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STABILITY ANALYSIS OF NETWORKED CONTROL SYSTEMS WITH NETWORKED DELAY AND PACKET DROPOUTS CONSTRAINTS

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Abstract. *The stability of networked control system based on the packet dropouts, networked delay and white noise is studied in this paper. The limitation value of signal-to-noise ratio (SNR) to stabilize networked control systems is obtained by the one-parameter compensator and the frequency domain analysis method. It is shown that the limitation value of SNR to stabilize networked control system is determined by the position of the non-minimum phase zeros and the unstable poles, the networked delay and the packet dropouts probability. Finally, the efficiency of the result is verified by using some typical examples.*

Keywords: Networked control systems, Networked delay, Packet dropouts probability, Non-minimum phase zeros

1. **Introduction.** In recent years, networked control systems have been widely used in many fields, such as industrial automation, distributed mobile communication and unmanned aerial vehicle [1]. The networked control systems bring a lot of convenience, but also bring a lot of new challenges to humans' lives. In the networked control systems, the data is transmitted through a communication network; however, the limitation of bandwidth and the channel capacity will lead to the date packet dropouts and networked delay. These problems often cause the decline of system performance, and even affect the instability of the system. However, the traditional control theory cannot be simply applied in the networked control systems. Faced with the new problems in the networked control systems, we should adopt a new analysis method to study and design the networked control systems. At present, more and more scholars have studied the stability of networked control systems. The problem of modeling and stabilization of a wireless networked control system with packet dropouts and networked delay was considered in paper [2]. A stochastic Lyapunov function approach to establish stability results of the networked control systems was proposed in paper [3]. The distributed finite-horizon filtering problem for a class of time-varying systems over lossy sensor networks was considered in paper [4]. The problem of stabilization about uncertain networked control systems with random but bounded delays was discussed in paper [5].

Above all of these are from the perspective of the state space to study the stability of the networked system, and the condition to stabilize the system is obtained by solving the linear matrix inequality (LMI). However, in the process of the actual networked control system communication, the signal transmitted in the channel is described by its frequency domain characteristics. However, the stability of the networked systems is analyzed rarely by using the frequency domain analysis method.

At present, some achievements have been obtained to study the performance of networked control systems by the frequency domain method [6], and the research results showed that the required value to stabilize the networked control system is decided by the system internal structure characteristics and network channel parameters. It was pointed out that the optimal tracking performance of the multi-variable discrete control system was determined by the basic characteristics of the system and the bandwidth of the network channel in paper [7]. The stability of networked control systems based on the effect of the SNR and the networked delay was studied in paper [8]. Therefore, it has a great advantage to study the networked control system by using the frequency domain analysis method.

In this paper, the stability of networked control systems is studied based on the effect of communication networked delay and packet dropouts. The limitation value of the SNR to stabilize networked control systems is obtained by the method of frequency domain analysis and spectral decomposition. The limitation value is determined by the position of the non-minimum phase zeros and the unstable poles, the networked delay and the packet dropouts probability. The obtained results show that there is a close relationship between the minimum value of the SNR to stabilize networked control systems and the essential characteristics of the system and communication network parameters, theoretically, which will be used to guide the design of networked control systems.

This paper is organized as follows. Section 2 introduces the problem formulation. The stability of networked control systems based on the packet dropouts, networked delay and white noise is studied in Section 3. A typical example is given to illustrate the results in Section 4. The paper conclusion and future research direction are presented in Section 5.

