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# OPTIMAL DESIGN OF A PERMANENT MAGNET SYNCHRONOUS GENERATOR USING AN EXTENICS-BASED TAGUCHI APPROACH

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Abstract. *This paper presents a novel approach to deal with multi-objective design optimization of a permanent magnet synchronous generator (PMSG) using a combination of Taguchi method and extension theory. PMSGs are widely used in green energy technologies including small scale wind-turbing generators and micro-hydro generation systems due to their high power density and high efficiency. For comparison purposes, a five-objective optimum design for a PMSG is handled with a Taguchi method and the presented approach, and an equivalent magnetic circuit analysis is made using RMxprt software. The presented extenics-based Taguchi approach is experimentally validated as an efficient way in the design of a PMSG for a majority of objective function optimizations, and is found to outperform a typical Taguchi counterpart in terms of reliability.*
**Keywords:** Permanent magnet synchronous generator (PMSG), Multi-objective optimal design, Taguchi method, Extenics-based Taguchi approach

1. **Introduction.** A surface-mounted PMSG is a type of the most attractive brushless DC (BLDC) generators applied in green energy technologies, including small scale wind-turbine generators and micro-hydro generation systems [1]. Since the air gap magnetic field of a PMSG is built by surface-mounted permanent magnets, it gives a high power density and a high efficiency. Besides, the energy conversion is produced by the interaction between the rotor permanent magnets and the stator teeth, and configuration thereof has a great effect on the performance of a PMSG [2]. Hence the optimization on the permanent magnet shape and stator teeth is seen as required to improve the performance of a PMSG.

The Taguchi method has been known as powerful for parameter design optimization in single, but not multi, objective optimization problems [3]. For optimizing a multi-objective problem, the combined use of the statistical analysis of variance (ANOVA) and a typical Taguchi method abbreviated as the Taguchi-ANOVA method, is a widely accepted approach [4,5], but unfortunately leads to erroneous estimation. As an improved version of the Taguchi-ANOVA method, this paper addresses the PMSG optimization of a mirco-hydro generation system using an extenics-based Taguchi approach. The performance is compared with the design optimized by the Taguchi-ANOVA method, and a good agreement is seen between the measurement and the optimized design as well.

The rest of this paper is outlined as follows. A PMSG is modeled in Section 2. A typical Taguchi method and the presented extenics-based Taguchi approach are detailed in Section 3 for subsequent PMSG parameter optimization. Simulation results are discussed in Section 4 and this paper is finally concluded in Section 5.

2. **Generator Modeling.** The proposed technique is applied to the optimization of a PMSG for a micro-hydro generation system. It is a 3-phase, 210V, 1800VA, 60rpm, 66 poles-72 slots, $Y$-connected permanent magnet synchronous generator, and photos of the stator and the rotor are presented in Figure 1. As illustrated in Figure 2, there are four variables involved in a PMSG design. Factor $A$ refers to the thickness of the permanent magnet in mm (levels 3.2, 3.6 and 4), factor $B$ the air gap length in mm (levels 1.5, 1.8 and 2.1), factor $C$ the stator teeth thickness in mm (levels 6.8, 7.3 and 7.8), and factor $D$ the stator yoke thickness in mm (levels 9.7, 10.2 and 10.7). Table 1 gives the actual values of three settings of the four design parameters, and hence an initial design, symbolized as ($A2$, $B1$, $C1$, $D1$), represents that $A = 3.6$mm, $B = 1.5$mm, $C = 6.8$mm and $D = 9.7$mm. The five objective functions herein are the minimization of the cogging torque and the voltage regulation both, and the maximization of the output voltage, the output power and the efficiency.

