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AN IMPROVED MULTIPLE EXTENDED TARGET TRACKING ALGORITHM BASED ON VARIATIONAL BAYESIAN CARDINALITY EQUILIBRIUM MULTI-OBJECTIVE BERNOULLI FILTERING

Jie Liu, Shoulin Yin* and Lin Teng

Software College
Shenyang Normal University
No. 253, Huanghe North Street, Huanggu District, Shenyang 110034, P. R. China
nan127@sohu.com; *Corresponding author: 352720214@qq.com; 1532554069@qq.com

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Abstract. *The tracking performance of traditional multiple extended target tracking algorithm declines sharply under the unknown measurement noise covariance. So this paper proposes an improved multiple extended target tracking algorithm based on variational Bayesian cardinality equilibrium multi-objective Bernoulli filtering (VBCEM-OB). Under the unknown measurement noise covariance conditions, the improved algorithm shows that the measurement is generated by the measurement manufacturer which distributes on the extended target randomly. The new scheme uses variational Bayesian method to approximatively solve state joint probability density and measurement noise covariance of each measurement producer. We can obtain its recursive form to estimate measurement producer. And we adopt clustering method to get the state of extended target for the state of measurement producer. Finally, simulation experiments show that the improved algorithm can track extended target with unknown number and unknown measurement noise covariance. Compared with traditional cardinality equilibrium multi-objective Bernoulli filtering (CEM-OB), the tracking precision of VBCEM-OB is improved further.*
Keywords: Multiple extended target tracking, Joint probability density, Variational Bayesian, Cardinality equilibrium multi-objective Bernoulli filtering, Recursive form

1. **Introduction.** In the traditional target tracking [1] field, we also regard target as a point target. Each target produces a measurement at most. In the practical application, we are unable to predict the number of target. So it needs to track the unknown number of multiple target. Multiple target tracking [2] algorithm based on random set theory has attracted widespread attention which does not have a complex data correlation. Many researchers have studied the multiple target tracking. Bocca et al. [3] combined real-time multiple target tracking with RF sensor networks and used RSS measurements on multiple frequency channels on each link, combining them with a fade level-based weighted average. Niedfeldt and Beard [4] proposed a recursive random sample consensus algorithm to robustly estimate the states of an unknown number of dynamic targets. CEM-OB can track multiple maneuvering targets and evolve to linear Gaussian observation model within a time.

In order to improve the tracking accuracy, we apply variational Bayesian method into cardinality equilibrium multi-objective Bernoulli tracking algorithm. Variational Bayesian [5,6] is an approximate calculation and complex integral method used for Bayesian estimation and machine learning which has a lower computation complexity compared with traditional sampling methods. After combining variational Bayesian with CEM-OB, we not only make parameterized approximation for joint posterior density of measurement noise covariance and measurement producers, but can also obtain the recursive form. This paper firstly introduces the system model of multi-extended target and variational Bayesian method at random set filtering framework. Then we deduce Gaussian closed solution of multi-extended target tracking under the unknown measurement noise covariance. Compared with traditional CEM-OB, the new scheme can estimate the measurement noise covariance adaptively. Also it can track the multi-extended target with a higher precision. The following are the structures of this paper. Section 2 introduces the fundamental theory. Section 3 represents the new algorithm's process and flow. Simulation and analysis are given in Section 4. The conclusions are drawn in Section 5.

2. **Multi-Extended Target System Based on Random Set Filtering Framework.** For extended target, we suppose that at k time, $N(k)$ targets, $M(k)$ measurement. State set X_k and measurement set Z_k are:

$$X_k = \{x_{k,1}, \ldots, x_{x,N(k)}\} \in F(X) \tag{1}$$

$$Z_k = \{z_{k,1}, \ldots, z_{x,M(k)}\} \in F(Z) \tag{2}$$

where $F(X)$ and $F(Z)$ are state space and measurement space respectively.

Set state equation and measurement equation of one extended target:

$$x_k = Fx_{k-1} + Gw_k \tag{3}$$

$$z_k^p = Hx_k^p + v_k \tag{4}$$

where x_{k-1} is the target state at $k-1$ time. F and G are state transition matrix and input matrix respectively. H is measurement matrix. w_k and v_k are process noise and measurement noise respectively. Their covariance is Q_k and R_k respectively. R_k is unknown. x_k^p is measurement producer. z_k^p is the measurement value at k time.

