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AN IMPROVED ANT COLONY ALGORITHM USED FOR UNSCENTED KALMAN FILTER

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Abstract. *Traditional unscented Kalman filter has some disadvantages such as slow convergence speed, big time complexity and high filter error, which leads to unsatisfied real-time requirement. What's more, abnormal disturbances error can greatly affect the accuracy and stability of unscented Kalman filter. In order to perfect those problems and improve unscented Kalman filter, we put forward an improved ant colony algorithm to improve the accuracy of unscented Kalman filter. We use ant colony algorithm with grid division method to find the optimal combination of system error and measurement error in unscented Kalman filter. It realizes the optimization for unscented Kalman filter. Finally, experiment results show that the accuracy of unscented Kalman filter based on improved ant colony algorithm is better than traditional unscented Kalman filter algorithm.*

Keywords: Unscented Kalman filter, Ant colony algorithm, Abnormal disturbances, Grid division method

1. **Introduction.** Extended Kalman filter (EKF) [1-3] is very useful for solving nonlinear system optimal state estimation problems as a common estimation method based on linear approximation for Kalman filter theory. The main purpose of filtering is to be able to forecast and estimate the state and the error statistics of nonlinear system in real time. EKF is widely used in nonlinear filtering system, which is a typical representative for function approximation nonlinear filtering. However, when the higher-order terms of Taylor expansion of nonlinear function cannot be ignored, the linearized system will produce large errors and even filter is not stable, the result is easy to diverge. So it introduces unscented Kalman filter (UKF) [4,5]. UKF is based on unscented transformation (UT) and uses linear Kalman filter framework. However, traditional UKF cannot control process gain matrix in time according to filtering effect system covariance. Thus the estimated value of the filter cannot fast-track system status. To solve the problems of traditional UKF, Liu and Yin [6] proposed an improved unscented Kalman filter using a minimum skewness monomorphic sampling strategy to reduce the amount of calculation of unscented Kalman filter and improve the accuracy of unscented Kalman filter. Also Liu et al. [7] proposed an improved square root unscented Kalman filter (SRUKF) according to square root unscented Kalman filter and backward smoothing algorithm. This new scheme adopted equilateral triangle decomposition. Meanwhile, this new algorithm expanded dimension for state vector and propagated process noise and observation noise by nonlinear system. Kong et al. [8] presented that a modified square-root unscented Kalman filter (SR-UKF) algorithm was employed in BDS and GPS conditions. However, authors perfect UKF only in one aspect. Meng et al. [9] showed a novel approach using the moving window method with AUKF and LSSVM to accurately establish the battery model with limited initial training samples. Zhou et al. [10] described a new adaptive filtering approach for nonlinear systems with additive noise. Based on the square-root

unscented KF (SRUKF), traditional Maybeck's estimator was modified and extended to nonlinear systems. The square root of the process noise covariance matrix Q or that of the measurement noise covariance matrix R was estimated straightforwardly.

In this paper, we propose a new UKF based on ant colony algorithm. This new method contains two aspects: using grid division method to improve ant colony algorithm; using the new ant colony to optimize the prediction and measurement error, which can reduce the total error when adopting UKF filter. Finally, we conduct experiments to prove the new scheme's feasibility. The following is the structure of this paper. In Section 2, we briefly introduce UKF algorithm. We detailedly show the new UKF based on improved ant colony algorithm in Section 3. In Section 4, we make experiments to verify the new scheme's advantage. In Section 5, we give conclusions for this paper.

2. Overview of UKF Algorithm.

UKF compensates the deficiency of the EKF algorithm based on Kalman filter and UT [11,12]. It uses deterministic sampling strategy to approximate nonlinear distribution. UKF algorithm can dispose the nonlinear non-Gaussian system filtering problems with higher accuracy and faster calculation speed. The following is the process of UKF algorithm.

Step 1. Initialization.

$$\hat{x}_0 = E\left[x_0\right] \tag{1}$$

$$P_0 = E\left[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\right] \tag{2}$$

where x_0 is initial state matrix, and P_0 is initial covariance matrix. E is expectation function.