(a) (b)

Figure 1. Photos of (a) the rotor, and (b) the stator for a PMSG

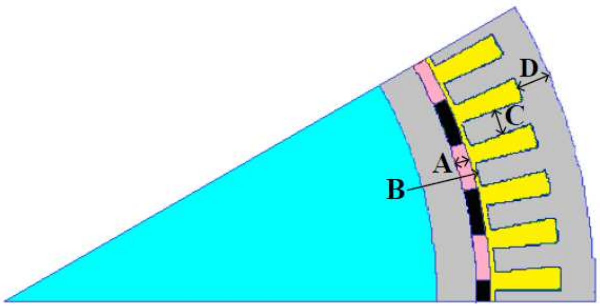


Figure 2. Four design variables of a PMSG

Table 1. Actual values of the four involved

| $L_9$ | $A$ | $B$ | $C$ | $D$ | $A$ [mm] | $B$ [mm] | $C$ [mm] | $D$ [mm] |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 3.2 | 1.5 | 6.8 | 9.7 |
| 2 | 1 | 2 | 2 | 2 | 3.2 | 1.8 | 7.3 | 10.2 |
| 3 | 1 | 3 | 3 | 3 | 3.2 | 2.1 | 7.8 | 10.7 |
| 4 | 2 | 1 | 2 | 3 | 3.6 | 1.5 | 7.3 | 10.7 |
| 5 | 2 | 2 | 3 | 1 | 3.6 | 1.8 | 7.8 | 9.7 |
| 6 | 2 | 3 | 1 | 2 | 3.6 | 2.1 | 6.8 | 10.2 |
| 7 | 3 | 1 | 3 | 2 | 4 | 1.5 | 7.8 | 10.2 |
| 8 | 3 | 2 | 1 | 3 | 4 | 1.8 | 6.8 | 10.7 |
| 9 | 3 | 3 | 2 | 1 | 4 | 2.1 | 7.3 | 9.7 |

## 3. Analysis Approaches.

### 3.1. Taguchi method.

Taguchi method is developed as a statistics-based approach, and offers a simple means of analysis and design parameter optimization [4,5]. After a designed experiment is conducted and an equivalent magnetic circuit analysis is performed on the experimental results for collecting all the required data, analysis of means (ANOM) and analysis of variance (ANOVA) are carried out to estimate the effects and determine the relative significance of the four design parameters. The optimal settings for each design variables are then available from the plot of main factor effect.

3.1.1. *Analysis of means (ANOM).* The overall mean $m(y)$ is defined as

$$m(y) = \frac{1}{n}\sum_{j=1}^{n} y(j) \tag{1}$$

where $y(j)$ and $n$ represent the $j$th experimental result and the number of experiments, respectively.

For an $L_9$ orthogonal array (OA), the average effect of the first performance characteristic $y_1$ of the design variable $A$ at level one, represented as $m_{A1}(y_1)$, is given as

$$m_{A1}(y_1) = \frac{1}{3}\left[y_1(1) + y_1(2) + y_1(3)\right] \tag{2}$$

where the variable $A$ is set to level one only in experiments 1, 2 and 3 in an $L_9$ OA. The other performance characteristics for all variables at each level are defined in the same way as in Equation (2).

3.1.2. *Analysis of variance (ANOVA).* In an attempt to evaluate relative significance of a design parameter in an $L_9$ orthogonal array, the sum of squares (SS) of the factor $A$ (SSA) describes a major portion of the total variation of $y_k$, defined as

$$\text{SSA} = 3\sum_{l=1}^{3}\left(m_{Al}(y_k) - m(y)\right)^2 \tag{3}$$

where $l$ and $y_k$ denote the number of levels corresponding to the variable $A$, and the $k$th performance characteristic, respectively. SSB, SSC and SSD are defined in the same way as in Equation (3).

### 3.2. Extenics-based Taguchi approach.

The extension theory [6] converts a multi-objective problem to a single objective problem in a qualitative and a quantitative manner. Matter element analysis is the key to extension theory [7], and a thing or even a mutative process can be well described as a set $(N, C, V)$ composed of three unitary arrays, where $N$, $C$ and $V$ denote the name, the characteristic and the corresponding quantitative value, respectively, and extension theory is applied to resolving contradiction [8-10]. Then, use of a Taguchi orthogonal array requires the least number of experiments to optimize the design parameters in wide diversity of disciplines.

