ICIC Express Letters
Part B: Applications
Volume **7**, Number **1**, January **2016**

ICIC International ©2016 ISSN 2185-2766
pp. 151–158

FUZZY OPTIMIZATION OF IRON AND STEEL SUPPLY CHAIN NETWORK WITH RISKS

Zhuo Dai
School of Electronic Commerce
Jiujiang University
No. 551, East Qianjin Road, Jiujiang 332005, P. R. China
Daizhuo1@126.com

Received July 2015; accepted October 2015

Abstract. *This paper proposed a model of iron and steel supply chain network with two objectives, which are to minimize the total costs and risks. The supply chain network is composed of iron ore suppliers, iron and steel factories, and distribution centers. The total costs include variable costs, fixed costs, and transportation costs. Different from previous studies, this paper simultaneously considers the risks and costs of iron and steel supply chain network. The model is a mixed integer nonlinear programming model and hard to be solved. However, when an auxiliary variable is introduced, this bi-objective model is transformed into single objective model and is solved by CPLEX 12.6 perfectly.*
Keywords: Fuzzy optimization, Iron and steel, Supply chain network

1. **Introduction.** Iron and steel industry is the pillar industry of the national economy. It supplies materials for other important sectors of the national economy, such as machinery industry, construction industry, and automobile industry. It plays an important and basic role in the development of the national economy. With the integration of global economy and the rapid development of high and new technology, iron and steel enterprises face increasing competition. The rules of competition have been changed. The competition of single enterprise has become the competition of overall supply chain. Supply chain management is not only an important means of iron and steel enterprises to win the competition, but also a research hotspot. In the process of supply chain management, the optimization of supply chain management has important significance because it affects the performance of the supply chain management and is related to the efficiency and profits of enterprises. Therefore, studying the optimization of the iron and steel supply chain network has important theoretical and practical significance.

Some scholars optimize the iron and steel supply chain network. Zadeh et al. designed a steel supply chain network in 2013. A mixed integer nonlinear programming model and a mixed integer linear programming model are presented and solved by using a commercial solver [1]. Wang established a bidirectional VRP model on the iron and steel enterprises' external logistics and distribution network and solved it using improved immune genetic algorithm in 2011 [2]. Zheng proposed an optimization model to minimize the total cost, which is composed of transportation cost, inventory cost and the operation cost and solved this model using genetic algorithm in 2012 [3]. Alawneh et al. developed a linear programming model to describe Qatar steel manufacturing supply chain from suppliers to consumers in 2014. The model is validated and solved using GAMS software [4]. However, the above researches have the following two issues. First, uncertain environment especially the fuzzy environment is not involved in above literature. Second, previous studies have only a single objective, which is minimizing the total costs or total transportation distance. In summary, this paper will study the optimization problem of iron and steel supply chain network under fuzzy environment. There are two objectives. One is minimizing the total cost of supply chain network. The other is minimizing the risks of iron and steel supply chain network.

The main contributions of this paper can be summarized as follows. First, the risks of supply chain are considered. Minimizing the risks is the objective of this study. Few previous studies regard risks as their objective due to the computational difficulties. Second, the risks are measured by lower possibilistic semivariance, which is more appropriate than variance. Third, the optimization of iron and steel supply chain network is studied. In the past, supply chain networks of home appliances, agricultural products, and electronic products were designed and optimized. However, the researches on iron and steel supply chain network are few. Fourth, the bi-objective problem is solved. Previous studies only have single objective, i.e., minimizing the costs or maximizing the profits. Fifth, this paper solves the optimization problem under fuzzy environment. Uncertainties in most previous literature come from random environment rather than fuzzy environment.

The rest of this paper is as follows. In Section 2, fundamental definitions and theorem are introduced. Mathematical formulation of the proposed model is presented in Section 3. In Section 4, the solution method is proposed. Section 5 presents a numerical example. Finally, the conclusion of this article and future research are given in Section 6.

2. **Fundamental Definitions and Theorem.** A fuzzy number C is a fuzzy set of the real line X with a normal, fuzzy convex and continuous membership function $\eta_c(x)$ of bounded support. γ-level set of a fuzzy number C is denoted by $[C]_\gamma = \{x \in X | \eta_c(x) \geq \gamma\}$ (the closure of the support of C) if $\gamma = 0$. If C is a fuzzy number, $[C]_\gamma$ is a subset on X for all $\gamma \in [0, 1]$.