Step 2. Calculating UT transformation and Sigma-Points.

Step 3. Time updating. It utilizes nonlinear state equation $f(*)$ and the Sigma-Points obtained by Step 2 to convert Sigma-Points into $x_{k+1|k}$. The state prediction value $\hat{x}_{k+1}$ and error covariance matrix $\hat{P}_{x,k+1}$ can be calculated by $x_{k+1|k}$. Q_{k+1} is measurement noise matrix. W_i is state matrix.

$$X^i_{k+1|k} = f(x^i_k) \tag{3}$$

$$\hat{x}_{k+1} = \sum_{i=0}^{2n} W_i^m X^i_{k+1|k} \tag{4}$$

$$\hat{P}_{x,k+1} = \sum_{i=0}^{2n} W_i^c \left(X^i_{k+1|k} - \hat{x}_{k+1}\right)\left(X^i_{k+1|k} - \hat{x}_{k+1}\right)^T + Q_{k+1} \tag{5}$$

Step 4. Measurement updating. It utilizes nonlinear state equation $h(*)$ and the Sigma-Points obtained by Step 2 to convert Sigma-Points into $\xi_{k+1|k}$. The measurement prediction value $\hat{z}_{k+1}$ and error covariance matrix $\hat{P}_{z,k+1}$, $\hat{P}_{x,z}$ can be calculated by $\xi_{k+1|k}$.

$$\xi^i_{k+1|k} = h(\xi^i_k) \tag{6}$$

$$\hat{z}_{k+1} = \sum_{i=0}^{2n} W_i^m \xi^i_{k+1|k} \tag{7}$$

$$\hat{P}_{z,k+1} = \sum_{i=0}^{2n} W_i^c (\xi^i_{k+1|k} - \hat{z}_{k+1})(\xi^i_{k+1|k} - \hat{z}_{k+1})^T + R_{k+1} \tag{8}$$

$$\hat{P}_{x,z} = \sum_{i=0}^{2n} W_i^c (X^i_{k+1|k} - \hat{x}_{k+1})(\xi^i_{k+1|k} - \hat{z}_{k+1})^T \tag{9}$$

$$K_{k+1} = \hat{P}_{x,z}/\hat{P}_{z,k+1} \tag{10}$$

$$x_{k+1} = \hat{x}_{k+1} + K_{k+1}(z_{k+1} - \hat{z}_{k+1}) \tag{11}$$

$$P_{k+1} = P_{k+1|k} - K_{k+1}\hat{P}_{z,k+1}K_{k+1}^T \tag{12}$$

where W_i^m and W_i^c are mean value and variance weight coefficient respectively. $\hat{z}_{k+1}$ is observed estimation value. R_{k+1} is measurement noise matrix. K_{k+1} is Kalman gain matrix.

3. **The New UKF Based on Improved Ant Colony Algorithm.** To reduce the main factors affecting the accuracy of UKF and improve the efficiency of traditional UKF algorithm, we introduce the improved ant colony algorithm into UKF. In new UKF algorithm, we reduce the error of system estimation and measurement by ant colony with grid division method. New optimized results will be applied into UKF, which greatly reduces the filter error.

3.1. **Improved ant colony algorithm based on grid partitioning strategy.** Ant colony algorithm [13-15] is an intelligence algorithm by simulating ant colony foraging behavior, which uses the bionic principle to simulate ant looking for food, and then finds optimal path. Ant colony algorithm also can be used to solve the continuous space optimization. Using an improved ant colony algorithm based on grid partitioning strategy finds optimal solutions of system error and measurement error in continuous space, which makes Kalman prediction effect approximate optimal.