3.2.1. *Linear normalization of an experimental result.* A linear normalization of an experimental result is performed first in the range between zero and one. Subsequently, the higher-the-better $x^*_{ih}(k)$ and the lower-the-better $x^*_{il}(k)$ are respectively written as

$$\begin{cases} x^*_{ih}(k) = \dfrac{x_i^{(0)}(k) - \min .x_i^{(0)}(k)/\zeta}{\zeta \max .x_i^{(0)}(k) - \min .x_i^{(0)}(k)/\zeta} \\[2ex] x^*_{il}(k) = \dfrac{\zeta \max .x_i^{(0)}(k) - x_i^{(0)}(k)}{\zeta \max .x_i^{(0)}(k) - \min .x_i^{(0)}(k)/\zeta} \end{cases} \tag{4}$$

where $x_i^{(0)}(k)$ denotes a reference sequence, $\min .x_i^{(0)}(k)$ and $\max .x_i^{(0)}(k)$ the smallest and the largest values respectively, and $\zeta$ the expansive coefficient with a recommended value of 1.2 approximately.

3.2.2. *Definition of a multidimensional matter-element.* In extension theory [7], a multi-dimensional matter-element $R$ is defined as

$$R = (N, C, V) = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix} = \begin{bmatrix} N, c_1, v_1 \\ c_2, v_2 \\ \vdots \\ c_n, v_n \end{bmatrix} \tag{5}$$

where $N$ represents a matter, $C = [c_1, c_2, \dots, c_n]$ a characteristic vector and $V = [v_1, v_2, \dots, v_n]$ a value vector of $C$.

3.2.3. *Degree of dependence computation according to an extended correlation function.* An illustration of extended correlation functions is presented in Figure 3 [8-10]. If $X_o =< a, b >$ and $X =< c, d >$ are two intervals in the field of real numbers, then $X_o \subset X$, where $X_o$ and $X$ are the classical and the limited domains, respectively. An extended correlation function can be defined as

$$K(x) = \begin{cases} -\rho(x, X_0), & x \in X_0 \\ \dfrac{\rho(x, X_0)}{\rho(x, X) - \rho(x, X_0)}, & x \notin X_0 \end{cases} \tag{6}$$

where

$$\rho(x, X_o) = \left| x - \frac{a+b}{2} \right| - \frac{b-a}{2} \tag{7}$$

represents the distance between a point $x$ and a given real interval $X_o$, and likewise

$$\rho(x, X) = \left| x - \frac{c+d}{2} \right| - \frac{d-c}{2} \tag{8}$$

symbolizes the distance between a point $x$ and a given real interval $X$.

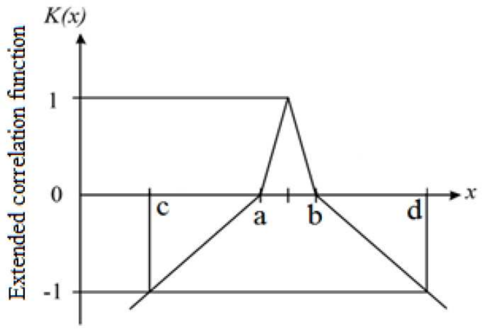


FIGURE 3. An illustration of extended correlation functions

The extended correlation function can be used to calculate the dependent degree between $x$ and $X_o$. If $K(x) \geq 0$, it can describe the degree to which $x$ belongs to $X_o$, while when $K(x) < 0$, it describes the degree to which $x$ does not belong to $X_o$.

3.2.4. *Determination of the optimal level combination.* According to ANOM, compute multiple performance characteristics index (MPCI) values of each design parameter, and the optimal parameter level combination can be determined for robust design and the factors that dominate the dependent degree can be found.

## 4. Results and Discussion.

**4.1. Results using Taguchi method.** Following the computation steps described in the previous section, the RMxprt simulation results are listed in Table 2. Figures 4-8 illustrate the factor response graphs of the cogging torque, the output voltage, the output power, the voltage regulation and the efficiency, respectively. It is seen that the factor-level combinations ($A2$, $B1$, $C1$, $D1$), ($A2$, $B1$, $C1$, $D1$) and ($A3$, $B1$, $C2$, $D1$) maximize the output voltage, the output power and the efficiency, respectively, while the factor-level combinations ($A3$, $B1$, $C1$, $D2$) and ($A3$, $B1$, $C2$, $D1$) minimize the cogging torque and the voltage regulation, respectively.