2. Problem Formulation.

The notation used throughout this paper is defined as follows. $\bar{z}$ denotes the conjugate of the complex number z. The expectation operator is defined as $E\{\cdot\}$. For any vector u, and matrix A, u^T, u^H, and A^T, A^H are their transpose and conjugate transpose, respectively; Euclidean norm of the vector u is $\|u\|$. Let the open right-half plane be denoted by $\mathcal{C}_+ := \{s : \mathrm{Re}(s) > 0\}$, the open left-half plane is $\mathcal{C}_- := \{s : \mathrm{Re}(s) < 0\}$. Denote the Euclidean vector norm is $\|\cdot\|_2$ and $\|\cdot\|_F$ as the Frobenius norm, in addition $\|G\|_F = tr(G^H G)$. $\mathcal{L}_2$ is Lebesgue space standard frequency range with the inner product $< f, g >:= (1/2\pi)\int_{-\infty}^{+\infty} tr[f^H(jw)g(jw)]dw$, which further induces the $\mathcal{L}_2$ norm $\|f\|_2^2 =< f, f >$. Next, $\mathcal{L}_2$ is equivalent to two orthogonal subspaces H_2 and $H_2^{\perp}$ which are given in paper [9]. Finally, define $\mathbb{R}\mathcal{H}_\infty$ are all stable, proper rational function matrices.

We establish the networked control systems as depicted in Figure 1, where the problem is to investigate the stability of networked control systems.

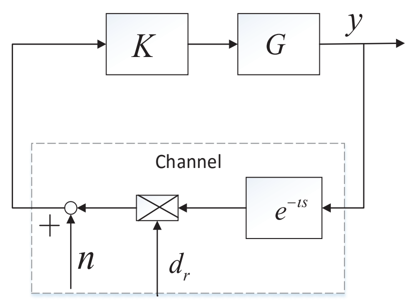

Figure 1. Networked control systems with packet dropouts and networked delay

In Figure 1, G represents the plant model, K represents the single degree compensator, and y represents the system output, whose transfer functions are denoted as $G(s)$, $K(s)$ and $Y(s)$, respectively. The characteristics of the network channel are reflected by the data packet dropouts, networked delay and white noise, which are denoted as d_r, ι and n, respectively. The random process of a Bernoulli distribution can be used to simulate the process of data packet dropouts. The parameter d_r represents whether or not a packet is dropped.

$$d_r = \begin{cases} 0 & \text{if the systems output is not successfully transmitted to the controller} \\ 1 & \text{if the systems output is successfully transmitted to the controller} \end{cases}$$

And the distribution probability for d_r is: $P\{d_r = 1\} = 1 - q$, $P\{d_r = 0\} = q$, $0 \leq q < 1$, and q represents the packet dropouts probability. In following analysis, this is an assumption that the packet dropouts and additive white noise are completely independent. The input energy of the network channel is limited, and then $E\left\{\|Y\|^2\right\} < \Gamma$ where Γ is the maximum value of the input energy of the network channel.

Making the networked control systems achieve stability, the input energy of the network channel must be greater than a certain limitation of the SNR. In order to obtain the limitation value of the SNR to stabilize networked control systems, we first derive the expression of the output power spectral density of the system. According to Figure 1, we can obtain

$$Y = \left(Y d_r e^{-\iota s} + n\right) GK \tag{1}$$

Firstly, $S_Y(j\omega)$ represents the output frequency characteristics of communication network channel, and $S_{nY}(j\omega)$ represents the frequency characteristics from the white noise of communication network to the system output. According to the method in paper [10] and a simple calculation, (1) can be converted to

$$S_Y(j\omega) = \frac{G(j\omega)K(j\omega)}{1 - qe^{-j\omega\iota}(j\omega)G(j\omega)K(j\omega)} S_{nY}(j\omega)$$

Further, according to paper [10], we can obtain

$$E\left\{\|Y\|^2\right\} = P = \left\| \frac{KG}{1 - e^{-\iota s}K(1-q)G} \right\|_2^2 \Phi$$

where P represents the input power of the communication network channel, and Φ represents the power spectral density of white noise. $\gamma = \frac{P}{\Phi}$ represents the SNR of network channel. Therefore, the SNR of the system must be satisfied

$$\left\| \frac{KG}{1 - e^{-\iota s}K(1-q)G} \right\|_2^2 < \frac{P}{\Phi} \tag{2}$$