Assuming the random multi-extended target state set is X_{k-1} at $k-1$ time. And each $x_{k-1} \in X_{k-1}$ will exist next time with a probability of $p_{S,k}(x_{k-1})$. x_{k-1} can transfer to a new state x_k with a probability of $f_{k|k-1}(x_k|x_{k-1})$ or disappear with a probability of $1 - p_{S,k}(x_{k-1})$. So we can build Bernoulli random finite set for this state: $S_{k|k-1}(x_{k-1})$. Existence probability of Bernoulli set is $r = p_{S,k}(x_{k-1})$, and probability density function is $p(\cdot) = f_{k|k-1}(\cdot|x_{k-1})$. So we can obtain the random finite set of multi-extended target state:

$$X_k = \left(\bigcup_{x_{k-1} \in X_{k-1}} S_{k|k-1}(x_{k-1}) \right) \cup \Gamma_k \tag{5}$$

where Γ_k denotes multi-Bernoulli random finite set of new produced target.

One extended target has the state: $x_k \in X_k$. At k time, it has M_k measurement points x_k^p. $p = 1, 2, \ldots, M_k$. The sets of all the target measurement producer points are: X_k^p. Each point can be detected with a probability of $p_{D,k}(x_k^p)$ or missed detection with a probability of $1 - p_{D,k}(x_k^p)$. Each measurement producer point can generate Bernoulli random finite set: $\Theta_k(x_k^p)$. Existence probability of measurement producer point is $r = p_{D,k}(x_k^p)$, and probability density function is $p(\cdot) = g_k(\cdot|x_k^p)$. Due to the influence of clutter, sensor will produce some false measurement. They can be seen as Poisson random set K_k. So the random finite set of multi-extended target measurement is:

$$Z_k = \left(\bigcup_{x_k^p \in X_k^p} \Theta_k\left(x_k^p\right) \right) \cup K_k \tag{6}$$

(6) contains unpredictability and clutter of detection. If the random finite set is independent, Z_k is a multi-Bernoulli random finite set.

We need to joint estimate the probability density of x_k^p and R_k. Supposing their dynamic models is independent. The predicted joint probability density is:

$$p\left(x_k^p, R_k|z_{1:k-1}^p\right) = \int p\left(x_k^p|x_{k-1}^p\right) p\left(R_k|R_{k-1}\right) \times p\left(x_{k-1}^p, R_{k-1}|z_{1:k-1}^p\right) dx_{k-1}^p dR_{k-1} \tag{7}$$

where $p\left(x_k^p|x_{k-1}^p\right)$ can be obtained by dynamic equation of system. $p\left(R_k|R_{k-1}\right)$ is difficult to calculate. According to the Bayesian principle, the updated joint posterior probability density $p\left(x_k^p, R_k|z_{1:k}^p\right)$ can be expressed as:

$$p\left(x_k^p, R_k|z_{1:k}^p\right)=\frac{p\left(z_k^p|x_k^p, R_k\right)p\left(x_k^p, R_k|z_{1:k-1}^p\right)}{\int p\left(z_k^p|x_k^p, R_k\right)p\left(x_k^p, R_k|z_{1:k-1}^p\right)dx_k^p dR_k} \tag{8}$$

where $p\left(z_k^p|x_k^p, R_k\right)$ is likelihood function associated with R_k.

3. **Improved Multi-Extended Target Tracking Algorithm Based on VBCEM-OB and Gaussian Implementation Process.** Supposing measurement producer set is Bernoulli random set, joint posterior probability density of measurement producer point state x_k^p and R_k is parameter of Bernoulli random set. We use Gaussian inverse gamma distribution to approximate this joint posterior probability density. The measurement of every time conducts filtering for this distribution and gets the updated x_k^p. Then it makes cluster for x_k^p. The center point of this cluster is estimation state of extended target. The process is this paper's new algorithm.

The detailed processes of this improved algorithm are as follows.

1) Predicting.

At $k-1$ time, the posterior probability density function of measurement producer random set filter is expressed as:

$$\pi_{k-1}^{\left(x_{k-1}^p, R_{k-1}\right)}=\left\{\left(r_{k-1}^{(i)}, p_{k-1}^{(i)}\left(x_{k-1}^p, R_{k-1}\right)\right)\right\}_{i=1}^{M_{k-1}} \tag{9}$$

where $\pi_{k-1}^{\left(x_{k-1}^p, R_{k-1}\right)}$ denotes multi-Bernoulli parameter sets. So the probability density function of predicted measurement producer random set filter is:

$$\begin{aligned}&\pi_{k|k-1}^{\left(x_{k|k-1}^p, R_{k|k-1}\right)}\\&=\left\{\left(r_{P,k|k-1}^{(i)}, p_{P,k|k-1}^{(i)}(x_{k|k-1}^p, R_{k|k-1})\right)\right\}_{i=1}^{M_{k-1}}\bigcup\left\{\left(r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)}\left(x_k^p, R_k\right)\right)\right\}_{i=1}^{M_{\Gamma,k}}\end{aligned} \tag{10}$$

