*ICIC Express Letters*
*Part B: Applications*
Volume **7**, Number **1**, January **2016**


pp. 189–195

# EVALUATING THE AGGREGATIVE RISK RATE IN SOFTWARE DEVELOPMENT BASED ON LEVEL $(\lambda, 1)$ INTERVAL-VALUED FUZZY NUMBERS

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Received June 2015; accepted August 2015

Abstract. *Since we could not know the error between the evaluated value $v_{ijq}^{(k)}$ and the population objective value $V_{ijq}^{(k)}$ (unknown), we therefore cannot take the confidence level as 1, that is, we cannot take the membership grade of $\widetilde{v}_{ijq}^{*(k)}$ at $v_{ijq}^{(k)}$ as 1. Thus, it should be more reasonable to consider the membership grade of $\widetilde{v}_{ijq}^{*(k)}$ within the interval $(\lambda, 1)$, $0 < \lambda < 1$. In this study, we presented level $(\lambda, 1)$ interval-value fuzzy numbers and compositional inference rule to tackle the aggregative risk rate in the fuzzy sense.*
**Keywords:** Level $(\lambda, 1)$ interval-valued fuzzy number, Signed distance, Aggregative risk rate

1. **Introduction.** During the past decades, computer technologies have changed so fast that the need of large software system becomes much more intensive. There will be many problems occuring in the software system development life cycle, such as postponed schedule, increased cost, inefficiency and abandonment [9]. The risk evaluation and management is an important issue. Up to now, there are many authors investing risk identification, risk analysis [1-6,17], and tackling the rate of aggregative risk [9,11-14].

Due to the complexity of risk factors and the compounding uncertainty associated with future sources of risk, risk is normally not treated with mathematical rigor during the early life cycle phases [1]. Lee [9] classified the risk factors [1-6,17] into six attributes, divided each attribute into some risk items, and built up the hierarchical structured model of aggregative risk and the evaluating procedure of structured model, and proposed the procedure to evaluate the rate of aggregative risk using two stages fuzzy assessment method. In [11], Lee et al. showed that the computing result will have some errors unless the triangular or trapezoid is isosceles and presented the other algorithm to evaluate the rate of aggregative risk. Lee and Lin [12] presented the other methods to tackle the risk rate in software development in fuzzy circumstances. Lee et al. [13] presented the fuzzy sense of interval value $[m_{ij} - \Delta_{ij1}, m_{ij} + \Delta_{ij2}]$ of $m_{ij}$ instead of single value $m_{ij}$ on assessment for the sub-item $X_{ij}$ to do the rate of aggregated risk. Lee and Lin [14] presented the evaluating risk rate based on the statistical confidence interval and defuzzified by the signed distance method.

Based on [10,15], we use the level $(\lambda, 1)$ interval-valued fuzzy numbers and compositional inference rule to evaluate the aggregative risk rate.

Section 2 presents some properties of fuzzy sets. Section 3 is the proposed method. We make conclusions in Section 4.

2. **Preliminaries.** This section contains some definitions and propositions used in Section 3, [7,8,10,15,16,18,19].

**Definition 2.1.** *A fuzzy set $\widetilde{A}$ defined on $R$ is called the level $\lambda$ triangular fuzzy number if its membership function is*

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{\lambda(x-a)}{b-a}, & a \le x \le b \\ \dfrac{\lambda(c-x)}{c-b}, & b \le x \le c \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

*where $a < b < c$, $0 < \lambda \le 1$, then $\widetilde{A}$ is called the level $\lambda$ fuzzy number and denoted by $\widetilde{A} = (a,b,c;\lambda)$. If $\lambda = 1$, $\widetilde{A}$ is called a normal triangular fuzzy number and denoted by $\widetilde{A} = (a,b,c)$.*

**Definition 2.2.** **[7]** *Suppose that $\widetilde{B}^L = (a,b,c;\lambda)$, $\widetilde{B}^U = (p,b,r;\rho)$, where $p < a < b < c < r$, $a$, $b$, $c$, $p$, $r \in R$, $0 < \lambda < \rho \le 1$. Let $\widetilde{B} = \left[\widetilde{B}^L, \widetilde{B}^U\right]$. $\widetilde{B}$ is called a level $(\lambda,\rho)$ interval-valued fuzzy number.*

