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# FINITE TIME STABILIZATION OF UNCERTAIN SWITCHED SYSTEMS WITH TIME-VARYING DELAYS

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Abstract. *This paper deals with finite time stabilization (FTS) for a class of switched systems. Based on multiply Lyapunov Krasovskii function and average dwell time approach, the sufficient conditions on the existence of stabilization controller are established. Furthermore, the conditions are transformed into LMIs via a novel method. Finally, a robust state feedback controller is built such that the switched system is finite time bounded. The simulation experiment illustrates the validities of the obtained results.*
**Keywords:** Switched system, FTS, Model uncertainty, Time-varying delay

1. **Introduction.** The switched system is an important kind of hybrid systems [1]. It is widely applied in industrial fields and extensively investigated by researchers [2]. In particular case, if system state is bounded in specified time interval, then an unstable system may meet application requirements [3]. Thus, the concept of finite time boundedness (FTB) is of great significance. Recently, the FTB and FTS have attracted attention of researchers. Based on multiply Lyapunov function, Lin et al. analyzed FTB for switched system with fixed delay [4]. Since time delay usually varies with time, the problems of FTB and FTS were studied for switched linear systems with time varying-delay [5]. Switching laws could be divided into time-dependent and state-dependent switching law. Most of the existing literature discuss FTB and FTS under time-dependent switching law. In [6], FTS was investigated for switched system under state-dependent switching law. Although the obtained results in above literature are valid, their mathematical calculations are very complicated. Zhong and Chen presented a new FTB definition which could simplify the analysis of FTB and FTS [7]. For switched system, switching behavior has great impact on system state. Thus, when FTB and FTS are studied, it is necessary to analyze the effect of switching behavior on state. In [8], the switching behavior's impact on system state was analyzed and an FTS controller was built.

In practice, model uncertainty is universal and has great impacts on system state. However, in most of literature, FTB and FTS are discussed without taking it into account. In this paper, the problem of FTS is studied for uncertain switched system. Since model uncertainties exist in system, the obtained results usually are not LMIs. Therefore, a novel method is presented to convert the obtained results into LMIs. Finally, a robust FTS controller is built such that the switched system is finite time bounded. The contributions in this paper are as follows: (1) a novel method which could convert no-LMIs into LMIs is presented; (2) the analysis of FTS is greatly simplified by utilizing a new definition on FTB.

In the rest of the paper, Section 2 will introduce some necessary definitions and lemmas. The main results are given in Section 3. In Section 4, the validity of the obtained results is illustrated by an example. Finally, the paper is concluded by Section 5.

2. **Problem Statement and Preliminaries.** Consider the following switched system:

$$\begin{aligned}\dot{x}(t) = &\left(A_{\sigma(t)} + \Delta A_{\sigma(t)}\right) x(t) + \left(B_{\sigma(t)} + \Delta B_{\sigma(t)}\right) x(t-h(t)) \\ &+F_{\sigma(t)}u(t) + G_{\sigma(t)}w(t) \quad 0 \le h(t) \le d, \quad 0 \le t,\end{aligned} \tag{1}$$

$$x(t) = \varphi(t) \quad t \in [-d, 0), \tag{2}$$

where $x(t) \in R^n$ represents system state; $\sigma(t) \in \underline{m}$ denotes the switching law; $\underline{m} = [1, \cdots, m]$; $A_{\sigma(t)}$, $B_{\sigma(t)}$, $F_{\sigma(t)}$ and $G_{\sigma(t)}$ are known matrices; $\Delta A_{\sigma(t)}$ and $\Delta B_{\sigma(t)}$ denote uncertainties satisfying $\left[\begin{array}{cc} \Delta A_{\sigma(t)} & \Delta B_{\sigma(t)} \end{array}\right] = D_{\sigma(t)} M_{\sigma(t)} \left[\begin{array}{cc} E_{\sigma(t)1} & E_{\sigma(t)2} \end{array}\right]$; $h(t)$ is time-varying delay; $u(t)$ represents system input; $w(t)$ is external disturbance; $\varphi(t)$ denotes the vector-valued initial function; $d$ is a positive constant. The desired feedback controller is written as:

$$u(t) = H_{\sigma(t)} x(t). \tag{3}$$

Our task is to choose appropriate gain matrix $H_{\sigma(t)}$ such that system (1) and (2) is finite-time bounded. A closed-loop system is obtained via combining (1), (2) and (3).