3. **Stability Analysis of Networked Control Systems.** For any transfer function $(1-q)G$, consider a coprime factorization of $(1-q)G$ as

$$(1-q)G = \frac{N}{M} \tag{3}$$

where N, $M \in \mathbb{R}\mathcal{H}_\infty$, and it meets Bezout identity

$$MX - NYe^{-\iota s} = 1 \tag{4}$$

where X, $Y \in \mathbb{R}\mathcal{H}_\infty$. It is well known that every stabilizing compensator $\mathcal{K}$ can be characterized by Youla parameterization [2].

$$\mathcal{K} = \left\{ K : K = \frac{(Y - MQ)}{X - e^{-\iota s}NQ},\ Q \in \mathbb{R}\mathcal{H}_\infty \right\} \tag{5}$$

We also know that a non-minimum phase transfer function can be decomposed into a minimum phase part and an all-pass factor [11]. Then

$$N = (1-q)L_z N_m, \quad M = B_p M_m \tag{6}$$

where L_z and B_p are all-pass factors, N_m and M_m are the minimum phase parts, L_z includes all non-minimum phase zeros z_i ($z_i \in \mathbb{C}_+$, $i = 1, \cdots, n$) of the given plant, and

B_p includes all unstable poles p_j ($p_j \in \mathbb{C}_+, j = 1, \cdots, m$) of the given plant [2]. L_z and B_p can be expressed as

$$L_z(s) = \prod_{i=1}^{n_s} \frac{s - z_i}{s + \bar{z}_i}, \quad B_p(s) = \prod_{j=1}^{m} \frac{s - p_j}{s + \bar{p}_j} \tag{7}$$

Theorem 3.1. *As shown in Figure 1, assuming that the plant has unstable poles p_j ($p_j \in \mathbb{C}_+, j = 1, \cdots, m$) and non-minimum phase zeros z_i ($z_i \in \mathbb{C}_+, i = 1, \cdots, n$) in order to stabilize the networked control systems, the SNR of the communication network channel must be satisfied*

$$\frac{P}{\Phi} > \frac{1}{(1-q)^2} \sum_{j,i \in m} \frac{4Re(p_j)Re(p_i)}{\bar{p}_j + p_i} \frac{e^{\iota p_j} e^{\iota p_i} L_z^{-1}(p_i) L_z^{-H}(p_j)}{\bar{b}_j b_i} \tag{8}$$

where $R_1 \in \mathbb{R}\mathcal{H}_\infty$, $b_j = \prod\limits_{i \in N, i \neq j} \frac{p_i - p_j}{p_j + \bar{p}_i}$.

Proof: T_{yn} represents the transfer function of the noise and the output signal, and according to (2), we can get $T_{yn} = \frac{KG}{1 - e^{\iota s} K(1-q)G}$, and get $\inf\limits_{K \in \mathcal{K}} \left\| \frac{KG}{1 - e^{-\iota s} K(1-q)G} \right\|_2^2$.

According to (2), (3), (4) and (5), we can get

$$T_{Yn} = (1-q)^{-1}(Y - MQ)N$$

Define $J_1^* = \inf\limits_{Q \in \mathbb{R}\mathcal{H}_\infty} \|T_{Yn}\|_2^2$, and then

$$J_1^* = \inf_{K \in \mathcal{K}} \left\| \frac{(Y - MQ)}{1 - q} N \right\|_2^2 \tag{9}$$

Because L_z is an all-pass factor, and according to (6) and (9), J_1^* can be converted to

$$J_1^* = \inf_{K \in \mathcal{K}} \left\| (1-q)^{-1}(Y - MQ)N_m \right\|_2^2$$

Because B_p is an all-pass factor, J_1^* can be converted to

$$J_1^* = \frac{1}{(1-q)^2} \inf_{K \in \mathcal{K}} \left\| \frac{N_m Y}{B_p} - M_m N_m Q \right\|_2^2$$