The $\alpha$-level set of $\widetilde{B} = [(a,b,c;\lambda),(p,b,r;\rho)]$ is defined as follows. If $0 \le \alpha \le \lambda$, $\alpha$-level set of $\widetilde{B}$ is defined as

$$B(\alpha) = \left\{x|\mu_{\widetilde{B}^U}(x) \ge \alpha\right\} - \left\{x|\mu_{\widetilde{B}^L}(x) > \alpha\right\} = \left[\widetilde{B}_l^U(\alpha), \widetilde{B}_l^L(\alpha)\right] \cup \left[\widetilde{B}_r^L(\alpha), \widetilde{B}_r^U(\alpha)\right] \tag{2}$$

where

$$\begin{aligned} \widetilde{B}_l^L(\alpha) &= a + (b-a)\frac{\alpha}{\lambda},\ \widetilde{B}_r^L(\alpha) = c - (c-b)\frac{\alpha}{\lambda} \\ \widetilde{B}_l^U(\alpha) &= p + (b-p)\frac{\alpha}{\rho},\ \widetilde{B}_r^U(\alpha) = r - (r-b)\frac{\alpha}{\rho} \end{aligned} \tag{3}$$

If $\lambda \le \alpha \le \rho$, then

$$B(\alpha) = \left[\widetilde{B}_l^U(\alpha), \widetilde{B}_r^U(\alpha)\right] \tag{4}$$

$$\widetilde{B}_l^U(\alpha) = p + (b-p)\frac{\alpha}{\rho},\ \widetilde{B}_r^U(\alpha) = r - (r-b)\frac{\alpha}{\rho} \tag{5}$$

According to decomposition theory [18,19] and (3)-(5), $\widetilde{B}$ can be denoted as follows:

$$\widetilde{B} = \bigcup_{0 \le \alpha < \lambda} \left(\left[\widetilde{B}_l^U(\alpha), \widetilde{B}_l^L(\alpha); \alpha\right] \cup \left[\widetilde{B}_r^L(\alpha), \widetilde{B}_r^U(\alpha); \alpha\right]\right) \cup \bigcup_{\lambda \le \alpha \le 1} \left[\widetilde{B}_l^U(\alpha), \widetilde{B}_r^U(\alpha); \alpha\right] \tag{6}$$

Let $F_{IV}(\lambda,\rho) = \{[(a,b,c;\lambda),(p,b,r;\rho)]|p < a < b < c < r,\ a,b,c \in R\}$, $0 < \lambda < \rho \le 1$.

As mentioned by [16], when the signed distance and ranking of level $(\lambda,\rho)$ interval-valued fuzzy number on $F_{IV}(\lambda,\rho)$ are taken into consideration, we should first illustrate the signed distance [18] as follows.

**Definition 2.3.** **[18]** *For each $a$, $0 \in R$ we define the signed distance from $a$ to $0$ by $d_0(a,0) = a$.*

**Definition 2.4.** **[10,18]** *(a) Let $\widetilde{B} = [(a,b,c;\lambda),(p,b,r;\rho)] \in F_{IV}(\lambda,\rho)$, the signed distance from $\widetilde{B}$ to $\widetilde{0}$ is defined by*

$$d\left(\widetilde{B},\widetilde{0}\right) = \frac{1}{16}\left[6b + a + c + 4p + 4r + 3(2b-p-r)\frac{\lambda}{\rho}\right] \tag{7}$$

*(b) The signed distance of the $\rho$ triangular fuzzy number $\widetilde{A} = (p, b, r; \rho)$ is defined by*

$$d\left(\widetilde{A}, \widetilde{0}\right) = \frac{1}{2\rho}\int_0^{\rho}\left(\widetilde{A}_l(\alpha) + \widetilde{A}_r(\alpha)\right)d\alpha = \frac{1}{4}(2b + p + r) \tag{8}$$