$$\begin{aligned}\dot{x}(t) = &\left(A_{\sigma(t)} + \Delta A_{\sigma(t)} + F_{\sigma(t)} H_{\sigma(t)}\right) x(t) + \left(B_{\sigma(t)} + \Delta B_{\sigma(t)}\right) x(t-h(t)) \\ &+G_{\sigma(t)}w(t) \quad 0 \le h(t) \le d, \quad 0 \le t,\end{aligned} \tag{4}$$

$$x(t) = \varphi(t) \quad t \in [-d, 0). \tag{5}$$

Some assumptions, definitions and lemmas are firstly introduced.

**Assumption 2.1.** *System (1) and (2) is a continuous system with $w^T(t)w(t) < \gamma$.*

**Assumption 2.2.** *For time-varying delay in system (1) and (2), $\dot{h}(t) \le \rho$, $0 \le h(t) \le d$, $\rho \le 1$.*

**Definition 2.1.** [9] *For given positive constants $C_1 < C_2$, $T_f$, $\gamma$ and switching signal $\sigma(t)$, if $\|x(t_0)\| \le C_1 \Rightarrow \|x(t)\| < C_2$ for $t \in [0, T_f]$, then system (1) and (2) is finite-time bounded with $(C_1, C_2, T_f, \sigma(t))$, where $\|x\| = \sum_{i=1}^{n} x_i^2$; $x_i$ is the ith element of $x$. Specify $C_1 = \sup\limits_{t\in[-d,0)} \|x(t)\|$.*

**Definition 2.2.** *For $T \ge t \ge 0$, $N_{\sigma(t)}(t, T)$ denotes the switching number of $\sigma(t)$ over $(t, T]$. If $N_{\sigma(t)}(t, T) \le N_0 + \frac{T-t}{\tau_a}$ holds for $\tau_a \ge 0$ and an integer $N_0 \ge 0$, then $\tau_a$ is called as average dwell-time [3].*

**Lemma 2.1.** [10] *For matrices $D$, $E$, and symmetric matrix $Y$, $Y + DFE + E^T F^T D^T < 0$ holds for $F^T F \le I$ if and only if there exists $\varepsilon > 0$ such that $Y + \varepsilon DD^T + \varepsilon^{-1} E^T E < 0$.*

3. **Main Results.**

**Lemma 3.1.** *For given $d > 0$, $\lambda > 0$, and $\rho \le 1$, if there exist positive symmetric matrices $P_i$, $Q_i$, $R_i$, $J$ and positive scalars $a$ and $b$ such that*

$$\left[\begin{array}{ccccc} \psi_{11} & P_i B_i & P_i G_i & E_{i,1}^T b & P_i \\ * & (\rho-1)Q_i & 0 & E_{i,2}^T b & 0 \\ * & * & J & 0 & 0 \\ * & * & * & (a-2b)I & 0 \\ * & * & * & * & -aI \end{array}\right] < 0, \tag{6}$$

*then*

$$\begin{bmatrix} \psi_{11} & P_iB_i & P_iG_i & E_{i,1}^T & P_i \\ * & (\rho-1)Q_i & 0 & E_{i,2}^T & 0 \\ * & * & J & 0 & 0 \\ * & * & * & -\varepsilon I & 0 \\ * & * & * & * & -\varepsilon^{-1}I \end{bmatrix} < 0, \tag{7}$$

*where* $\psi_{11} = (A_i + F_iH_i)^TP_i + P_i(A_i + F_iH_i) + Q_i + dR_i - \lambda P_i$.

