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NOVEL MATHEMATICAL MODELS FOR TWO-STAGE ASSEMBLY FLOW SHOP SCHEDULING PROBLEM WITH DETERIORATION AND PREVENTIVE MAINTENANCE ACTIVITIES

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Abstract. *In this study, we derive novel mathematical models for the two-stage assembly flow shop scheduling problem (TSAFSP) with deterioration and preventive maintenance activities (PMAs). The TSAFSP having a single machining machine (MM) produces various types of components to assemble products in the first stage. Each component has different deterioration rate during the machining process and an appropriate schedule of PMAs restores the deteriorations of component-manufacturing times in the MM. When the required components are available for the associated product, a single assembly machine can start to assemble in the second stage. The main decisions in this study are to simultaneously determine the optimal number and position of PMAs component-manufacturing sequence and the optimal product-assembly sequence to minimize the makespan. To solve the problem, we develop mixed integer linear programming (MILP) models for deterioration without PMAs (DTR) and extend them to the case for deterioration with PMAs (DTRPM). The performance of DTRPM is compared with DTR.*

Keywords: Two-stage assembly flow shop scheduling problem, Preventive maintenance activity, Deterioration, Mixed integer linear programming

1. **Introduction.** In general, production scheduling is one of the most important issues in the planning and scheduling of the manufacturing system. In this regard, many previous studies have been conducted to minimize the makespan in various manufacturing systems. Among them, the two-stage assembly flow shop problem (TSAFSP) is applied to many real-life industrial applications such as fire engine assembly system [1] and personal computer manufacturing system [2]. However, the TASFSP is computationally very complex and difficult to find the optimal solution even if it is applied to many real-life industrial applications. Yan et al. [3] studied a TASFSP in which m identical parallel machines are processed in the first stage, each of which can produce only one component for a product. A single assembly machine in the second stage assembles the m components together to complete the product. Three-types of objective functions are considered which minimize the weighed sum of maximum makespan, earliness, and lateness. To find the near-optimal solution for TASFSP within a reasonable time, they proposed a hybrid variable neighborhood search-electromagnetism-like mechanism (VNS-EM) algorithm. Liao et al. [4] considered a TSAFSP with batch setup times to minimize the makespan. In this study, they considered a TSAFSP with a machining machine producing all kinds of components in the first stage and an assembly machine in the second stage. They developed a mixed integer linear programming (MILP) model, and found several optimality properties to enhance to find the optimal solution. Furthermore, an efficient heuristic based on these optimal properties is proposed. Meanwhile, the studies related to the deterioration and PMAs of the machine scheduling were also conducted to minimize the

makespan in various manufacturing systems. Deterioration means that tools wear is processed or production efficiency is decreased due to the twist of tools fixing product over time. The deterioration effect decreases the production efficiency of machines due to various reasons such as a mal-position of tools, mal-alignment of jobs, abrasion of tools, and scraps of operations. The deteriorated processing times of jobs are recovered to an original processing time by the preventive maintenance activities (PMAs) for tool changing process or cleaning process of machines. In recent years, several studies have conducted meta-heuristic algorithms for a machine scheduling with complex deterioration schemes and PMAs. Balhalke et al. [6] proposed several hybrid meta-heuristic algorithms for the single-machine scheduling problem with sequence dependent setup time and deteriorating jobs to minimize the makespan. Joo and Kim [5] proposed genetic algorithms for single machine scheduling problems with time-dependent linear deterioration and RMAs. In that study, they assumed that the number of RMAs was predetermined and that both the position of RMAs and the jobs are scheduled based on the RMAs.

Based on the above-mentioned studies, no studies considered the TSAFSP to assemble products with dynamic component-sizes, setup time, deterioration, and PMA simultaneously. The application of dynamic component-sizes, setup time, deterioration and PMA for scheduling problems enables to apply real-life industries in detail by considering the various kinds and number of components, setup time of machine, and wear of tools and activity that restore the wear of the tool. Due to this reason, in this paper, we developed novel MILP models for TSAFSP with dynamic component-sizes, setup time, deterioration and PMAs. The paper is organized as follows. Section 2 describes the mathematical models and problem statement of this problem. In Section 3, computational experiments are conducted for identifying the relationship between variables by randomly generated test problems. Finally, the conclusions and future studies are discussed in Section 4.