**Definition 2.5.** *For $\widetilde{B} = [(a, b, c; \lambda), (p, b, r; \rho)]$, $\widetilde{C} = [(u, v, w; \lambda), (g, v, h; \rho)] \in F_{IV}$, the level $(\lambda, \rho)$ interval-valued fuzzy number $\widetilde{B} \oplus \widetilde{C}$ and $k\widetilde{B}$ are defined as follows:*

$$\widetilde{B} \oplus \widetilde{C} = [(a + u, b + v, c + w; \lambda), (p + g, b + v, r + h; \rho)]$$

$$k\widetilde{B} = \begin{cases} [(ka, kb, kc; \lambda), (kp, kb, kr; \rho)], & k > 0 \\ [(kc, kb, ka; \lambda), (kr, kb, kp; \rho)], & k < 0 \end{cases}$$

Using Definitions 2.4 and 2.5, we have the following proposition.

**Proposition 2.1.** *If $\widetilde{B}$, $\widetilde{C} \in F_{IV}(\lambda, \rho)$, $k \in R$, then we have*

$$d\left(\widetilde{B} \oplus \widetilde{C}, \widetilde{0}\right) = d\left(\widetilde{B}, \widetilde{0}\right) + d\left(\widetilde{C}, \widetilde{0}\right)$$
$$d\left(k\widetilde{B}, \widetilde{0}\right) = kd\left(\widetilde{B}, \widetilde{0}\right).$$

3. **The Proposed Method.** We present the fuzzy assessment method as follows.

Step 1: Assessment form for the risk items:

Based on the hierarchical structure model of aggregative risk and contents of structure model in [9,11-14], we proposed the assessment form in Table 1 and the algorithm as follows.

Referring to [9,11-14], the criteria ratings of risk are linguistic variables with linguistic values $V_1, V_2, \ldots, V_7$, where $V_1$ = extra low, $V_2$ = very low, $V_3$ = low, $V_4$ = middle, $V_5$ = high, $V_6$ = very high, and $V_7$ = extra high.

These linguistic values are treated as triangular fuzzy numbers as follows:

$$\widetilde{V}_1 = \left(0, 0, \frac{1}{6}\right),$$
$$\widetilde{V}_k = \left(\frac{k-2}{6}, \frac{k-1}{6}, \frac{k}{6}\right), \text{ for } k = 2, 3, \ldots, 6$$
$$\widetilde{V}_7 = \left(\frac{5}{6}, 1, 1\right)$$

In Table 1, $W_2(i)$ is the weight of the risk attribute $X_i$, for $i = 1, 2, \ldots, 6$, and satisfies

$$0 \leq W_2(i) \leq 1, \quad \sum_{i=1}^{6} W_2(i) = 1 \tag{9}$$

$W_1(i, j)$ is the weight of the risk item $X_{ij}$, and satisfies

$$0 \leq W_1(i, j) \leq 1, \quad \sum_{j=1}^{n_i} W_1(i, j) = 1 \tag{10}$$

for $i = 1, 2, \ldots, 6$; $j = 1, 2, \ldots, n_i$; $n_1 = 1$, $n_2 = 4$, $n_3 = 2$, $n_4 = 4$, $n_5 = 2$, $n_6 = 1$.

Step 2: Suppose that there are $r$ experts to evaluate the aggregative risk rate in software development.

Let $v_{ijq}^{(k)} \in [0, 1]$ be the assessment for the sub-item $X_{ij}$ given by the evaluator $q$ with respective to the criteria $V_k$, where $i = 1, 2, \ldots, 6$, $j = 1, 2, \ldots, n_i$, $q = 1, 2, \ldots, r$, $k = 1, 2, \ldots, 7$, and

$$\sum_{k=1}^{7} v_{ijq}^{(k)} = 1, \quad 0 \leq v_{ijq}^{(k)} \leq 1 \tag{11}$$

Table 1. Contents of the proposed assessment form [9,11-14]