**Proof:** Since $a > 0$, $(a-b)^2a^{-1} \geq 0$. Furthermore, $a - 2b \geq -ba^{-1}b$. Then from (6), we get

$$\begin{bmatrix} \psi_{11} & P_iB_i & P_iG_i & E_{i,1}^Tb & P_i \\ * & (\rho-1)Q_i & 0 & E_{i,2}^Tb & 0 \\ * & * & J & 0 & 0 \\ * & * & * & -ba^{-1}bI & 0 \\ * & * & * & * & -aI \end{bmatrix} < 0. \tag{8}$$

Pre-multiplying $diag\{I,\ I,\ I,\ b^{-1},\ I\}$ and post-multiplying $diag\{I,\ I,\ I,\ b^{-1},\ I\}$ to (8) yield

$$\begin{bmatrix} \psi_{11} & P_iB_i & P_iG_i & E_{i,1}^T & P_i \\ * & (\rho-1)Q_i & 0 & E_{i,2}^T & 0 \\ * & * & J & 0 & 0 \\ * & * & * & -a^{-1}I & 0 \\ * & * & * & * & -aI \end{bmatrix} < 0. \tag{9}$$

If letting $a = \varepsilon^{-1}$, then inequality (7) is obtained. This completes the proof.

**Remark 3.1.** *Since $\varepsilon^{-1}$ exists in inequality (7), inequality (7) is not LMIs. Lemma 3.1 could successfully convert inequality (7) into inequality (6) which is LMIs. By introducing positive scalars a and b, inequality (7) is transformed into LMIs. The whole transformation process is very concise.*

**Theorem 3.1.** *For given positive constants $C_1$, $C_1 < C_2$, $T_f$, $\gamma$, $\rho \leq 1$, $\lambda$, $d$, and $\beta > 1$, if there exist positive symmetric matrices $P_i$, $Q_i$, $R_i$, $J$, and matrix $K_i$, and positive scalars a and b such that*

$$\begin{bmatrix} \psi'_{11} & P_iB_i & P_iG_i & E_{i,1}^Tb & P_i \\ * & (\rho-1)Q_i & 0 & E_{i,2}^Tb & 0 \\ * & * & -J & 0 & 0 \\ * & * & * & (a-2b)I & 0 \\ * & * & * & * & -aI \end{bmatrix} < 0, \tag{10}$$

$$P_i \leq \beta P_j, \quad Q_i \leq \beta Q_j, \quad R_i \leq \beta R_j, \quad i \in \underline{m}, \quad j \in \underline{m}, \tag{11}$$

$$\tau_a > \frac{T_f \ln \beta}{\ln C_2 + \ln \lambda_5 + \ln(\Phi_1 + \Phi_2)}, \tag{12}$$

*then under the controller $u(t) = H_ix(t)$, system (4) and (5) is finite-time bounded under the switching law satisfying (12), where $\Phi_1 = e^{2\lambda T_f}\lambda_4T_f\gamma$, $\Phi_2 = e^{\lambda T_f}(\lambda_1 + d\lambda_2e^{\lambda d} + d^2\lambda_3e^{\lambda d})C_1$, $\psi'_{11} = A_i^TP_i + K_i^T + P_iA_i + K_i + Q_i + dR_i - \lambda P_i$, $H_i = F_i^{-1}P_i^{-1}K_i$, $\lambda_1 = \max\lambda(P_i)$, $\lambda_2 = \max\lambda(Q_i)$, $\lambda_3 = \max\lambda(R_i)$, $\lambda_4 = \max\lambda(J)$, $\lambda_5 = \min\lambda(P_i)$, $\max\lambda(P_i)$ denotes the maximum eigenvalue of $P_i$, and $\min\lambda(P_i)$ represents the minimum eigenvalue of $P_i$.*