2. **Problem Statement and Mathematical Models.**

In this study, we focused on two-stage flow shop assembly system with deterioration and PMAs. This system contains two successive stages. In the first stage, a single machining machine produces various types of components to assemble products. During machining process, a setup time is required whenever machining machine starts to process a new component or process a different component. When the required components are available for the associated product, a single assembly machine can assemble these components into the product in the second stage. The components have different deterioration rates, respectively, the deterioration has a major effect on component-manufacturing time, and an appropriate schedule of PMAs restores the deteriorations of component-manufacturing times in the machining machine. To verify the effect of PMA, we proposed two novel MILP models: the MILP model with only deterioration (DTR) and the MILP model with deterioration and PMAs (DTRPM).

The notations and constraints for DTR are defined as follows.

Parameter

P	the number of products
C	the number of components
S	the length of component-manufacturing sequence (CMS)
i	index for product type i, $i = 1, 2, \ldots, P$
j	index for component type j, $j = 1, 2, \ldots, C$
k	index for product assembly-sequence (PAS) k, $k = 1, 2, \ldots, P$
l	index for CMS l, $l = 1, 2, \ldots, S$
ST	setup time in the machining machine

CT_j	manufacturing time for component type j
PT_i	assembly time for product type i
RCP_{ij}	the required number of component type j for product type i
DTR_j	the deterioration rate for component type j
B	big number

Binary variables

$$W_{jl} = \begin{cases} 1, & \text{if component type } j \text{ is scheduled at CMS } l \\ 0, & \text{otherwise} \end{cases}$$

$$X_{ik} = \begin{cases} 1, & \text{if product type } i \text{ is scheduled at PAS } k \\ 0, & \text{otherwise} \end{cases}$$

$$Y_l = \begin{cases} 1, & \text{if CMS } l \text{ and } l-1 \text{ are the same} \\ 0, & \text{otherwise} \end{cases}$$

$$Z_{kl} = \begin{cases} 1, & \text{if there is a lack of components for assembling any product} \\ & \text{assigned to PAS } k \text{ at CMS } l \\ 0, & \text{otherwise} \end{cases}$$

Continuous variables

R_{jkl}	lack of components with type j for assembling any product assigned to PAS k at CMS l
CN_{jl}	the cumulated number of manufactured components with type j at CMS l
PN_{jk}	the cumulated number of used components with type j for PAS k
CCT_l	completion time of component at CMS l
PCT_k	completion time of any product assigned to PAS k
AVT_{kl}	available time for assembling any product assigned to PAS k at CMS l
PMT_l	cumulative time of setup at CMS $l = 1, 2, \ldots, s$
$C_{\max}$	makespan for the two-stage assembly flow shop scheduling

$$Min \quad C_{\max} \tag{1}$$

$$\text{s.t.} \quad \sum_{j=1}^{C} W_{jl} = 1, \quad l = 1, 2, \ldots, S \tag{2}$$

$$\sum_{l=1}^{S} W_{jl} = \sum_{i=1}^{P} RCP_{ij}, \quad j = 1, 2, \ldots, C \tag{3}$$

$$\sum_{i=1}^{P} X_{ik} = 1, \quad k = 1, 2, \ldots, P \tag{4}$$

$$\sum_{k=1}^{P} X_{ik} = 1, \quad i = 1, 2, \ldots, P \tag{5}$$

$$Y_l = \max\nolimits_{j=1}^{C} (W_{jl-1} + W_{jl}, 1) - 1, \quad j = 2, 3, \ldots C \tag{6}$$