| Risk attribute | Risk item | Weight-2 | Weight-1 | Linguistic variables | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $V_6$ | $V_7$ |
| $X_1$: Personal | | $W_2(1)$ | | | | | | | | |
| | $X_{11}$: Personal shortfalls, key person(s) quit | | $W_1(1,1)$ | $\widetilde{v}_{11}^{(1)}$ | $\widetilde{v}_{11}^{(2)}$ | $\widetilde{v}_{11}^{(3)}$ | $\widetilde{v}_{11}^{(4)}$ | $\widetilde{v}_{11}^{(5)}$ | $\widetilde{v}_{11}^{(6)}$ | $\widetilde{v}_{11}^{(7)}$ |
| $X_2$: System requirement | | $W_2(2)$ | | | | | | | | |
| | $X_{21}$: Requirement ambiguity | | $W_1(2,1)$ | $\widetilde{v}_{21}^{(1)}$ | $\widetilde{v}_{21}^{(2)}$ | $\widetilde{v}_{21}^{(3)}$ | $\widetilde{v}_{21}^{(4)}$ | $\widetilde{v}_{21}^{(5)}$ | $\widetilde{v}_{21}^{(6)}$ | $\widetilde{v}_{21}^{(7)}$ |
| | $X_{22}$: Developing the wrong software function | | $W_1(2,2)$ | $\widetilde{v}_{22}^{(1)}$ | $\widetilde{v}_{22}^{(2)}$ | $\widetilde{v}_{22}^{(3)}$ | $\widetilde{v}_{22}^{(4)}$ | $\widetilde{v}_{22}^{(5)}$ | $\widetilde{v}_{22}^{(6)}$ | $\widetilde{v}_{22}^{(7)}$ |
| | $X_{23}$: Developing the wrong user interface | | $W_1(2,3)$ | $\widetilde{v}_{23}^{(1)}$ | $\widetilde{v}_{23}^{(2)}$ | $\widetilde{v}_{23}^{(3)}$ | $\widetilde{v}_{23}^{(4)}$ | $\widetilde{v}_{23}^{(5)}$ | $\widetilde{v}_{23}^{(6)}$ | $\widetilde{v}_{23}^{(7)}$ |
| | $X_{24}$: Continuing stream requirement changes | | $W_1(2,4)$ | $\widetilde{v}_{24}^{(1)}$ | $\widetilde{v}_{24}^{(2)}$ | $\widetilde{v}_{24}^{(3)}$ | $\widetilde{v}_{24}^{(4)}$ | $\widetilde{v}_{24}^{(5)}$ | $\widetilde{v}_{24}^{(6)}$ | $\widetilde{v}_{24}^{(7)}$ |
| $X_3$: Schedules and budgets | | $W_2(3)$ | | | | | | | | |
| | $X_{31}$: Schedule not accurate | | $W_1(3,1)$ | $\widetilde{v}_{31}^{(1)}$ | $\widetilde{v}_{31}^{(2)}$ | $\widetilde{v}_{31}^{(3)}$ | $\widetilde{v}_{31}^{(4)}$ | $\widetilde{v}_{31}^{(5)}$ | $\widetilde{v}_{31}^{(6)}$ | $\widetilde{v}_{31}^{(7)}$ |
| | $X_{32}$: Budget not sufficient | | $W_1(3,2)$ | $\widetilde{v}_{32}^{(1)}$ | $\widetilde{v}_{32}^{(2)}$ | $\widetilde{v}_{32}^{(3)}$ | $\widetilde{v}_{32}^{(4)}$ | $\widetilde{v}_{32}^{(5)}$ | $\widetilde{v}_{32}^{(6)}$ | $\widetilde{v}_{32}^{(7)}$ |
| $X_4$: Developing technology | | $W_2(4)$ | | | | | | | | |
| | $X_{41}$: Gold-plating | | $W_1(4,1)$ | $\widetilde{v}_{41}^{(1)}$ | $\widetilde{v}_{41}^{(2)}$ | $\widetilde{v}_{41}^{(3)}$ | $\widetilde{v}_{41}^{(4)}$ | $\widetilde{v}_{41}^{(5)}$ | $\widetilde{v}_{41}^{(6)}$ | $\widetilde{v}_{41}^{(7)}$ |
| | $X_{42}$: Skill levels inadequate | | $W_1(4,2)$ | $\widetilde{v}_{42}^{(1)}$ | $\widetilde{v}_{42}^{(2)}$ | $\widetilde{v}_{42}^{(3)}$ | $\widetilde{v}_{42}^{(4)}$ | $\widetilde{v}_{42}^{(5)}$ | $\widetilde{v}_{42}^{(6)}$ | $\widetilde{v}_{42}^{(7)}$ |
| | $X_{43}$: Straining hardware | | $W_1(4,3)$ | $\widetilde{v}_{43}^{(1)}$ | $\widetilde{v}_{43}^{(2)}$ | $\widetilde{v}_{43}^{(3)}$ | $\widetilde{v}_{43}^{(4)}$ | $\widetilde{v}_{43}^{(5)}$ | $\widetilde{v}_{43}^{(6)}$ | $\widetilde{v}_{43}^{(7)}$ |
| | $X_{44}$: Straining software | | $W_1(4,4)$ | $\widetilde{v}_{44}^{(1)}$ | $\widetilde{v}_{44}^{(2)}$ | $\widetilde{v}_{44}^{(3)}$ | $\widetilde{v}_{44}^{(4)}$ | $\widetilde{v}_{44}^{(5)}$ | $\widetilde{v}_{44}^{(6)}$ | $\widetilde{v}_{44}^{(7)}$ |
| $X_5$: External | | $W_2(5)$ | | | | | | | | |
| | $X_{51}$: Shortfalls in externally furnished components | | $W_1(5,1)$ | $\widetilde{v}_{51}^{(1)}$ | $\widetilde{v}_{51}^{(2)}$ | $\widetilde{v}_{51}^{(3)}$ | $\widetilde{v}_{51}^{(4)}$ | $\widetilde{v}_{51}^{(5)}$ | $\widetilde{v}_{51}^{(6)}$ | $\widetilde{v}_{51}^{(7)}$ |
| | $X_{52}$: Shortfalls in externally performed tasks | | $W_1(5,2)$ | $\widetilde{v}_{52}^{(1)}$ | $\widetilde{v}_{52}^{(2)}$ | $\widetilde{v}_{52}^{(3)}$ | $\widetilde{v}_{52}^{(4)}$ | $\widetilde{v}_{52}^{(5)}$ | $\widetilde{v}_{52}^{(6)}$ | $\widetilde{v}_{52}^{(7)}$ |
| $X_6$: Performance | | $W_2(6)$ | | | | | | | | |
| | $X_{61}$: Real-time performance shortfalls | | $W_1(6,1)$ | $\widetilde{v}_{61}^{(1)}$ | $\widetilde{v}_{61}^{(2)}$ | $\widetilde{v}_{61}^{(3)}$ | $\widetilde{v}_{61}^{(4)}$ | $\widetilde{v}_{61}^{(5)}$ | $\widetilde{v}_{61}^{(6)}$ | $\widetilde{v}_{61}^{(7)}$ |