According to the decomposition of the partial fraction

$$\frac{N_m Y}{B_p} = \sum_{j \in m} \left(\frac{\bar{p}_j + s}{s - p_j} \right) \frac{N_m(p_j) Y(p_j)}{b_j} + R_1$$

where $R_1 \in \mathbb{R}\mathcal{H}_\infty$, $b_j = \prod\limits_{i \in N, i \neq j} \frac{p_i - p_j}{p_j + \bar{p}_i}$. Therefore

$$\begin{aligned} J_1^* &= \frac{1}{(1-q)^2} \inf_{Q \in \mathbb{R}\mathcal{H}_\infty} \left\| \sum_{j \in m} \left(\frac{\bar{p}_j + s}{s - p_j} \right) \frac{N_m(p_j) Y(p_j)}{b_j} + R_1 - N_m Q M_m \right\|_2^2 \\ &= \frac{1}{(1-q)^2} \inf_{Q \in \mathbb{R}\mathcal{H}_\infty} \left\| \sum_{j \in m} \left(\frac{\bar{p}_j + s}{s - p_j} - 1 \right) \frac{N_m(p_j) Y(p_j)}{b_j} + R_1 + \frac{N_m(p_j) Y(p_j)}{b_j} - N_m Q M_m \right\|_2^2 \end{aligned}$$

Because $\sum_{j\in m}\left(\frac{\bar{p}_j+s}{s-p_j}-1\right)\frac{N_m(p_j)Y(p_j)}{b_j}$ is in $\mathcal{H}_2^{\perp}$ and $R_1+\frac{N_m(p_j)Y(p_j)}{b_j}-N_mQM_m$ is in $\mathcal{H}_2$, then

$$\begin{aligned} J_1^* = {} & \frac{1}{(1-q)^2}\left\|\sum_{j\in m}\frac{2\,\mathrm{Re}(p_j)}{s-p_j}\frac{N_m(p_j)Y(p_j)}{b_j}\right\|_2^2 \\ & + \frac{1}{(1-q)^2}\inf_{Q\in\mathbb{R}\mathcal{H}_\infty}\left\|R_1+\frac{N_m(p_j)Y(p_j)}{b_j}-N_mQM_m\right\|_2^2 \end{aligned} \tag{10}$$

According to (4) and $M(p_j)=0$, we can obtain

$$N_m(p_j)Y(p_j)=e^{p_j\iota}(p_j)L_z^{-1}(p_j) \tag{11}$$

We plug (11) into (10), then

$$\begin{aligned} J_1^* = {} & \frac{1}{(1-q)^2}\left\|\sum_{j\in m}\frac{2\,\mathrm{Re}(p_j)}{s-p_j}\frac{e^{\iota p_j}(p_j)L_z^{-1}(p_j)}{b_j}\right\|_2^2 \\ & + \frac{1}{(1-q)^2}\inf_{Q\in\mathbb{R}\mathcal{H}_\infty}\left\|R_1+\frac{e^{\iota p_j}L_z^{-1}(p_j)}{b_j}-N_mQM_m\right\|_2^2 \end{aligned}$$

Because N_m and M_m are the minimum phase parts, and $R_1\in\mathbb{R}\mathcal{H}_\infty$, $\mathcal{Q}\in\mathbb{R}\mathcal{H}_\infty$, $\inf_{Q\in\mathbb{R}\mathcal{H}_\infty}\left\|R_1+\frac{e^{\iota p_j}L_z^{-1}(p_j)}{b_j}-N_mQM_m\right\|_2^2$ can be made arbitrarily small by choosing, then

$$\inf_{Q\in\mathbb{R}\mathcal{H}_\infty}\left\|R_1+\frac{e^{\iota p_j}L_z^{-1}(p_j)}{b_j}-N_mQM_m\right\|_2^2=0$$

Therefore

$$J_1^*=\frac{1}{(1-q)^2}\left\|\sum_{j\in m}\frac{2\,\mathrm{Re}(p_j)}{s-p_j}\frac{e^{\iota p_j}L_z^{-1}(p_j)}{b_j}\right\|_2^2$$

By a simple calculation, we can get

$$J_1^*=\frac{1}{(1-q)^2}\sum_{j,i\in m}\frac{4Re(p_j)Re(p_i)}{\bar{p}_j+p_i}\frac{e^{\iota p_j}e^{\iota p_i}L_z^{-1}(p_i)L_z^{-H}(p_j)}{\bar{b}_jb_i}$$

The proof is completed.