TABLE 2. RMxprt simulations in an $L_9$ OA

| $L_9$ | Design variables | | | | Cogging torque (N.m) | Output voltage (V) | Output power (W) | Voltage regulation (%) | Efficiency (%) |
|---|---|---|---|---|---|---|---|---|---|
| | $A$ | $B$ | $C$ | $D$ | | | | | |
| 1 | 1 | 1 | 1 | 1 | 0.574 | 200.34 | 1781.2 | 41.07 | 73.66 |
| 2 | 1 | 2 | 2 | 2 | 0.211 | 192.13 | 1636.6 | 45.27 | 71.33 |
| 3 | 1 | 3 | 3 | 3 | 1.066 | 181.56 | 1461.2 | 51.61 | 68.43 |
| 4 | 2 | 1 | 2 | 3 | 0.851 | 199.13 | 1756.1 | 33.63 | 75.84 |
| 5 | 2 | 2 | 3 | 1 | 0.974 | 198.52 | 1744.7 | 42.16 | 72.26 |
| 6 | 2 | 3 | 1 | 2 | 0.253 | 191.04 | 1618.6 | 49.85 | 69.71 |
| 7 | 3 | 1 | 3 | 2 | 0.245 | 199.35 | 1754.6 | 30.22 | 76.99 |
| 8 | 3 | 2 | 1 | 3 | 0.636 | 196.37 | 1708.0 | 36.63 | 74.52 |
| 9 | 3 | 3 | 2 | 1 | 0.627 | 191.29 | 1616.6 | 36.33 | 74.64 |