Corresponding to the interval $\left[v_{ijq}^{(k)} - \Delta_{ijq1}^{(k)}, v_{ijq}^{(k)} + \Delta_{ijq2}^{(k)}\right]$, we have the triangular fuzzy number as follows:

$$\widetilde{v}_{ijq}^{*(k)} = \left(v_{ijq}^{(k)} - \Delta_{ijq1}^{(k)}, v_{ijq}^{(k)}, v_{ijq}^{(k)} + \Delta_{ijq2}^{(k)}\right) \tag{12}$$

where

$$0 < v_{ijq}^{(k)} - \Delta_{ijq1}^{(k)} < v_{ijq}^{(k)} + \Delta_{ijq2}^{(k)} \leq 1;\ \Delta_{ijq1}^{(k)} > 0,\ \Delta_{ijq2}^{(k)} > 0. \tag{13}$$

Defuzzifying $\widetilde{v}_{ijq}^{*(k)}$ by signed distance method, we have

$$d\left(\widetilde{v}_{ijq}^{*(k)}, \widetilde{0}\right) = v_{ijq}^{(k)} + \frac{1}{4}\left(\Delta_{ijq2}^{(k)} - \Delta_{ijq1}^{(k)}\right) \in \left[v_{ijq}^{(k)} - \Delta_{ijq1}^{(k)}, v_{ijq}^{(k)} + \Delta_{ijq2}^{(k)}\right] \tag{14}$$