**Proof:** Construct multiply Lyapunov Krasovskii function for system (4) and (5).

$$V_i(t) = x^T(t)P_ix(t) + \int_{t-h(t)}^{t} e^{\lambda(t-s)}x^T(s)Q_ix(s)ds + \int_{-d}^{0}\int_{t+\theta}^{t} e^{\lambda(t-s)}x^T(s)R_ix(s)dsd\theta. \tag{13}$$

Take the derivative of $V_i(t)$ along the trajectory of system (4) and (5) on $t \in [t_k, t_{k+1})$.

$$\begin{aligned}
\dot{V}_i(t) &= \lambda V_i(t) + \dot{x}^T(t)P_i x(t) + x^T(t)P_i\dot{x}(t) - \lambda x^T(t)P_i x(t) + x^T(t)Q_i x(t) \\
&\quad - \left(1-\dot{h}(t)\right)e^{\lambda h(t)}x^T(t-h(t))Q_i x(t-h(t)) + dx^T(t)R_i x(t) \\
&\quad - \int_{t-d}^{t} e^{\lambda(t-s)}x^T(s)R_i x(s)ds \\
&\le \lambda V_i(t) + \dot{x}^T(t)P_i x(t) + x^T(t)P_i\dot{x}(t) + x^T(t)(Q_i + dR_i - \lambda P_i)x(t) \\
&\quad -(1-\rho)x^T(t-h(t))Q_i x(t-h(t)) \\
&= \lambda V_i(t) + \begin{bmatrix} x(t) \\ x(t-h(t)) \\ w(t) \end{bmatrix}^T \begin{bmatrix} \psi_{11} & P_i(B_i+\Delta B_i) & P_iG_i \\ * & (\rho-1)Q_i & 0 \\ * & * & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ w(t) \end{bmatrix} \\
&= \lambda V_i(t) + \begin{bmatrix} x(t) \\ x(t-h(t)) \\ w(t) \end{bmatrix}^T \left\{ \begin{bmatrix} \psi'_{11} & P_iB_i & P_iG_i \\ * & (\rho-1)Q_i & 0 \\ * & * & 0 \end{bmatrix} + \begin{bmatrix} P_i \\ 0 \\ 0 \end{bmatrix} M_i \begin{bmatrix} E_1^T \\ E_2^T \\ 0 \end{bmatrix}^T \right. \\
&\quad \left. + \begin{bmatrix} E_1^T \\ E_2^T \\ 0 \end{bmatrix} M_i^T \begin{bmatrix} P_i \\ 0 \\ 0 \end{bmatrix}^T \right\} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ w(t) \end{bmatrix}.
\end{aligned} \tag{14}$$

According to Lemma 2.1, Lemma 3.1 and (10), we get

$$\begin{aligned}
\dot{V}_i(t) &\le \lambda V + w^T(t)Jw(t), \\
\frac{d}{dt}(e^{-\lambda t}V_i(t)) &< e^{-\lambda t}w^T(t)Jw(t).
\end{aligned} \tag{15}$$

Let $t_k$ denote the instant of the $K$th switching and $t_{k^-}$ denote the instant just before $t_k$. Integrate from $t_k$ to $t$ on both sides of (15).

$$\begin{aligned}
V_i(t) &< e^{\lambda(t-t_k)}V_i(t_k) + \int_{t_k}^{t} w^T(s)e^{\lambda(t-s)}Jw(s)ds \\
V_i(t) &< \beta e^{\lambda(t-t_k)}V_i(t_{k^-}) + \int_{t_k}^{t} w^T(s)e^{\lambda(t-s)}Jw(s)ds \\
V_i(t) &< \beta^2 e^{\lambda(t-t_{k-1})}V_i\left(t_{(k-1)^-}\right) + \beta e^{\lambda(t-t_k)}\int_{t_{k-1}}^{t_k} w^T(s)e^{\lambda(t-s)}Jw(s)ds \\
&\quad + \int_{t_k}^{t} w^T(s)e^{\lambda(t-s)}Jw(s)ds.
\end{aligned} \tag{16}$$