where

$$r_{P,k|k-1}^{(i)}=r_{k-1}^{(i)}\left\langle p_{k-1}^{(i)}\left(x_{k-1}^p, R_{k-1}\right), p_{S,k}\right\rangle \tag{11}$$

$$\begin{aligned}&p_{P,k|k-1}^{(i)}(x_{k|k-1}^p, R_{k|k-1})\\&=\left\langle f_{k|k-1}\left(x_{k|k-1}^p\middle|\cdot\right)p_{k|k-1}\left(R_{k|k-1}|\cdot\right), p_{k-1}^{(i)}\left(x_{k-1}^p, R_{k-1}\right)p_{S,k}\right\rangle \Big/ \left\langle p_{k-1}^{(i)}\left(x_{k-1}^p, R_{k-1}\right)p_{S,k}\right\rangle\end{aligned} \tag{12}$$

And $p_{S,k}$ denotes measurement producer survival probability, $f_{k|k-1}\left(x_{k|k-1}^p\middle|\cdot\right)$ is multi-extended target measurement producer state transition probability density function. $p_{k|k-1}\left(R_{k|k-1}|\cdot\right)$ is the dynamic transfer model of R_k. $\left\{\left(r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)}\left(x_k^p, R_k\right)\right)\right\}_{i=1}^{M_{\Gamma,k}}$ denotes multi-Bernoulli parameter sets of new target at k time.

2) Updating.

Supposing probability density function of measurement producer random set filter can be expressed by multi-Bernoulli parameter set:

$$\pi_{k|k-1}^{\left(x_{k|k-1}^p, R_{k|k-1}\right)}=\left\{\left(r_{k|k-1}^{(i)}, p_{k|k-1}^{(i)}\left(x_{k|k-1}^p, R_{k|k-1}\right)\right)\right\}_{i=1}^{M_{k|k-1}} \tag{13}$$

The posterior probability density function of updated measurement producer random set filter can be approximated as:

$$\pi_k^{\left(x_k^p, R_k\right)}\approx\left\{\left(r_{L,k}^{(i)}, p_{L,k}^{(i)}\left(x_k^p, R_k\right)\right)\right\}_{i=1}^{M_{k|k-1}}\bigcup\left\{\left(r_{U,k}^{*}(z_k^p), p_{U,k}^{*}\left(x_k^p, R_k; z_k^p\right)\right)\right\}_{z_k^p\in Z_k} \tag{14}$$

where

$$r_{L,k}^{(i)} = r_{k|k-1}^{(i)} \frac{1 - \left\langle p_{k|k-1}^{(i)}\left(x_{k|k-1}^{p}, R_{k|k-1}\right), p_{D,k}\right\rangle}{1 - r_{k|k-1}^{(i)}\left\langle p_{k|k-1}^{(i)}\left(x_{k|k-1}^{p}, R_{k|k-1}\right), p_{D,k}\right\rangle} \tag{15}$$

$$p_{L,k}^{(i)}\left(x_k^p, R_k\right) = p_{k|k-1}^{(i)}\left(x_{k|k-1}^{p}, R_{k|k-1}\right) \cdot \frac{1 - p_{D,k}}{1 - \left\langle p_{k|k-1}^{(i)}\left(x_{k|k-1}^{p}, R_{k|k-1}\right), p_{D,k}\right\rangle} \tag{16}$$

$p_{D,k}$ denotes detection probability. Z_k is observation set. $r_{U,k}^{*}\left(z_k^p\right)$ is clutter density function. Under the independent x_k^p and R_k conditions, $\eta_k\left(x_k^p, R_k; z\right)$ can be represented as a new expression: $\eta_k\left(x_k^p, R_k; z\right) \approx Q_x\left(x_k^p\right) Q_R\left(R_k\right)$. $Q_x\left(x_k^p\right)$ is Gaussian distribution. $Q_R(R_k)$ is product of the inverse gamma distribution, and their expression is as (13) and (14). Because R_k is unknown, we need to iterate and update R_k. That can correct measurement points state and covariance. Before iterating, we must set predicted value of current time as initial value. If the measurement producer error between previous iteration and current iteration is less than a certain value. The process will stop. And the current iteration value is regarded as updated measurement producer state.

4. **Simulation Experiment and Analysis.** Traditional CEM-OB and this paper's new scheme VBCEM-OB are compared for multi-extended target tracking under simulation implementation clutter background.