$$CCT_l \geq ST + \sum_{j=1}^{C} CT_j \cdot W_{jl}, \quad l = 1 \tag{7}$$

$$\begin{array}{l} CCT_l \geq CCT_{l-1} + ST \cdot (1 - Y_l) \\ \qquad +CT_j \cdot W_{jl} - B \cdot (-W_{jl}) \\ \qquad +DTR_j \cdot (CCT_{l-1} - PMT_{l-1}) \end{array}, \quad j = 1, 2, \ldots, C;\ l = 2, 3, \ldots, S \tag{8}$$

$$ST \geq PMT_l, \quad l = 1 \tag{9}$$

$$PMT_l \leq PMT_{l-1} + ST \cdot (1 - Y_l), \quad l = 2, 3, \ldots, S \tag{10}$$

$$CN_{jl} = \sum_{a=1}^{S} X_{ja}, \quad j = 1, 2, \ldots, C; \quad l = 1, 2, \ldots, S \tag{11}$$

$$PN_{jk} = \sum_{i=1}^{P} \left(CT_j \cdot \sum_{a=1}^{P} Y_{ia} \right), \quad i = 1, 2, \ldots, P; \quad j = 1, 2, \ldots, C \tag{12}$$

$$R_{jkl} = \max \left(PN_{jk} - CN_{jl}, 0 \right), \quad \begin{aligned} & j = 1, 2, \ldots, C; \quad k = 1, 2, \ldots, P; \\ & l = 1, 2, \ldots, S \end{aligned} \tag{13}$$

$$AVT_{kl} = CCT_l + B \cdot Z_{kl}, \quad j = 1, 2, \ldots, C; \quad l = 1, 2, \ldots, S \tag{14}$$

$$B \cdot Z_{kl} \geq \sum_{j=1}^{C} R_{jkl}, \quad k = 1, 2, \ldots, P; \quad l = 1, 2, \ldots, S \tag{15}$$

$$PCT_k \geq \min\nolimits_{l=1}^{S} (AVT_{kl}) + \sum_{k=1}^{P} PT_i \cdot X_{ik}, \quad k = 1 \tag{16}$$

$$PCT_k \geq \max \left(PCT_{k-1}, \min\nolimits_{l=1}^{S} (AVT_{kl}) \right), \quad k = 2, 3, \ldots, P \tag{17}$$

$$C_{\max} \geq PCT_k, \quad k = 1, 2, \ldots, P \tag{18}$$

The objective function in Equation (1) is to minimize the makespan of the TSAFSP with deterioration and PMAs of machining machine. The constraint (2) ensures that machining machine must produce only one component at each sequence. The constraint (3) ensures that the number of each component produced by machining machine must be equal to the number of necessary component of each product. The constraints (4) and (5) ensure that assembly machine must produce only one product at each sequence. Equation (6) defines the relationship between the setup times and the components. The constraints (7) and (8) define the completion time of each component. The constraint (8) applies a deterioration rate for each component. The constraints (9) and (10) define the net process time with deterioration rate except setup time. The constraints (11), (12) and (13) determine the number of used component at each sequence. The constraint (14) determines the available assembly time for a product at each CMS. The constraint (15) ensures that Z_{jl} is equal to 1, if sum of R_{jkl} is non-negative value. The constraints (16) and (17) define the completion time of each product. Finally, the constraint (18) calculates the makespan of the TSAFSP. The additional notations and constraints for DTRPM are defined as follows.

Parameter

PVT the time of preventive maintenance activity

Binary variables

$$PM_l = \begin{cases} 1, & \text{if there is preventive maintenance to } l \\ 0, & \text{otherwise} \end{cases}$$

The equations in constraints (7), (8), (9) and (10) change as below:

$$CCT_l \geq PVT \cdot PM_l + ST + \sum_{j=1}^{C} CT_j \cdot W_{jl}, \quad l = 1 \tag{7.1}$$

$$\begin{aligned} CCT_l \geq\ & CCT_{l-1} + PVT \cdot PM_l \cdot (1 - Y_l) + DTR_j \cdot (CCT_{l-1} - PMT_{l-1}) \\ & + ST + CT_j \cdot W_{jl} - B \cdot (1 - W_{jl}), \\ & j = 1, 2, \ldots, C; \quad l = 2, 3, \ldots, S \end{aligned} \tag{8.1}$$

$$PMT_l \leq \min \left(B \cdot PM_l, CCT_l \right) + ST, \quad l = 1 \tag{9.1}$$

$$PMT_l \leq PMT_{l-1} + ST \cdot Y_l + \min \left(B \cdot PM_l, CCT_l - PMT_{l-1} \right), \quad l = 2, 3, \ldots, S \tag{10.1}$$