Firstly, it needs to determine the range of variable (i.e., size of the continuous domain), and it makes grid partitioning for this space. It should determine the lower x_{ilower} and upper bounds x_{iupper} of each component x_i in the solution $x = (x_1, x_2, \ldots, x_n)^T$ and divide x_i into N same blocks written as:

$$h_i = (x_{iupper} - x_{ilower})/N \tag{13}$$

In n-dimension space, grid is composed of $nN+1$ points. In this model, each ant selects a point from the first row to the n-th row respectively in divided grim to form a solution. So m ants can constitute m solutions in one iteration. Ant chooses the point according to pheromone, and selection probability is:

$$p_{ij} = \frac{\tau_{ij}(t)}{\tau_{i0}(t) + \tau_{i1}(t) + \ldots + \tau_{iN}(t)} \tag{14}$$

where $\tau_{ij}(t)$ is size of pheromone of coordinate (i, j). t is running time of algorithm. The process of ant constructing an intact solution can be called a moment. If ant selects a point at the i-th row and j-column, it denotes that the j-th value in x_i is selected, and

$$x_{ij} = x_{ilower} + jh_i \tag{15}$$

Set the total sum of pheromone in every path as 1. Initializing initial value of pheromone, it will be split averagely on each point.

$$\tau_{ij}(0) = 1/(N+1) \tag{16}$$

Meanwhile,

$$p_{ij} = \tau_{ij}(t) \tag{17}$$

Namely, the probability of ant selecting each point is equal to the size of pheromone in this point, which provides convenience for the algorithm.

When updating pheromone, we put the m solution into objective function to solve the function value, and then we can determine a global optimal solution S^{gb}. Finally, we update pheromone of every point in each row,

$$\begin{aligned} \tau_{ij}(t+1) &= (1-\rho)\tau_{ij}(t) + \Delta\tau_{ij}(t+m) \\ \Delta\tau_{ij}(t+1) &= \rho \end{aligned} \tag{18}$$

where ρ is pheromone volatilization coefficient. $(1-\rho)\tau_{ij}(t)$ is pheromone volatilization process. $\Delta\tau_{ij}(t+m)$ is pheromone strengthen process only acting on the points in optimal solution S^{gb} path. Except S^{gb} path, other pheromones only carry out volatilization process.

When the iteration number of algorithm reaches at the maximum iteration number, it needs to find the corresponding column number $(c_1, c_2, \ldots, c_n)$ of the maximum points in each row of the matrix formed by τ_{ij}. Then it shrinks the range of variable. So after updating, the range of component x_i is,

$$x_{ilower} = \begin{cases} x_{ilower} + (c_i - \Delta)h_i, & x_{ilower} + (c_i - \Delta)h_i \geq x_{i0}; \\ x_{i0}, & x_{ilower} + (c_i - \Delta)h_i < x_{i0} \end{cases} \tag{19}$$

$$x_{iupper} = \begin{cases} x_{ilower} + (c_i + \Delta)h_i, & x_{ilower} + (c_i + \Delta)h_i \leq x_{iN}; \\ x_{iN}, & x_{ilower} + (c_i - \Delta)h_i > x_{iN} \end{cases} \tag{20}$$

where Δ is the shrinking degree of variable range.

After variable updating, it makes grid partitioning for continuous domain again according to (13) and initializes pheromone according to (16). In the shrunken space, it searches optimal solution, and repeats this process until satisfying accuracy requirement.

3.2. **Process of UKF based on improved ant colony algorithm.**

Step 1. Solving upper and lower values of system estimation error and measurement error. When we compute maximum values of system estimation error, we get the maximum and minimum value in each iteration. Then we calculate the extremum variance. Similarly, when estimating the maximum measurement error, assuming that there is no system error, measured data are in reasonable frequency range. We select the two biggest values as loop sequence respectively from the forward bias and reverse bias, and find the extremum variance. Set minimum value of the two error parameters as 0, namely no system error and measurement error.

Step 2. Initializing parameters of UKF and ant colony algorithm. Set $\hat{x}_0 = P_0 = 1$, $Q_1 = R_1 = 0$. Parameters in ant colony algorithm are as in Reference [16]. And the objective function is the error square sum of UKF system prediction and measurement.

Step 3. Running ant colony algorithm. It searches the optimal solution of system estimation error and measurement error in the known upper and lower value space. Then it gets mean value.

Step 4. We apply the optimal value into state prediction value and error covariance matrix of UKF. Finally, it improves the filter accuracy.