Since we could not know the error between the evaluated value $v_{ijq}^{(k)}$ and the population objective value $V_{ijq}^{(k)}$ (unknown), we therefore cannot take the confidence level as 1, that is, we cannot take the membership grade of $\widetilde{v}_{ijq}^{*(k)}$ at $v_{ijq}^{(k)}$ as 1. Thus, it should be more reasonable to consider the membership grade of $\widetilde{v}_{ijq}^{*(k)}$ within the interval $[\lambda, 1]$, $0 < \lambda < 1$. Hence, we may rewrite the triangular fuzzy number (12) to level $(\lambda, 1)$ interval-valued fuzzy number (17) as follows:

$$\widetilde{v}_{ijq}^{(k)U} = \left(v_{ijq}^{(k)} - \Delta_{ijq1}^{(k)}, v_{ijq}^{(k)}, v_{ijq}^{(k)} + \Delta_{ijq2}^{(k)}\right) \left(= \widetilde{v}_{ijq}^{*(k)}\right) \tag{15}$$

$$\widetilde{v}_{ijq}^{(k)L} = \left(v_{ijq}^{(k)} - \Delta_{ijq3}^{(k)}, v_{ijq}^{(k)}, v_{ijq}^{(k)} + \Delta_{ijq4}^{(k)}; \lambda\right), \ 0 < \lambda < 1 \tag{16}$$

From (15) and (16), we have the level $(\lambda, 1)$ interval-valued fuzzy number $\widetilde{v}_{ijq}^{(k)}$ as follows:

$$\widetilde{v}_{ijq}^{(k)} = \left[\widetilde{v}_{ijq}^{(k)L}, \widetilde{v}_{ijq}^{(k)U}\right] \tag{17}$$

$$0 < \Delta_{ijq3}^{(k)} < \Delta_{ijq1}^{(k)} < v_{ijq}^{(k)}, \ 0 < \Delta_{ijq4}^{(k)} < \Delta_{ijq2}^{(k)} \tag{18}$$

**Remark 3.1.** *If $\Delta_{ijq3}^{(k)} = \Delta_{ijq4}^{(k)} = 0$, $\lambda = 0$, then the level $(\lambda, 1)$ interval-valued fuzzy number $\widetilde{v}_{ijq}^{(k)}$ becomes triangular fuzzy number $\widetilde{v}_{ijq}^{(k)U}$ $(= \widetilde{v}_{ijq}^{*(k)})$ in (15). Thus, the triangular fuzzy number is a special case of level $(\lambda, 1)$ interval-valued fuzzy number as follows:*

$$d\left(\widetilde{v}_{ijq}^{(k)}, \widetilde{0}\right) = v_{ijq}^{(k)} + \frac{1}{4}\left(\Delta_{ijq2}^{(k)} - \Delta_{ijq1}^{(k)}\right) = d\left(\widetilde{v}_{ijq}^{*(k)}, \widetilde{0}\right) \tag{19}$$

From (15), (16) and Definition 2.5, we have (20) as follows:

$$\widetilde{v}_{ij}^{(k)} = \frac{1}{r}\left(\widetilde{v}_{ij1}^{(k)} + \widetilde{v}_{ij2}^{(k)} + \cdots + \widetilde{v}_{ijq}^{(k)}\right) = \left[\widetilde{v}_{ij}^{(k)L}, \widetilde{v}_{ij}^{(k)U}\right] \tag{20}$$

where

$$\widetilde{v}_{ij}^{(k)L} = \left(\frac{1}{r}\sum_{q=1}^{r}\left(v_{ijq}^{(k)} - \Delta_{ijq3}^{(k)}\right), \frac{1}{r}\sum_{q=1}^{r} v_{ijq}^{(k)}, \frac{1}{r}\sum_{q=1}^{r} v_{ijq}^{(k)} + \Delta_{ijq4}^{(k)}; \lambda\right) \tag{21}$$

$$\widetilde{v}_{ij}^{(k)U} = \left(\frac{1}{r}\sum_{q=1}^{r}\left(v_{ijq}^{(k)} - \Delta_{ijq1}^{(k)}\right), \frac{1}{r}\sum_{q=1}^{r} v_{ijq}^{(k)}, \frac{1}{r}\sum_{q=1}^{r} v_{ijq}^{(k)} + \Delta_{ijq2}^{(k)}\right) \tag{22}$$

