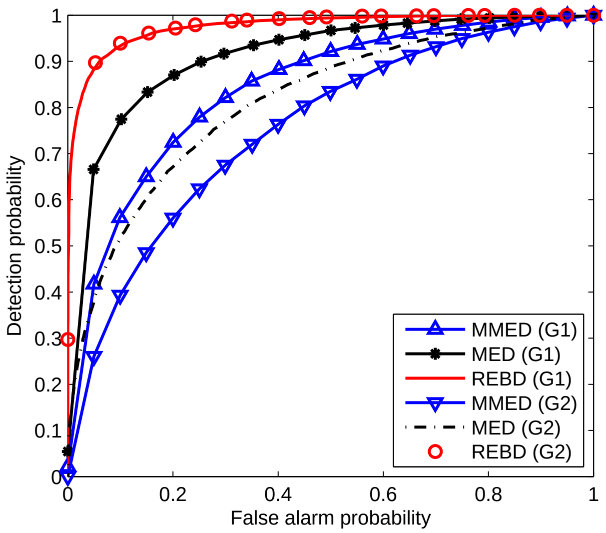

FIGURE 3. ROC curves of two groups of randomly generated noise levels. The terms G1 and G2 denote the first and second group of noise levels.

independent of the distribution of noise levels, while MMED and MED depend on the distribution of noise levels. Furthermore, it can also be seen that REBD outperforms MMED and MED. For $P_f = 0.1$, REBD has a detection probability over 0.95, while the detection probability of MED is below 0.8, and that of MMED is smaller than 0.7.

5. **Conclusions.** In this letter, we studied the case of non-identical noise levels among antennas CR systems, which lead to the thresholds of MMED and MED untraceable for a CFAR. By exploiting all the eigenvalues and the diagonal elements of covariance matrices, we proposed a robust eigenvalue-based detection. Numerical results have demonstrated that the proposed REBD is robust to the non-identical noise levels among antennas, which outperforms the MMED and MED. Deriving the theoretical expression for the decision threshold of the proposed detector will be the focus of our future work.

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REFERENCES

[1] S. Haykin, Cognitive radio: Brain-empowered wireless communications, *IEEE Journal on Selected Areas in Communications*, vol.23, no.2, pp.201-220, 2005.

[2] K. Srisomboon, A. Prayote and W. Lee, Two-stage spectrum sensing for cognitive radio under noise uncertainty, *The 8th IEEE International Conference on Mobile Computing and Ubiquitous Networking*, Hokkaido, Japan, pp.19-24, 2015.

[3] A. Mariant, A. Giorgetti and M. Chiani, Effects of noise power estimation on energy detection for cognitive radio applications, *IEEE Trans. Communications*, vol.59, no.12, pp.3410-3420, 2011.

[4] S. Debenath, Improved p-norm energy detector in awgn channel for spectrum sensing in cognitive radio, *The 8th International Conference on Wireless Internet*, Lisbon, Portugal, pp.13-14, 2014.

[5] W. Jouini, Energy detection limits under log-normal approximated noise uncertainty, *IEEE Signal Processing Letters*, vol.18, no.7, pp.423-426, 2011.

[6] S. Upadhyay and S. Deshmukh, Blind parameter estimation based matched filter detection for cognitive radio networks, *IEEE International Conference on Communications and Signal Processing*, Chengdu, China, 2015.

[7] Y. Liu, Z. Zhong, G. Wang et al., Cyclostationary detection based spectrum sensing for cognitive radio networks, *Journal of Communications*, vol.10, no.1, pp.74-79, 2015.

[8] Y. Zeng and Y. C. Liang, Eigenvalue-based spectrum sensing algorithms for cognitive radio, *IEEE Trans. Communications*, vol.57, no.6, pp.1784-1793, 2009.

[9] Y. Zeng, C. L. Koh and Y. C. Liang, Maximum eigenvalue detection: Theory and application, *IEEE International Conference on Communications*, Beijing, China, pp.4160-4164, 2008.

[10] S. M. Putri and I. Sugihartono, Energy efficiency in cognitive radio with cooperative MME (Maximum to Minimum Eigenvalue) spectrum sensing method, *IEEE International Seminar on Intelligent Technology and Its Applications*, ITS Surabaya, Indonesia, pp.379-384, 2015.

[11] S. S. Kalamkar, A. Banerjee and A. K. Gupta, SNR wall for generalized energy detection under noise uncertainty in cognitive radio, *The 19th IEEE Asia-Pacific Conference on Communications*, Bali, Indonesia, pp.375-380, 2013.

[12] R. Tandra and A. Sahai, SNR walls for signal detection, *IEEE Journal of Selected Topics in Signal Processing*, vol.2, no.1, pp.4-17, 2008.

[13] M. Jin, Y. Li and H. G. Ryu, On the performance of covariance based spectrum sensing for cognitive radio, *IEEE Trans. Signal Processing*, vol.60, no.7, pp.3670-3682, 2012.