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AN EFFICIENT INFINITE IMPULSE RESPONSE EQUALIZATION APPROACH FOR LOUDSPEAKER SYSTEM USING VECTOR FITTING

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Abstract. *Traditional loudspeaker equalization algorithms suffer from large computational complexity when high accurate results are expected, which may lead to unacceptable time delay in real-time applications. In this paper, an efficient approach with less cost is presented for loudspeaker system using vector fitting (VF). An approximated infinite impulse response (IIR) model of loudspeaker system is built. As well, a stable IIR equalization filter is derived from such a model. The poles and zeros of this IIR equalizer can be computed and iteratively relocated to ensure accuracy and efficiency of loudspeaker equalization. Comparative experiments are presented and results show that the proposed approach can substantially reduce the time consumption without deterioration in accuracy of loudspeaker equalization.*
Keywords: Loudspeaker equalization, Vector fitting, Time consumption

1. **Introduction.** Loudspeaker systems, even though with careful manufacture, still remain one of weakest links of audio systems. The use of loudspeaker equalization is an attractive way to improve their non-ideal responses. It is a common assumption that the deviation from the flat loudspeaker frequency response introduces "color" in reproduced sound as well as "harm" in subjective listening experience [1]. So the equalization filter, known as inverse filter, essentially requires an inverse model of a loudspeaker frequency response. However, the loudspeaker systems are in general non-minimum phase systems, which means any inverse operation is not allowed. Hence, only approximate inverse models can be found.

Compared to infinite impulse response (IIR) filters, the finite impulse response (FIR) filters, even designed with modified structure [2], show less efficiency and more computational complexity [3]. That is the reason why IIR method is employed in equalization. The key step of designing corresponding IIR equalizer is to translate a loudspeaker system to an IIR model. The stability and efficiency of the IIR model and equalizer must be guaranteed. In addition, as the importance of phase equalization has already been recognized by many researchers [4-6], the IIR filter should provide simultaneous magnitude and phase equalization.

Many different methods [7-11] can be used to build IIR model of loudspeaker system for equalization; however, different problems exist with them as well. Typically, the order of IIR model cannot be properly decided by almost all methods [8-11] and the phase response is neglected in some magnitude equalization methods. In addition, a convergence problem appears in some methods [9-11] since they include a parameter search procedure. Moreover, excess-phase part of loudspeaker has been ignored by some magnitude equalization methods [9,11]. Furthermore, a common disadvantage among these methods [7-11] is that the computational cost or delay will increase too much if loudspeaker system needs to be equalized with enough accuracy, and this excessive delay cannot be permissible in live application, where delay above 10 ms is noticeable and annoying to the speakers and musicians.

In this paper, an efficient IIR equalization approach for loudspeaker system is presented. The main contributions of proposed approach are: (i) a modified VF algorithm, that models loudspeaker system as IIR structure; (ii) an initial poles location strategy, used to speedup the modeling; (iii) an iterative optimization procedure, improving the accuracy of IIR model; (iv) simultaneous magnitude and phase equalization. The advantages of this approach in loudspeaker equalization are demonstrated with examples.

The organization of the paper is as follows. In Section 2 the steps of the method are described detailedly, concentrating on IIR model and IIR equalizer. And comparisons with other equalization methods are carried out and simulation results are illustrated in Section 3. Finally, Section 4 concludes the paper and discusses the future work.

2. IIR Equalization.

2.1. IIR model.

The transfer function of the loudspeaker system $H(z)$ can be approximated by a causal and stable IIR system $\hat{H}(z)$ with numerator polynomial $P(z)$ and denominator polynomial $Q(z)$, as follows,

$$\hat{H}(z) = \frac{P(z)}{Q(z)} \tag{1}$$

Therefore, all poles of $\hat{H}(z)$ (zeros of $Q(z)$) must lie inside unit circle. Rewriting the $\hat{H}(z)$ to a poles-residues form, we have,

$$\hat{H}(z) = \sum_{n=1}^{N} \frac{c_n}{z - a_n} + d \tag{2}$$

The problem at hand is to estimate all coefficients (residues c_n, poles a_n and constant d) in (2), which VF algorithm can solve efficiently. Only a brief outline of the VF technique is given here, while a detailed description can be found in [12] and [13]. A scaling rational function $\sigma(z)$ in (3) is introduced, which has the same poles with and can be approximated by the product of $\sigma(z)$ and $\hat{H}(z)$ in (4).