Definition 2.1. *Let C be a fuzzy number with γ-level set $[C]_\gamma = [\underline{c}(\gamma), \overline{c}(\gamma)]$, $\gamma \in [0, 1]$. Then the possibilistic mean value of C is defined as follows [5].*

$$E(C) = \int_0^1 \gamma(\underline{c}(\gamma) + \overline{c}(\gamma))d\gamma \tag{1}$$

If the weighted function $f(\gamma) = 2\gamma$, the definitions of the lower and upper possibilistic semivariances of C are as follows.

Definition 2.2. *Let C be a fuzzy number with γ-level set $[C]_\gamma = [\underline{c}(\gamma), \overline{c}(\gamma)]$, $\gamma \in [0, 1]$. Let $E(C)$ be the possibilistic mean value of C. Then the upper and lower possibilistic semivariances of fuzzy number C are as follows [6].*

$$\mathrm{var}^+(C) = \int_0^1 2\gamma(E(C) - \overline{c}(\gamma))^2 d\gamma \tag{2}$$

$$\mathrm{var}^-(C) = \int_0^1 2\gamma(E(C) - \underline{c}(\gamma))^2 d\gamma \tag{3}$$

Definition 2.3. *If any two fuzzy numbers with $[C]_\gamma = [\underline{c}(\gamma), \overline{c}(\gamma)]$, $\gamma \in [0, 1]$ and $[D]_\gamma = \left[\underline{d}(\gamma), \overline{d}(\gamma)\right]$, $\gamma \in [0, 1]$ are given, the upper and lower possibilistic semicovariances between C and D are as follows.*

$$\mathrm{cov}^+(C, D) = \int_0^1 2\gamma(E(C) - \overline{c}(\gamma))\left(E(D) - \overline{d}(\gamma)\right) d\gamma \tag{4}$$

$$\mathrm{cov}^-(C, D) = \int_0^1 2\gamma(E(C) - \underline{c}(\gamma))(E(D) - \underline{d}(\gamma))d\gamma \tag{5}$$

Theorem 2.1. [7] *Let C_1, C_2, $\cdots$, C_n be n fuzzy numbers, and let λ_1, λ_2, $\cdots$, λ_n be n positive real numbers.*

$$\mathrm{var}^+\left(\sum_{i=1}^n \lambda_i C_i\right) = \sum_{i=1}^n \lambda_i^2 \mathrm{var}^+(C_i) + 2\sum_{i<j=1}^n \lambda_i\lambda_j \mathrm{cov}^+(C_i, C_j) \tag{6}$$

$$\mathrm{var}^-\left(\sum_{i=1}^{n}\lambda_i C_i\right)=\sum_{i=1}^{n}\lambda_i^2\mathrm{var}^-(C_i)+2\sum_{i<j=1}^{n}\lambda_i\lambda_j\mathrm{cov}^-(C_i,C_j) \tag{7}$$

3. **Formulation of Model.**

3.1. **Notation and index.**

S set of iron ore suppliers
M set of iron and steel factories
D set of distribution centers
C set of consumer zones
s index of iron ore suppliers $s \in S$
m index of iron and steel factories $m \in M$
d index of distribution centers $d \in D$
c index of consumer zones $c \in C$
Parameters:
f_s the trapezoidal fuzzy fixed cost of iron ore suppliers s, $f_s \sim (e_{f,s}, h_{f,s}, s_{f,s}, k_{f,s})$
f_m the trapezoidal fuzzy fixed cost of iron and steel factories, $f_m \sim (e_{f,m}, h_{f,m}, s_{f,m}, k_{f,m})$
f_d the trapezoidal fuzzy fixed cost of distribution centers d, $f_d \sim (e_{f,d}, h_{f,d}, s_{f,d}, k_{f,d})$
v_s unit iron ore cost of supplier s
v_m unit production cost of iron and steel factory m
v_d unit distribution cost of distribution center d
r_{sm} unit trapezoidal fuzzy freight rate from iron ore supplier s to factories, $r_{sm} \sim (e_{sm}, h_{sm}, s_{sm}, k_{sm})$
r_{md} unit trapezoidal fuzzy freight rate from iron and steel factory m to distribution center d, $r_{md} \sim (e_{md}, h_{md}, s_{md}, k_{md})$
r_{dc} unit trapezoidal fuzzy freight rate from distribution center d to consumer zone c, $r_{dc} \sim (e_{dc}, h_{dc}, s_{dc}, k_{dc})$
D_c demand of customer zone c
c_s capacity of iron ore supplier s
c_m capacity of iron and steel factory m
c_d capacity of distribution center d
Decision variable
y_s 1 if iron ore supplier s is open; 0 otherwise
y_m 1 if iron and steel factory m is open; 0 otherwise
y_d 1 if distribution center d is open; 0 otherwise
x_{sm} quantity of iron ore from the iron ore supplier s to iron and steel factory m
x_{md} quantity of product from iron and steel factory m to distribution center d
x_{dc} quantity of product from distribution center d to customer zone c