4. **Simulation Experiments.**

We make experiments under MATLAB platform testing object movement in indoor. Using ant colony algorithm (ACO) gets the optimal speed error value as Table 1 within 450s.

Table 1. Optimal speed error value based on ACO

Parameter	0-150s	151s-300s	301s-400s
System estimation error	0.08m/s	0.06m/s	0.07m/s
Measurement error	0.11m/s	0.10m/s	0.12m/s

We solve the mean value of the two errors respectively: system estimation error 0.07m/s, and measurement error 0.11m/s. Then we substitute them into UKF. In order to further verify the advantage of our new UKF, we make a comparison with traditional UKF, minimum skewness sampling UKF (MSUKF) [5] and the improved UKF based on ACO (ACO-UKF).

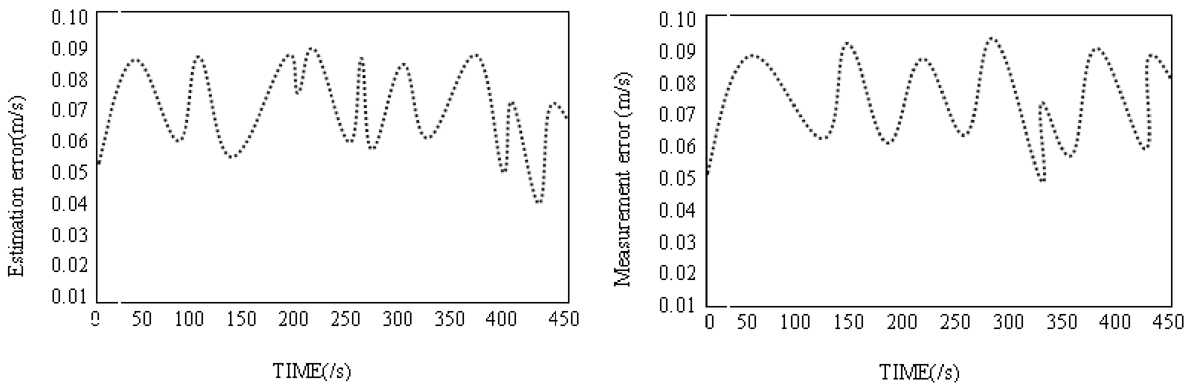

Figure 1. UKF speed error

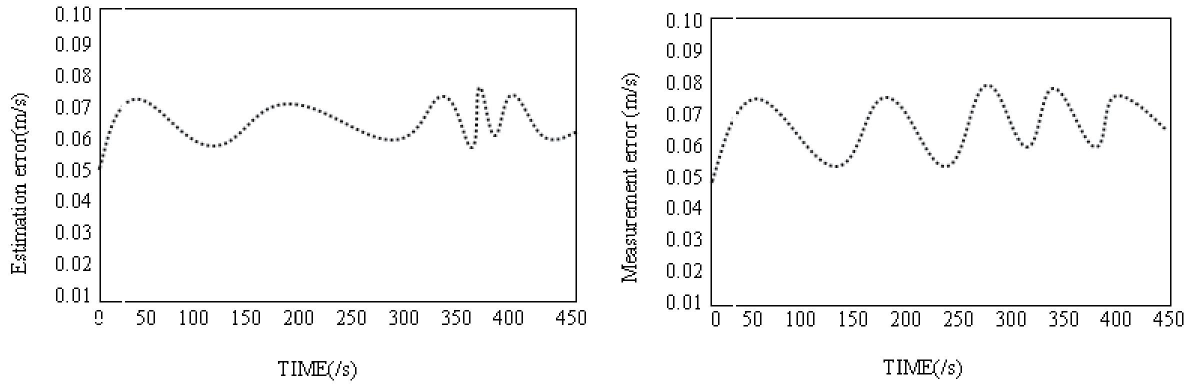

Figure 2. MSUKF speed error

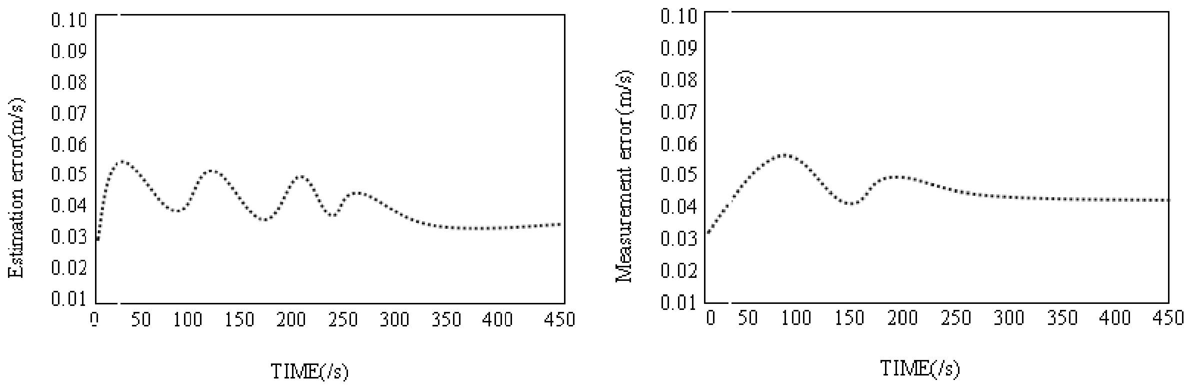

Figure 3. ACO-UKF speed error

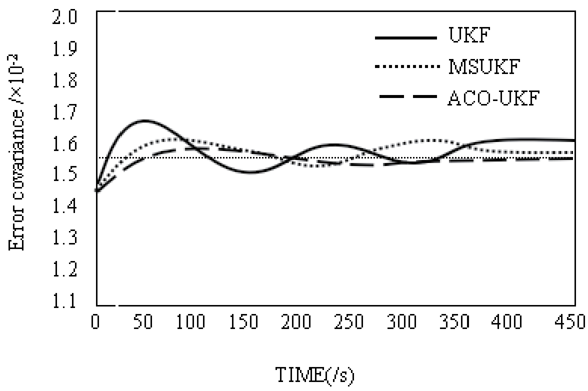

Figure 4. Error covariance with different methods

The comparison results are as Figures 1-3. From Figure 1, we can know that the estimation error of UKF is big and filtering measurement error is not reliable by the effect of dynamic model abnormal disturbance error. Figure 2 represents that when using sampling strategy to improve UKF, the accuracy of Figure 2 is better than Figure 1 and it can better approximate the mapping function of nonlinear dynamic model. The error of both is smaller than that of UKF. Through comparing Figure 2 and Figure 3, we can conduct a conclusion that the error and filter accuracy based