$$\sigma(z) = \sum_{n=1}^{N} \frac{\bar{c}_n}{z - \bar{a}_n} + 1 \tag{3}$$

where poles $\bar{a}_n$ and residues $\bar{c}_n$ are unknown, and N is number of poles or the order of IIR model. The order is decided by Hankel singular value decomposition (SVD) [7].

$$\sigma(z)\hat{H}(z) = \sum_{n=1}^{N} \frac{c_n}{z - \bar{a}_n} + d \tag{4}$$

Multiplying (3) with (2) and considering (4) yields the following relation:

$$\sum_{n=1}^{N} \frac{c_n}{z - \bar{a}_n} + d = \left(\sum_{n=1}^{N} \frac{\bar{c}_n}{z - \bar{a}_n} + 1 \right) \hat{H}(z) \tag{5}$$

or

$$\left(\sum_{n=1}^{N} \frac{c_n}{z - \bar{a}_n} + d \right) - \left(\sum_{n=1}^{N} \frac{\bar{c}_n}{z - \bar{a}_n} \right) \hat{H}(z) = \hat{H}(z) \tag{6}$$

There are N_s known samples $H(z_k)$, $k = 1, 2, \cdots, N_s$ at the N_s frequency points distributed over the entire frequency range (from 20 Hz to 20 KHz). Supposing the values for poles $\bar{a}_n$, then, a linear equation can be derived from (6), that is,

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{7}$$

where

$$\mathbf{x} = [c_1 \quad \cdots \quad c_N \quad d \quad \bar{c}_1 \quad \cdots \quad \bar{c}_N]^T \tag{8}$$

$$\mathbf{A} = \begin{bmatrix} \frac{1}{z_1-\bar{a}_1} & \vdots & \frac{1}{z_1-\bar{a}_N} & 1 & -\frac{H(z_1)}{z_1-\bar{a}_1} & \cdots & -\frac{H(z_1)}{z_1-\bar{a}_N} \\ \frac{1}{z_2-\bar{a}_1} & \vdots & \frac{1}{z_2-\bar{a}_N} & 1 & -\frac{H(z_2)}{z_2-\bar{a}_1} & \cdots & -\frac{H(z_2)}{z_2-\bar{a}_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \frac{1}{z_{N_s}-\bar{a}_1} & \vdots & \frac{1}{z_{N_s}-\bar{a}_N} & 1 & -\frac{H(z_{N_s})}{z_{N_s}-\bar{a}_1} & \cdots & -\frac{H(z_{N_s})}{z_{N_s}-\bar{a}_n} \end{bmatrix} \tag{9}$$

$$\mathbf{b} = [H(z_1) \quad H(z_2) \quad \cdots \quad H(z_{N_s})]^T \tag{10}$$

With reference to (8) and (5), we build the IIR model of loudspeaker, namely,

$$\hat{H}(z) = \frac{\sum_{n=1}^{N} \frac{c_n}{z-\bar{a}_n} + d}{\sum_{n=1}^{N} \frac{\bar{c}_n}{z-\bar{a}_n} + 1} = \frac{\prod_{n=1}^{N+1} (z-\lambda_n)}{\prod_{n=1}^{N+1} \left(z-\bar{\lambda}_n\right)} \tag{11}$$

where λ_n and $\bar{\lambda}_n$ denote estimated zeros and poles of $\hat{H}(z)$.