3.2. **Model formulation.**

Objective function

$$Minimize\,(var\,(total\ cost)) \tag{8}$$

$$Minimize(total\ cost\ ) \tag{9}$$

The total cost is the sum of the following costs, i.e., $total\ cost = fixed\ cost + variable\ cost + transportation\ cost$.

$$fixed\ cost = \sum_{s\in S} f_s y_s + \sum_{m\in M} f_m y_m + \sum_{d\in D} f_d y_d \tag{10}$$

$$variable\ cost = \sum_{s\in S}\sum_{m\in M} v_s x_{sm} + \sum_{m\in M}\sum_{d\in D} v_m x_{md} + \sum_{d\in D}\sum_{c\in C} v_d x_{dc} \tag{11}$$

$$transportation\ cost = \sum_{s\in S}\sum_{m\in M} r_{sm}x_{sm} + \sum_{m\in M}\sum_{d\in D} r_{md}x_{md} + \sum_{d\in D}\sum_{c\in C} r_{dc}x_{dc} \quad (12)$$

Constraints

All constraints of the proposed model are represented as follows.

$$\sum_{s\in S} x_{sm} = \sum_{d\in D} x_{md} \quad \forall m \quad (13)$$

$$\sum_{m\in M} x_{md} = \sum_{c\in C} x_{dc} \quad \forall d \quad (14)$$

$$\sum_{d} x_{dc} \geq D_c \quad \forall c \quad (15)$$

$$\sum_{m\in M} x_{sm} \leq c_s y_s \quad \forall s \quad (16)$$

$$\sum_{d\in D} x_{md} \leq c_m y_m \quad \forall m \quad (17)$$

$$\sum_{c\in C} x_{dc} \leq c_d y_d \quad \forall d \quad (18)$$

Constraints (13) and (14) are balanced constraints. Obviously, products or iron ores entering flows per node should be equal to all issuing flows of that node for the products or iron ores at each node. Therefore, constraints (13) and (14) are balance constraints of iron and steel factories, distribution centers, respectively. Constraint (15) ensures that demands for iron and steel must be fully met. Constraints (16) to (18) are capacity constraints. Capacity constraints control the maximum flows. Constraint (16) is for capacity of iron ore suppliers. Constraints (17) and (18) control output capacities of iron and steel factories, distribution centers, respectively.

According to Section 2, the crisp possibilistic mean value of fixed cost, variable cost, transportation cost, respectively, can be expressed as

$$\begin{aligned} E(fixed\ cost) = & \sum_{s\in S}\left(\frac{e_{f,s}+h_{f,s}}{2}+\frac{k_{f,s}-s_{f,s}}{6}\right)y_s \\ & + \sum_{m\in M}\left(\frac{e_{f,m}+h_{f,m}}{2}+\frac{k_{f,m}-s_{f,m}}{6}\right)y_m \\ & + \sum_{d\in D}\left(\frac{e_{f,d}+h_{f,d}}{2}+\frac{k_{f,d}-s_{f,d}}{6}\right)y_d \end{aligned} \quad (19)$$

$$\begin{aligned} E(transportation\ cost) = & \sum_{s\in S}\left(\frac{e_{sm}+h_{sm}}{2}+\frac{k_{sm}-s_{sm}}{6}\right)x_{sm} \\ & + \sum_{m\in M}\left(\frac{e_{md}+h_{md}}{2}+\frac{k_{md}-s_{md}}{6}\right)x_{md} \\ & + \sum_{d\in D}\left(\frac{e_{dc}+h_{dc}}{2}+\frac{k_{dc}-s_{dc}}{6}\right)x_{dc} \end{aligned} \quad (20)$$

$$E(variable\ cost) = \sum_{s\in S}\sum_{m\in M} v_s x_{sm} + \sum_{m\in M}\sum_{d\in D} v_m x_{md} + \sum_{d\in D}\sum_{c\in C} v_d x_{dc} \quad (21)$$

$$E(total\ cost) = E(fixed\ cost) + E(variable\ cost) + E(transportation\ cost) \quad (22)$$

Variance implies uncertainty of income. However, this uncertainty may lead to an additional loss or income. In general, risks mean the additional loss. Therefore, it is more appropriate to use the lower possibilistic semivariance of fuzzy total costs to measure the risks. Thus, according to definitions in Section 2, the risks of fuzzy total cost can be computed as follows.