on ant colony algorithm is prior to the improved sampling strategy and adaptive factor. In Figure 3, estimation error of ACO-UKF reaches the biggest value nearly 0.055 between 0s and 50s. After 50s, though the error changes range from 0.05 to 0.03, it will get convergence at 300s. It is a short time. As well as the measurement error, it has a very short convergence time. The two errors with ACO-UKF are less than those of UKF or MSUKF. Figure 4 shows that the new proposed method has the minimum covariance error. The ranges of covariance error with UKF, MSUKF and ACO-UKF methods are $[0.0151, 0.0170]$, $[0.0145, 0.0162]$ and $[0.0149, 0.0158]$ respectively. So the covariance error is more accepted by using the improved UKF based on ant colony algorithm. The new scheme not only solves the shortcomings of traditional UKF but introduces an improved ant colony which reduces the influence of dynamic model prediction error.

5. **Conclusions.** This paper proposes a new ant colony algorithm used for UKF. We use grid division method to improve ant colony to perfect the system prediction error and measurement error in UKF. The new ant colony UKF has a fast convergence. In addition, it has a significant effect on filter process and can better approximate the mapping function of nonlinear dynamics model. Through experiments, we illustrate that it is feasible in practical applications. In the future, we will evaluate our scheme with the existing improved algorithms and study new intelligence algorithms to improve UKF or EKF algorithm.

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