Assuming the switching times of $\sigma(t)$ over $[0, T_f]$ is $N$. By iterative calculation, it follows

$$\begin{aligned}
V_i(t) &< \beta^N e^{\lambda t}V_i(0) + \beta^N e^{\lambda(t-t_1)}\int_0^{t_1} w^T(s)e^{\lambda(t-s)}Jw(s)ds + \cdots \\
&\quad + \beta e^{\lambda(t-t_k)}\int_{t_{k-1}}^{t_k} w^T(s)e^{\lambda(t-s)}Jw(s)ds + \int_{t_k}^{T_f} w^T(s)e^{\lambda(t-s)}Jw(s)ds \\
&\le \beta^N e^{\lambda t}V_i(0) + \beta^N e^{\lambda t}\int_0^{t_1} w^T(s)e^{\lambda(t-s)}Jw(s)ds + \cdots \\
&\quad + \beta^N e^{\lambda t}\int_{t_{k-1}}^{t_k} w^T(s)e^{\lambda(t-s)}Jw(s)ds + \int_{t_k}^{T_f} w^T(s)e^{\lambda(t-s)}Jw(s)ds \\
&= \beta^N e^{\lambda T_f}V_i(0) + \beta^N e^{\lambda T_f}\int_0^{t} w^T(s)e^{\lambda(t-s)}Jw(s)ds \\
&< \beta^N e^{\lambda T_f}V_i(0) + \beta^N e^{2\lambda T_f}\lambda_{\max}(J)T_f\gamma.
\end{aligned} \tag{17}$$

$$\begin{aligned}
V_i(0) &= x^T(0)P_i x(0) + \int_{-h(0)}^{0} e^{-\lambda s}x^T(s)Q_i x(s)ds + \int_{-d}^{0}\int_{\theta}^{0} e^{-\lambda s}x^T(s)R_i x(s)dsd\theta \\
&\le \lambda_{\max}(P_i)\sup_{-d\le t\le 0}\left\{x^T(t)x(t)\right\} + d\lambda_{\max}(Q_i)e^{\lambda d}\sup_{-d\le t\le 0}\left\{x^T(t)x(t)\right\} \\
&\quad + d^2\lambda_{\max}(R_i)e^{\lambda d}\sup_{-d\le t\le 0}\left\{x^T(t)x(t)\right\} \\
&= \lambda_1 C_1 + d\lambda_2 e^{\lambda d}C_1 + d^2\lambda_3 e^{\lambda d}C_1.
\end{aligned} \tag{18}$$

$$V_i(t) < \beta^N e^{\lambda T_f} \left(\lambda_1 + d\lambda_2 e^{\lambda d} + d^2 \lambda_3 e^{\lambda d}\right) C_1 + \beta^N e^{2\lambda T_f} \lambda_4 T_f \gamma. \tag{19}$$

$$V_i(t) > x^T(t) P_i x(t) \geq \lambda_{\min}(P_i) x^T(t) x(t) \geq \min_{i \in M}\{\lambda_{\min}(P_i)\} x^T(t) x(t). \tag{20}$$

Combining (19) and (20), it is obtained that

$$x^T(t)x(t) < \frac{\beta^N e^{\lambda T_f} \left(\lambda_1 + d\lambda_2 e^{\lambda d} + d^2 \lambda_3 e^{\lambda d}\right) C_1 + \beta^N e^{2\lambda T_f} \lambda_4 T_f \gamma}{\lambda_5}. \tag{21}$$

On the other hand, it is inferred from (12) that

$$N < \frac{\ln C_2 + \ln \lambda_5 + \ln(\Phi_1 + \Phi_2)}{\ln \beta},$$

$$\beta^N(\Phi_1 + \Phi_2) < C_2 \lambda_5,$$

$$\frac{\beta^N e^{\lambda T_f} (\lambda_1 + d\lambda_2 e^{\lambda d} + d^2 \lambda_3 e^{\lambda d}) C_1 + \beta^N e^{2\lambda T_f} \lambda_4 T_f \gamma}{\lambda_5} < C_2. \tag{22}$$

Therefore, $x^T(t)x(t) < C_2$. The proof of Theorem 3.1 is completed.