The obtained theorem shows that the limitation value of SNR to stabilize networked control systems is determined by the position of the non-minimum phase zeros and the unstable poles, the networked delay and the packet dropouts probability.

4. **Numerical Example.** The unstable system model is considered as follows

$$G(s)=\frac{(s-2)(s+1)}{s(s-1)(s+5)}$$

The transfer function is non-minimum phase and contains an unstable pole at $p_1=1$, and a non-minimum phase zero at $z_1=2$.

The network data packet dropouts probability is $q\in(0\ \ 1)$. The ι^1, ι^2 and ι^3 represent different networked delays, respectively.

$$\iota^1=0.1,\quad \iota^2=0.2,\quad \iota^3=0.5$$

According to the theorem, the limitation value of SNR to stabilize networked control systems is obtained

$$J^*=\frac{18e^{\iota}}{(1-q)^2}$$

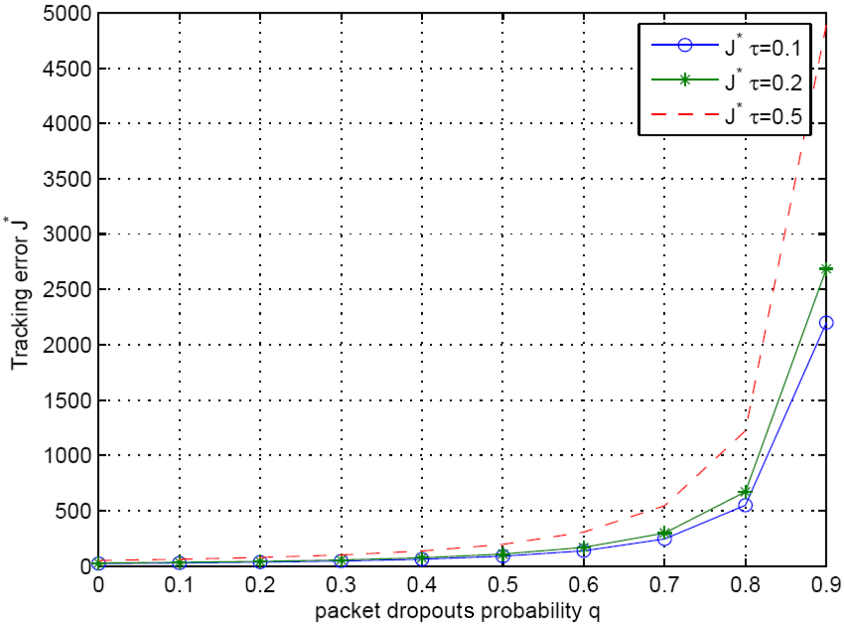

Figure 2. The limitation values of SNR under different networked delays

The limitation value of SNR to stabilize networked control systems under different packet dropouts probabilities and different networked delays is shown in Figure 2. It can be seen from Figure 2 that the limitation value of SNR to stabilize networked control systems is further increased with networked delay and packet dropouts, and it can also be seen that the smaller the network delay of the network channel, the lower the limitation value of SNR to stabilize networked control systems.

5. **Conclusion.** This paper studies the stability of networked control systems based on the networked delay and the packet dropouts constraints, and we can obtain the limitation values of SNR to stabilize networked control systems by using spectral decomposition technique. The limitation value is determined by the position of the non-minimum phase zeros and the unstable poles, the networked delay and the packet dropouts probability. The obtained result shows the limitation values of SNR to stabilize networked control systems which are determined by plant internal structure and networked parameters, no matter what compensator is adopted, which will be guidance for the design of networked control systems. An example has been given to illustrate the obtained results.

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