3. **Computational Experiment.** In order to identify the relationship between variables, computational experiments are conducted using randomly generated test problem. The proposed mixed integer linear programming (MILP) was coded in ILOG CPLEX 12.5. All experiments were executed on a PC with Intel core i7-4770 CPU, 4GB RAM and Windows 7 operating system and CPU time was limited in 7200 (sec.). We executed computational experiments which were classified into the experiments using DTR model and the experiments using DTRPM model. For comparing DTR with DTRPM, all parameters of experiments such as number of products, number of components, product type, component type, manufacturing time of components, and assembly time of products were the same for each CMS (e.g., $M = 6, 8, 10$). The setup-time rate (SR) and deterioration rate (DR) were set in two cases, low case and high case, respectively. The test results of MILP models are summarized in Table 1.

Table 1. The test result of DTR model and DTRPM model

			DTR			DTRPM			
M	SR	DR	Opt.	Time	No. of Setups	Opt.	Time	No. of Setups	No. of PMAs
6	High	High	248.58	4.66	3	215.70	3.71	4	3
6	High	Low	219.91	3.17	4	218.14	4.05	4	1
6	Low	High	215.30	2.39	4	193.35	4.37	5	2
6	Low	Low	192.27	1.71	5	192.25	3.91	4	0
8	High	High	249.42	36.82	4	246.48	87.93	5	2
8	High	Low	246.03	28.40	7	246.03	66.29	5	1
8	Low	High	232.50	23.57	6	232.48	338.97	7	3
8	Low	Low	232.05	16.27	10	232.03	182.53	7	0
10	High	High	248.67	244.80	4	N/A	7200.16		–
10	High	Low	236.16	565.90	6	236.16	896.10	6	1
10	Low	High	224.79	176.60	5	N/A	7200.15		–
10	Low	Low	224.16	273.84	7	N/A	7200.83		–

In Table 1, there are solutions of MILP models. We can summarize the main results into three. Firstly, the objective value of DTRPM model is less than or equal to the objective value of DTR model in all instances. This result indicates that an appropriate schedule of PMAs is critical to minimize the makespan by restoring the deteriorations of component-manufacturing times in the machining machine. Secondly, the numbers of PMAs for high deterioration rate instances are resulted in greater than or equal to 2. The numbers of PMAs for other instances which have low deterioration rate are resulted in less than 2. This result indicates that PMA is particularly an important activity in manufacturing environments with high deterioration rate. Finally, it can be seen that the number of setups for instances with high setup rate is less than the number of setups for instances which have low setup rate. This result indicates that decreasing the number of setups becomes important to minimize the makespan, as the portion of the setup time in the total manufacturing time is increased.

4. **Conclusions.** In this paper, a TSAFSP with deterioration and PMAs of machining machine (MM) to minimize the makespan was investigated. In the first stage, a single MM produces various types of components to assemble products. A setup time is required whenever MM starts to process a new component or process different components. When the required components are available for the associated product, a single assembly machine can assemble these components into the product in the second stage. The components have different deterioration rates, respectively, the deterioration has a major effect

on component manufacturing time, and an appropriate schedule of PMAs restores the deteriorations of component-manufacturing times in the MM. We formulated the TSAFSP into MILP models and investigated for the relationship between the variables in the case of DTRPM model and DTR model. The test results showed that an appropriate schedule of PMAs and the number of setup times had major effects on minimizing the makespan in TSAFSP.

In the future studies, we must propose the effective meta-heuristics to find a near-optimal solution for large-sized problems. Also, we can extend our current study into other manufacturing systems with parallel MM in two-stage or multi-stage scheme.

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