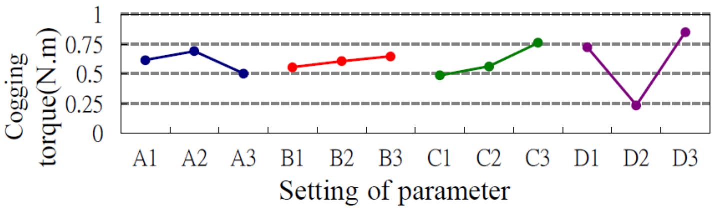


FIGURE 4. Factor response graphs of cogging torque

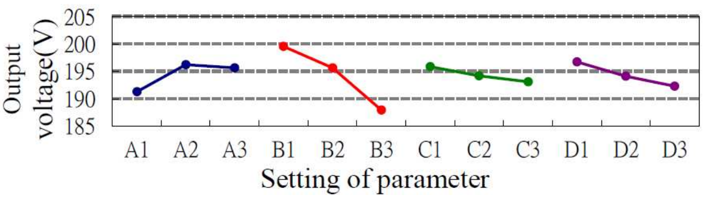


FIGURE 5. Factor response graphs of output voltage

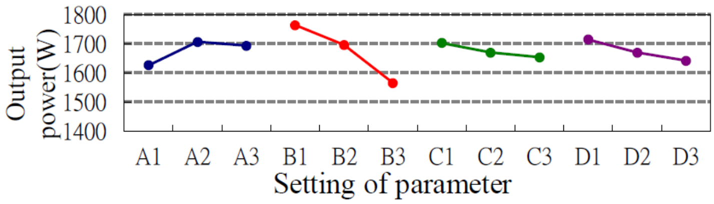


FIGURE 6. Factor response graphs of output power

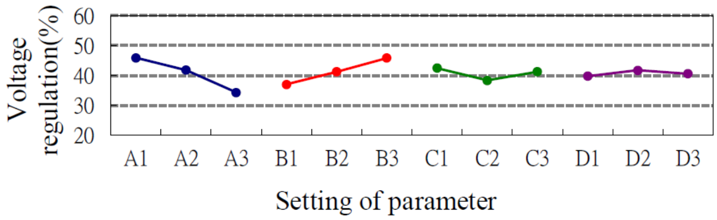


FIGURE 7. Factor response graphs of voltage regulation

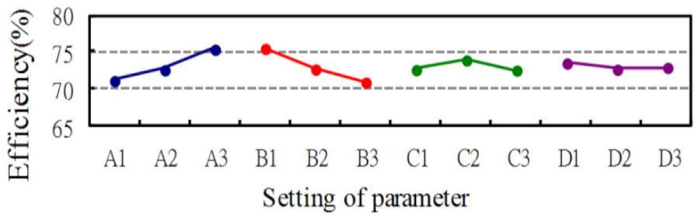


FIGURE 8. Factor response graphs of efficiency

TABLE 3. Effects of various design parameters using ANOVA

| Design variables | Cogging torque | | Output voltage | | Output power | | Voltage regulation | | Efficiency | |
|---|---|---|---|---|---|---|---|---|---|---|
| | SS | (%) | SS | (%) | SS | (%) | SS | (%) | SS | (%) |
| $A$ | 0.05 | 6.65 | 42.913 | 14.5 | 301440 | 24.2 | 207.215 | 57.4 | 27.8 | 43.2 |
| $B$ | 0.01 | 1.55 | 210.482 | 71.5 | 351493 | 28.2 | 121.309 | 33.6 | 31.8 | 49.4 |
| $C$ | 0.12 | 14.6 | 11.777 | 4.00 | 294144 | 23.6 | 26.799 | 7.42 | 3.60 | 5.59 |
| $D$ | 0.63 | 77.1 | 28.819 | 9.80 | 298367 | 23.9 | 5.6428 | 1.56 | 1.12 | 1.74 |
| Sum | 0.81 | 100 | 293.991 | 100 | 124544 | 100 | 669.656 | 100 | 64.4 | 100 |

4.2. **Results using Taguchi-ANOVA method.** The ANOVA analysis results are listed in Table 3. The optimal parameter level combination for the PMSG performance design is determined as ($A3$, $B1$, $C1$, $D2$).

4.3. **Results using extenics-based Taguchi approach.** With the linear normalization of experimental results shown in Table 4, and according to the performance characteristics of a PMSG, the classical field matter-element $R_o$ of interest can be written as

$$R_o = (N, c, v_o) = \begin{bmatrix} \text{Motor, Cogging Torque, } [0.5908,\ 1.1616] \\ \text{Output Voltage, } [0.5058,\ 0.6604] \\ \text{Output Power, } [0.5331,\ 0.7352] \\ \text{Voltage Regulation, } [0.5677,\ 1.0355] \\ \text{Efficiency, } [0.4704,\ 0.6775] \end{bmatrix} \tag{9}$$

and the limited field matter-element $R_p$ is expressed as

$$R_p = (N, c, v_p) = \begin{bmatrix} \text{Motor, Cogging Torque, } [0.1611,\ 1.1616] \\ \text{Output Voltage, } [0.2830,\ 0.6604] \\ \text{Output Power, } [0.2207,\ 0.7352] \\ \text{Voltage Regulation, } [0.2341,\ 1.0355] \\ \text{Efficiency, } [0.2688,\ 0.6775] \end{bmatrix} \tag{10}$$

Table 4. Linear normalization of the experimental results

| $L_9$ | Linear normalization | | | | |
|---|---|---|---|---|---|
| | Cogging torque (pu) | Output voltage (pu) | Output power (pu) | Voltage regulation (pu) | Efficiency (pu) |
| 1 | 0.6393 | 0.5503 | 0.6127 | 0.5677 | 0.4704 |
| 2 | 0.9680 | 0.4582 | 0.4554 | 0.4534 | 0.4045 |
| 3 | 0.1933 | 0.3396 | 0.2648 | 0.2809 | 0.3225 |
| 4 | 0.3885 | 0.5368 | 0.5854 | 0.7702 | 0.5321 |
| 5 | 0.2765 | 0.5299 | 0.5730 | 0.5380 | 0.4308 |
| 6 | 0.9305 | 0.4460 | 0.4359 | 0.3288 | 0.3587 |
| 7 | 0.9371 | 0.5392 | 0.5837 | 0.8629 | 0.5646 |
| 8 | 0.5828 | 0.5058 | 0.5331 | 0.6885 | 0.4947 |
| 9 | 0.5908 | 0.4488 | 0.4337 | 0.6967 | 0.4981 |