It is obvious that the results of $\hat{H}(z)$ in (11) depends much on the given poles $\bar{a}_n$ (called initial poles). An iteration procedure is introduced to improve the accuracy of the IIR model $\hat{H}(z)$ approximating the $H(z)$: let poles $\bar{\lambda}_n$ in (11) be the new initial poles $\bar{a}_n$, and recalculate (3)-(11) to complete one iteration, which can be continued until getting ideal results. It was shown that even if staring poles were poorly selected, a very accurate result was still achieved by reusing the new poles as starting poles in iterative procedure. Although, an advanced starting poles location strategy helps to speedup convergence by reducing number of iterations, the majority of initial poles should be set to complex conjugate with linearly distributed over loudspeaker frequency range, and some poles are essential when loudspeaker system function shows distinct resonance peaks representing corresponding conjugate poles. Occasionally unstable poles, appearing in iterations, undergo reciprocal flipping back inside the unit circle to guarantee stability, thus changing the phase without altering the magnitude. The effect of this phase change, however, is then suppressed through equalizer in Section 2.2.

2.2. **IIR equalizer.** It is well known that a mixed-phase system can be factorized by a minimum-phase system $H_{\min}(z)$ and an all-pass system $H_{ap}(z)$, viz,

$$\begin{aligned} H(z) &= \frac{(1-b_m z^{-1})(1-b_e z^{-1})}{(1-r_i z^{-1})} = \frac{(1-b_m z^{-1})(1-b_e^* z^{-1})}{(1-r_i z^{-1})} \cdot \frac{1-b_e z^{-1}}{1-b_e^* z^{-1}} \\ &= H_{\min}(z) \cdot H_{ap}(z) \end{aligned} \tag{12}$$

where r_i denote poles, b_m and b_e denote the zeros inside and outside the unit circle, and superscript $*$ denotes the complex conjugate. The separated minimum-phase part $H_{\min}(z)$ and all-pass part $H_{ap}(z)$ characterize magnitude and excess-phase of loudspeaker system response respectively.

Now, the minimum-phase function $H_{\min}(z)$ found in (12) is invertible whose zeros and poles are all inside unit circle. Therefore, the magnitude equalizer can be calculated as the reciprocal of $H_{\min}(z)$.

The excess-phase equalizer uses an all-pass filter proposed in [10], which is formed from the time-reversed, time-shifted, and time-windowed. A proper pure delay will generate after excess-phase equalization. The IIR equalizer is composed of the magnitude equalizer and the excess-phase equalizer together. It corrects the magnitude and phase of loudspeaker system at the same time.

3. Simulation Results. Experimental results are presented in this section, with a comparison between the proposed method and other well-known IIR equalization methods, such as the second order sections (SOS) method [2] and the balanced model truncation (BMT) [7] method. The objective measure used in this study is the mean absolute dB error [2] computed between the flat target and filter response. All experiments are done in Matlab 8.5 environment using a 4 GB-RAM 3.3-GHz laptop.

Figure 1 demonstrates the procedure of loudspeaker IIR modeling with 40th-order using VF method. As described in previous sections, a loudspeaker response including magnitude and phase can be approximated by such IIR model and an accurate matching will be accomplished by iterations. We have found experimentally, in general, well-pleasing solution appears less than fifteen iterations.

Figure 2(a) provides a comparison of the new method with BMT and SOS equalization techniques. Three filters are designed by the three methods with 40th-order loudspeaker's IIR model, and the responses have been scaled 12 dB to separate them for clarity. As

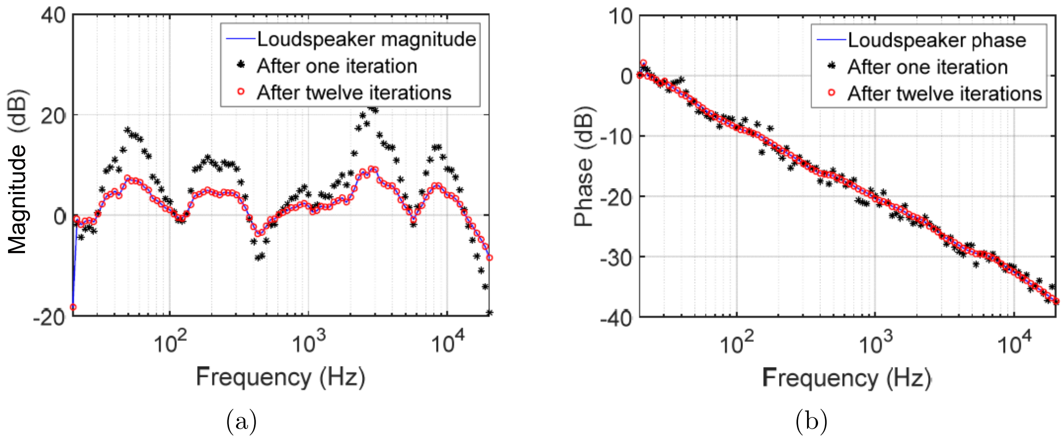

FIGURE 1. The fitting results of loudspeaker response by iterations: (a) the magnitude response fitting, (b) the phase response fitting