Suppose four target movement trajectories are uncrossed. Sampling period $T = 1$s. The whole process of observation lasts 40 sampling time. The target state equation and measurement equation are as Equations (3) and (4). Parameters setting is:

$$F = [1\ T\ 0\ 0, 0\ 1\ 0\ 0, 0\ 0\ 1\ T, 0\ 0\ 0\ 1]^T,\ G = [1/2\ 0, 1\ 0, 0\ 1/2, 0\ 1]^T,$$
$$H = [1\ 0\ 0\ 0, 0\ 0\ 1\ 0]^T.$$

Process noise covariance $Q_k = diag\left\{\sigma_{w1}^2, \sigma_{w2}^2\right\}$, $\sigma_{w1} = \sigma_{w2} = 0.5$. The number of measurement producer points follow a Poisson distribution. Mean value $\beta = 10$. New target random sets are expressed by multi-Bernoulli parameter sets $\pi_\Gamma = \left\{\left(r_\Gamma, p_\Gamma^{(i)}\right)\right\}_{i=1}^{3}$, and $r_\Gamma = 0.02$.

$$p_\Gamma^{(i)}(x, R) = N\left(x; m_\gamma^{(i)}, P_\gamma\right) \prod_{l=1}^{d} IG\left(\left(\sigma_{\gamma,l}^{(i)}\right)^2, \alpha_{\gamma,l}^{(i)}, \beta_{\gamma,l}^{(i)}\right), \quad i = 1, 2, 3 \tag{17}$$

where $m_\gamma^{(1)} = [40, 3.05, -40, 0.05]^T$, $m_\gamma^{(2)} = [-10, 3.2, -25, 0.08]^T$, $m_\gamma^{(3)} = [0, 5.8, -10, -0.06]^T$. $P_\gamma = diag\{2, 1, 2, 1\}$. Initial inverse gamma distribution is set as: $\alpha_0 = \beta_0 = 1$. Degenerate factor $\rho = 0.9$. Clutter modeling is Poisson random set, mean value $\lambda = 5$.

The survival probability and detection probability of target are $P_{S,k} = 0.99$ and $P_{D,k} = 0.98$ respectively. The real measurement noise standard deviation is $\sigma = \sigma_{v1} = \sigma_{v2} = 1$.

For the VBCEM-OB algorithm, $R_k = diag\left\{\sigma_{v1}^2, \sigma_{v2}^2\right\}$ is unknown. For CEM-OB, we make 100 Monte-Carlo simulation experiments independently and get the results to analysis when $\sigma = \sigma_{v1} = \sigma_{v2} = 0.5$, $\sigma = \sigma_{v1} = \sigma_{v2} = 1$, $\sigma = \sigma_{v1} = \sigma_{v2} = 8$.

Because this paper adopts multi-extended target tracking algorithm based on random set, this algorithm considers the corresponding relation between target set and measurement set. In this article, we adopt target number estimation mean value and optimal sub-pattern assignment distance to evaluate this algorithm. The optimal sub-pattern assignment distance can be expressed as:

$$\bar{d}_p^{(c)}(X, Z) = \left(\frac{1}{n}\left(\min_{\pi \in \Pi_n} \sum_{i=1}^{m} d^{(c)}\left(x_i, z_{\pi(i)}\right)^p + c^p(n - m)\right)\right)^{1/p} \tag{18}$$

And $p = 2$, $c = 60$. Figure 1 shows the simulation scene. We can know the movement of target and measurement of clutter in a single experiment. Figure 2 shows the result of GM-VBCEM-OB (Gaussian Mixture-VBCEM-OB) filter target state. Figure 3 and Figure 4 are the estimation target number comparison of GM-CEM-OB (Gaussian Mixture-CEM-OB) and GM-VBCEM-OB and comparison of optimal sub-pattern assignment distance with different measurement noise covariance respectively. When $\sigma = 1$, GM-CEM-OB and GM-VBCEM-OB have the same estimation results. They are also close to the true value. When $\sigma = 0.8$, GM-CEM-OB can produce overestimation. Because measurement noise covariance is smaller than true value, it results in that measurement producer points from one target cannot be clustered. And the optimal sub-pattern assignment distance will increase. When $\sigma = 8$, GM-CEM-OB can produce undervaluation due to the big error of measurement noise covariance. When $k = 8$s, $k = 12$s, $k = 26$s, tracking error of CEM-OB is bigger. Because new target at the first moment has a low existence probability, GM-CEM-OB confirms that there exists target at this position when existence probability is greater than a certain threshold. So tracking the new target will delay a moment. However, GM-VBCEM-OB can revise measurement producer points existence probability and not generate delay due to a process of iterative estimation.