**Remark 3.2.** *For inequality (14), if we only apply Lemma 2.1 to process it, the sufficient conditions on the existence of robust stabilization controller would also be obtained. However, the conditions must be not LMIs which imply they could not be solved by MATLAB. Therefore, we simultaneously apply Lemmas 2.1 and 3.1 to processing inequality (14). In this way, the sufficient conditions could be presented in form of LMIs.*

## 4. Numerical Example.

Consider a switched system with

**Subsystem 1:** $A1 = \begin{bmatrix} 0.2 & -0.4 \\ 0 & -0.4 \end{bmatrix}$, $B1 = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.3 \end{bmatrix}$, $F1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $G1 = \begin{bmatrix} 0.1 & -0.2 \\ 0 & 0.1 \end{bmatrix}$, $D1 = \begin{bmatrix} 0.1 & 0.2 \\ -0.2 & -0.1 \end{bmatrix}$, $M1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $E11 = \begin{bmatrix} -0.1 & 0.1 \\ -0.2 & 0.1 \end{bmatrix}$, $E12 = \begin{bmatrix} -0.1 & -0.1 \\ 1 & 0.1 \end{bmatrix}$;

**Subsystem 2:** $A2 = \begin{bmatrix} -0.1 & 0.1 \\ 0.1 & -0.3 \end{bmatrix}$, $B2 = \begin{bmatrix} 0.1 & 0 \\ 0.2 & 0.1 \end{bmatrix}$, $F2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$, $G2 = \begin{bmatrix} 0.2 & -0.1 \\ 0.6 & 0.1 \end{bmatrix}$, $D2 = \begin{bmatrix} -0.2 & 0.1 \\ 0.2 & 0.1 \end{bmatrix}$, $M2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $E21 = \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0.1 \end{bmatrix}$, $E22 = \begin{bmatrix} 0.1 & 0.1 \\ 1 & 0.1 \end{bmatrix}$;

**Time delay and initial function:** $\varphi(t) = [8 \quad 2]^T$, $h(t) = 0.5t$.

By given conditions, we get $\rho = 0.5$, $d = 5$, $C_1 = 68$. Specify $\lambda = 0.1$, $\beta = 1.1$, $T_f = 10$, $\gamma = 1$, $C_2 = 200$. Substituting these parameters into (10), (11) and (12) leads to $H1 = \begin{bmatrix} -0.656 & 0.1734 \\ 0.1860 & -0.1714 \end{bmatrix}$, $H2 = \begin{bmatrix} -0.2619 & -1.2313 \\ -1.3706 & -0.0821 \end{bmatrix}$, and $\tau_a > 0.0989$.

The simulation results of this example are shown in Figures 1 and 2. In Figure 1, the switching number is 38 over $t \in [0 \quad 10]$. Thus, $\tau_a = \frac{10}{38} > 0.0989$. In Figure 2, the curve of $\|x(t)\|$ is oscillatory, but $\|x(t)\| < 200 = C_2$ for $t \in [0 \quad 10]$. Therefore, under the designed controller (3), system (1) and (2) is finite time bounded. The validity of Theorem 3.1 is illustrated by the simulation experiment. Besides, although the system is finite time bounded, it could be inferred from Figure 2 that the system is not stable over $t \in [0 \quad 10]$. Thus, FTB is different from asymptotic stability. There are not any causal relationships between them.