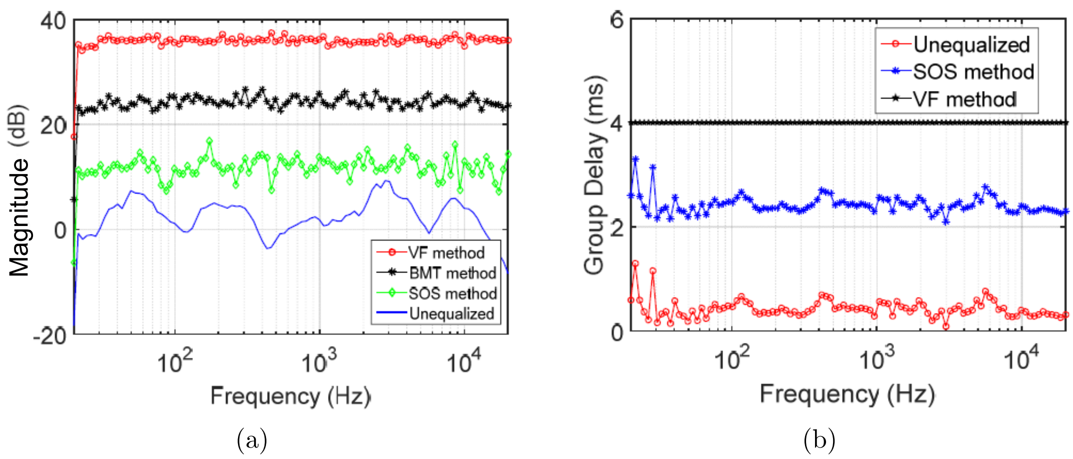

FIGURE 2. Loudspeaker equalization comparison for different equalizer types: (a) the magnitude equalization comparison, (b) the phase equalization comparison

shown in Figure 2(a), VF equalization method with ten iterations (the same number iterations below) performs much superiority to the other two and produces the best magnitude equalization. The experimental results prove that, in turn, SOS method has difficulty in finding convergent solution and BMT method is too vulnerable to algorithmic complexity to acquire accurate IIR model.

Figure 2(b) plots the group curves of loudspeaker response before and after equalization, which are offset by 1.5 ms for clarity. It is obvious that SOS equalization method does nothing to group time providing an evidence that it badly ignores the phase response of loudspeaker, while the VF equalization method generates flat group time curve that means loudspeaker response after equalization will be with linear phase although group time has increased to about 17 ms.

As mentioned before, the great advantage of proposed equalization technique is high-efficiency, verified by experimental results in Table 1. The error of unequalized loudspeaker response is calculated as 3.52 dB. With respect to different filter order, the VF equalization method always shows the lowest error and enjoys the fastest computed speed compared with BMT and SOS methods. Therefore, the time consumption of loudspeaker equalization can be substantially reduced using VF method. It is much helpful to carry out the real-time application especially when available hardware is constrained.

Table 1. Comparison of error and CPU time for different equalizer types

IIR order	Error (dB)			CPU time (sec)		
	VF	BMT	SOS	VF	BMT	SOS
30	0.484	0.574	1.061	0.239	1.876	0.857
40	0.290	0.338	0.621	0.408	2.963	1.986
50	0.255	0.287	0.499	0.596	4.617	3.561

4. Conclusions.

Application of VF technique to loudspeaker equalization allows to speedup the design of equalizer. Numerical examples then confirm the remarkable efficiency and accuracy of VF method over existing IIR equalization techniques in both magnitude and phase equalization. However, we have objectively noticed imperfection in low frequency of loudspeaker response even though with deep equalization, and this noticeable problem remains to be solved in future.

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