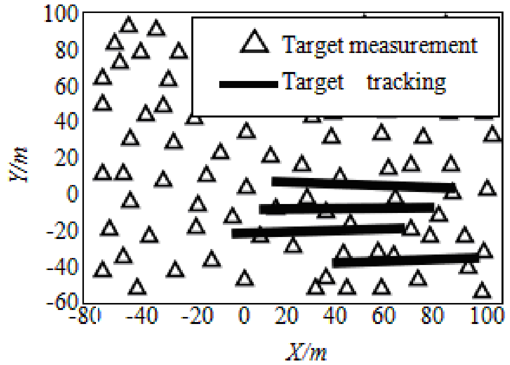

FIGURE 1. Target measurement and tracks

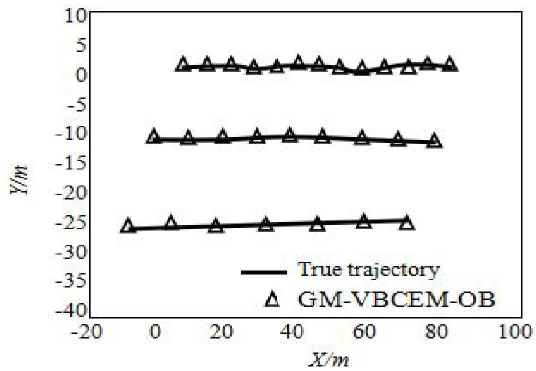

FIGURE 2. GM-VBCEM-OB filter estimation

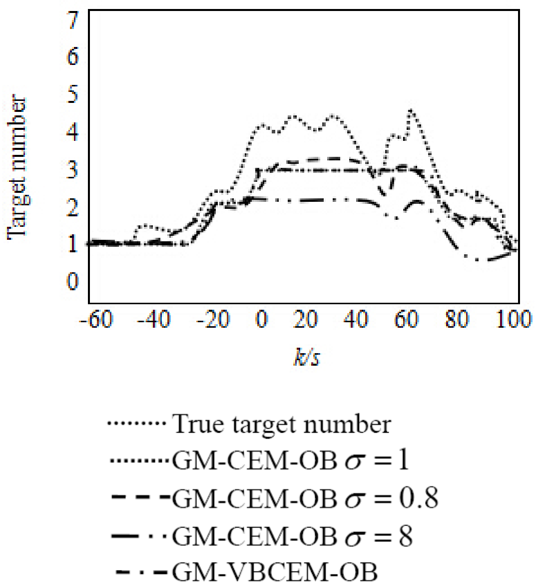

FIGURE 3. Comparison of target estimation number

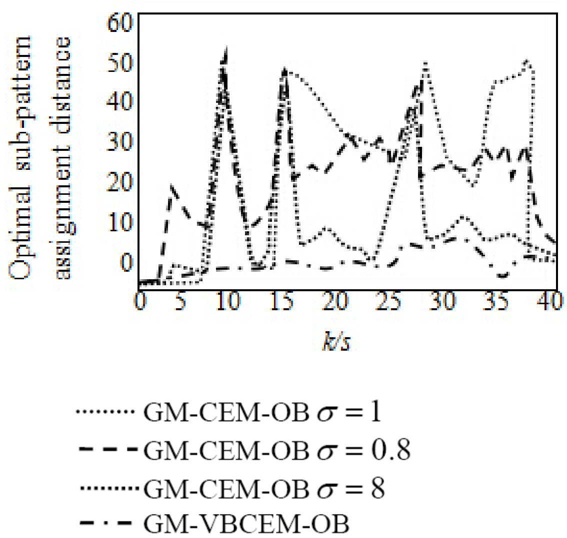

FIGURE 4. Comparison of distance

We can conclude that when the deviation of measurement noise covariance and the real value is very large, filter precision of GM-CEM-OB declines sharply. However, GM-VBCEM-OB not only adaptively estimates measurement noise covariance but can track multiple extended targets with a higher precision.

5. **Conclusions.** Extended target tracking is different from point target tracking. Multi-measurement may be from the same target. So we must estimate measurement producer point state. This paper proposes a new multi-extended target tracking algorithm and gives Gaussian implementation under the unknown measurement noise covariance and clutter tracking conditions. This new algorithm conducts joint estimation for probability density of measurement noise covariance and measurement producer points state. We use variational Bayesian method to approximate this joint probability density. It can get measurement producer points state after filter updating. The experiments show that this new algorithm can track multi-extended target effectively. In the future, we will study the crossed trajectory based on this paper's method to solve the target number undervaluation problem.

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