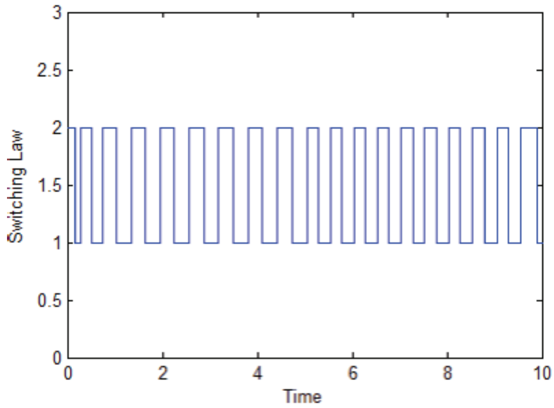


FIGURE 1. Switching law

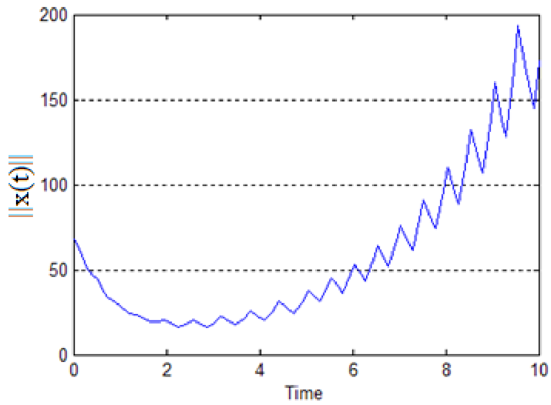


FIGURE 2. The function of $\|x\|$

5. **Conclusions.** In this paper, the problem of FTS is investigated for uncertain switched system by utilizing multiply Lyapunov Krasovskii function and average dwell time approach. The sufficient conditions on the existence of robust stabilization controller are established and are presented in form of LMIs by using a novel method. In simulation experiment, a robust FTS controller is successfully constructed such that system (1) and (2) is finite time bounded. Thus, the validity of obtained result is illustrated. In this paper, the time delay in system just exists in system state. However, time delay may exist in system state and switching law simultaneously in practice. This case would lead to great changes in switching instants and system state. The issue will be investigated in the future work.

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## REFERENCES

[1] D. Liberzon, *Switching in Systems and Control*, Springer Verlag, Boston, MA, 2003.
[2] Z. Li, Y. Soh and C. Wen, *Switched and Impulsive Systems: Analysis, Design, and Applications*, Springer-Verlag, Berlin, Germany, 2005.
[3] X. Lin, S. Li and Y. Zhou, Finite-time stability of switched linear systems with subsystems which are not finite-time stable, *IET Control Theory and Applications*, vol.8, no.12, pp.1137-1146, 2014.
[4] X. Lin, H. Du and S. Li, Finite-time boundedness and $L_2$ gain analysis for switched delay systems with norm-bounded disturbance, *Applied Mathematics and Computation*, vol.217, pp.5982-5993, 2011.
[5] M. Xiang and Z. Xiang, Finite-time $L_1$ control for positive switched linear systems with time-varying delay, *Commun Nonlinear Sci. Numer. Simulat.*, vol.18, pp.3158-3166, 2013.
[6] H. Liu, Y. Shen and X. Zhao, Finite-time stabilization and boundedness of switched linear system under state-dependent switching, *Journal of the Franklin Institute*, vol.350, pp.541-555, 2013.
[7] Y. Zhong and T. Chen, Finite-time boundedness analysis for a class of switched linear systems with time-varying delay, *Abstract and Applied Analysis*, vol.2014, no.2, pp.1-9, 2014.
[8] X. Wei and J. Xiao, $H_\infty$ finite time control for switched nonlinear discrete-time systems with norm-bounded disturbance, *Journal of the Franklin Institute*, vol.348, pp.331-352, 2011.
[9] P. Dorato, An overview of finite-time stability, *Current Trends in Nonlinear Systems and Control*, Birkhauser Boston, pp.185-194, 2006.
[10] X. Zhao, L. Zhang, P. Shi and M. Liu, Stability of switched positive linear systems with average dwell time switching, *Automatica*, vol.48, no.6, pp.1132-1137, 2012.