Step 3: Let

$$\widetilde{N}_i^{(k)} = \left(W_1(i,1)\widetilde{v}_{i1}^{(k)} \oplus W_1(i,2)\widetilde{v}_{i2}^{(k)} \oplus \cdots \oplus W_1(i,n_i)\widetilde{v}_{in_i}^{(k)}\right) = \left[\widetilde{N}_i^{(k)L}, \widetilde{N}_i^{(k)U}\right] \tag{23}$$

where

$$\begin{aligned}
\widetilde{N}_i^{(k)L} = &\left(\frac{1}{r}\sum_{j=1}^{n_i} W_1(i,j)\sum_{q=1}^{r}\left(v_{ijq}^{(k)} - \Delta_{ijq3}^{(k)}\right), \frac{1}{r}\sum_{j=1}^{n_i} W_1(i,j)\sum_{q=1}^{r} v_{ijq}^{(k)},\right.\\
&\left.\frac{1}{r}\sum_{j=1}^{n_i} W_1(i,j)\sum_{q=1}^{r}\left(v_{ijq}^{(k)} + \Delta_{ijq4}^{(k)}\right); \lambda\right)\\
\widetilde{N}_i^{(k)U} = &\left(\frac{1}{r}\sum_{j=1}^{n_i} W_1(i,j)\sum_{q=1}^{r}\left(v_{ijq}^{(k)} - \Delta_{ijq1}^{(k)}\right), \frac{1}{r}\sum_{j=1}^{n_i} W_1(i,j)\sum_{q=1}^{r} v_{ijq}^{(k)},\right.\\
&\left.\frac{1}{r}\sum_{j=1}^{n_i} W_1(i,j)\sum_{q=1}^{r}\left(v_{ijq}^{(k)} + \Delta_{ijq2}^{(k)}\right)\right)
\end{aligned}$$

for $i = 1, 2, \ldots, 6$; $k = 1, 2, \ldots, 7$.

Step 4: Defuzzifying (23) by signed distance method [18], we have

$$\begin{aligned} d\left(\widetilde{N}_i^{(k)}, \widetilde{0}\right) &= \frac{1}{r}\sum_{j=1}^{n_i} W_1(i,j)\sum_{q=1}^{r} v_{ijq}^{(k)} + \frac{1}{16r}\sum_{j=1}^{n_i} W_1(i,j)\sum_{q=1}^{r}\left(\Delta_{ijq4}^{(k)} - \Delta_{ijq3}^{(k)}\right) \\ &+ \frac{1}{4r}\sum_{j=1}^{n_i} W_1(i,j)\sum_{q=1}^{r}\left(\Delta_{ijq4}^{(k)} - \Delta_{ijq3}^{(k)}\right) \\ &+ \frac{3\lambda}{16r}\sum_{j=1}^{n_i} W_1(i,j)\sum_{q=1}^{r}\left(\Delta_{ijq2}^{(k)} - \Delta_{ijq1}^{(k)}\right) \end{aligned} \tag{24}$$

Let

$$N_i^{*,k} = d\left(\widetilde{N}_i^{*(k)}, \widetilde{0}\right) \tag{25}$$

Let

$$A_i = \sum_{k=1}^{7} d\left(V_k, \widetilde{0}\right)\cdot N_i^{*,k} \tag{26}$$

Let

$$S = \sum_{i=1}^{6} W_2(i)\cdot A_i \tag{27}$$

Then, we have the following proposition.

**Proposition 3.1.** *For the assessment form as shown in Table 1 and from Equation (24), we have*

*(A) The value of $N_i^{*,k}$ is the aggregative risk rate of the attribute $X_i$ with respect to the criteria $V_k$.*
*(B) The value of $A_i$ is the risk rate of the attribute $X_i$.*
*(C) The value of $S$ is the aggregative risk rate.*

## 4. Conclusions.

Evaluators are usually inconsistent regarding criteria and types of information, because they lack precise information in the evaluation criterion. To solve this problem, in this study we use the level $(\lambda, 1)$ interval-valued fuzzy numbers and the signed distance method to evaluate the aggregative risk rate. The proposed method can reflect the interviewee's incomplete and uncertain thoughts.

**Acknowledgment.** The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

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