Table 5. Degrees of dependence and MPCI values in each experiment

| $L_9$ | Cogging torque | Output voltage | Output power | Voltage regulation | Efficiency | MPCI |
|---|---|---|---|---|---|---|
| 1 | 0.0850 | 0.2882 | 0.3937 | 0 | 0 | 0.1534 |
| 2 | 0.3391 | −0.2136 | −0.2486 | −0.3426 | −0.3267 | −0.1585 |
| 3 | −0.9250 | −0.7460 | −0.8588 | −0.8597 | −0.7334 | −0.8246 |
| 4 | −0.4707 | 0.2003 | 0.2588 | 0.4328 | 0.2977 | 0.1438 |
| 5 | −0.7314 | 0.1560 | 0.1972 | −0.0889 | −0.1963 | −0.1327 |
| 6 | 0.4049 | −0.2685 | −0.311 | −0.7162 | −0.5539 | −0.2889 |
| 7 | 0.3933 | 0.2163 | 0.2504 | 0.3688 | 0.4547 | 0.3367 |
| 8 | −0.0185 | 0 | 0 | 0.2583 | 0.1174 | 0.0714 |
| 9 | 0 | −0.2559 | −0.3182 | 0.2757 | 0.1338 | −0.0329 |

Table 6. MPCI values and the optimal setting of each design parameter

| Settings of factor | $A$ | $B$ | $C$ | $D$ |
|---|---|---|---|---|
| Level1 | −0.2766 | 0.2113 | −0.0214 | −0.0041 |
| Level2 | −0.0926 | −0.0732 | −0.0159 | −0.0369 |
| Level3 | 0.1251 | −0.3821 | −0.2069 | −0.2031 |
| Optimal setting | 3 | 1 | 2 | 1 |

Through extension theory operations, the dependent degrees and MPCI values of each experiment are given in Table 5. The MPCI values and optimal setting of each design parameter are available in Table 6 and the optimal parameter level combination is determined as ($A3$, $B1$, $C2$, $D1$).

4.4. **Performance comparison.** For validation purposes, a PMSG is placed in a testbed, as illustrated in Figure 9. Table 7 gives a performance comparison among an initial design, the design optimized by the Taguchi-ANOVA method and the extenics-based Taguchi approach, and a good agreement is seen between the measured and the simulation results. Except that there is a slight increase of 0.2745N.m in the cogging torque as compared with the initial design (PMSG is specified to have a rated torque of 286.48N.m), the extenics-based Taguchi approach is validated to outperform the other two in the rest of objective optimizations.

5. **Conclusions.** This paper presents an extenics-based Taguchi approach to optimize five objective functions on a surface-mounted PMSG simulated using RMxprt software,

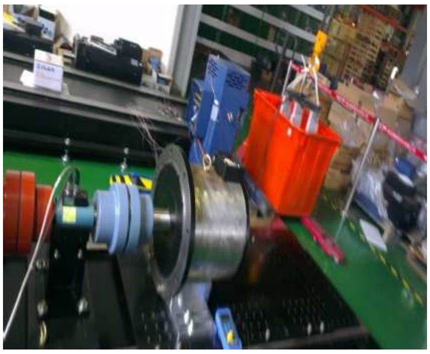

FIGURE 9. A PMSG during a test

TABLE 7. Performance comparison of a PMSG design

| Performance | Initial design | Taguchi-ANOVA | Extenics-based Taguchi | Test |
|---|---|---|---|---|
| Cogging torque (N.m) | 0.5999 | 0.6176 | 0.8744 | Not available |
| Output voltage (V) | 204.33 | 206.64 | 209.03 | 202 |
| Output power (W) | 1852.1 | 1893.5 | 1933.1 | 1806 |
| Voltage regulation (%) | 39.360 | 37.660 | 34.830 | 34.2 |
| Efficiency (%) | 73.960 | 74.280 | 75.090 | 76.3 |

and presents a simple way based on extension theory to estimate the MPCI of the PMSG as well. The proposed novel approach is found to outperform an initial design and the design optimized by the Taguchi-ANOVA method in four aspects and in terms of robustness. In the future, this proposed approach is expected to be widely applied to multi-objective optimum design in a diversity of disciplines, including the electrical engineering and manufacturing engineering.

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