$$
\begin{aligned}
&var^{-}(total\ cost)\\
&= \left[\sum_{s\in S}\sum_{m\in M}\left(\frac{e_{sm}+h_{sm}}{2}+\frac{k_{sm}-s_{sm}}{6}\right)x_{sm}+\sum_{m\in M}\sum_{d\in D}\left(\frac{e_{md}+h_{md}}{2}+\frac{k_{md}-s_{md}}{6}\right)x_{md}\right.\\
&+\sum_{d\in D}\sum_{c\in C}\left(\frac{e_{dc}+h_{dc}}{2}+\frac{k_{dc}-s_{dc}}{6}\right)x_{dc}+\sum_{s\in S}\left(\frac{e_{f,s}+h_{f,s}}{2}+\frac{k_{f,s}-s_{f,s}}{6}\right)y_{s}\\
&\left.+\sum_{m\in M}\left(\frac{e_{f,m}+h_{f,m}}{2}+\frac{k_{f,m}-s_{f,m}}{6}\right)y_{m}+\sum_{d\in D}\left(\frac{e_{f,d}+h_{f,d}}{2}+\frac{k_{f,d}-s_{f,d}}{6}\right)y_{d}\right]^{2}\\
&+\frac{1}{18}\left[\sum_{s\in S}\sum_{m\in M}s_{sm}x_{sm}+\sum_{m\in I}\sum_{d\in D}s_{md}x_{md}+\sum_{d\in D}\sum_{c\in C}s_{dc}x_{dc}+\sum_{s\in S}s_{f,s}y_{s}\right.\\
&\left.+\sum_{m\in M}s_{f,m}y_{m}+\sum_{d\in D}s_{f,d}y_{d}\right]^{2}
\end{aligned}
\tag{23}
$$

4. **The Solution Method.** The solution method is given as follows. Firstly, the positive ideal solution and negative ideal solution for each of the fuzzy objectives are solved. Secondly, the linear membership functions are determined. They can be expressed as follows.

$$
\mu_1(w_1)=\begin{cases}1 & \text{if } w_1 < w_1^{\alpha-PIS}\\ \dfrac{w_1^{\alpha-NIS}-w_1}{w_1^{\alpha-NIS}-w_1^{\alpha-PIS}} & \text{if } w_1^{\alpha-PIS}\le w_1\le w_1^{\alpha-NIS}\\ 0 & \text{if } w_1 > w_1^{\alpha-NIS}\end{cases}
\tag{24}
$$

$$
\mu_2(w_2)=\begin{cases}1 & \text{if } w_2 < w_2^{\alpha-PIS}\\ \dfrac{w_2^{\alpha-NIS}-w_2}{w_2^{\alpha-NIS}-w_2^{\alpha-PIS}} & \text{if } w_2^{\alpha-PIS}\le w_2\le w_2^{\alpha-NIS}\\ 0 & \text{if } w_2 > w_2^{\alpha-NIS}\end{cases}
\tag{25}
$$

$w_1^{\alpha-PIS}$ and $w_1^{\alpha-NIS}$ are minimum and maximum of semivariance of fuzzy total costs, respectively. $w_2^{\alpha-PIS}$ and $w_2^{\alpha-NIS}$ are minimum and maximum of fuzzy total costs, respectively. Thirdly, the auxiliary variable L which enables the fuzzy bi-objective problem to be converted into a single-objective problem is introduced. In general, L can be regarded as the level of satisfaction of decision-makers. If $L = 1$, then each objective is fully satisfied; if $0 < L < 1$, then all of the objectives are satisfied at the level L; if $L = 0$, then none of the objectives are satisfied. The single-objective problem can be expressed as follows.

$$\max L \text{ s.t. } L \le \mu_i(w_i),\ i = 1, 2,\ \text{Equations (13)-(18)}$$

5. **A Numerical Example.** The scale of the computational experiment is as follows: the number of potential locations for iron ore suppliers, iron and steel factories, distribution centers, and customer zones is three, three, three, six, respectively. Capacities of iron ore suppliers, iron and steel factories, distribution centers are all 600. Demand of iron and steel of each customer zone is 150. The variable costs of iron ore suppliers, iron and steel factories, and distribution centers are shown in Table 1. The fuzzy fixed costs of iron ore suppliers, iron and steel factories, distribution centers are shown in Table 2. The data about unit fuzzy freight rates among facilities are given from Table 3 to Table 5.

Table 1. The variable costs

	iron ore suppliers	iron and steel factories	distribution centers
1	110	65	75
2	110	40	140
3	110	60	110

Table 2. The possibility distributions of fixed costs

	iron ore suppliers	iron and steel factories	distribution centers
1	$(5000, 6560, 60, 60)$	$(1140, 1500, 90, 30)$	$(1000, 2360, 60, 60)$
2	$(5000, 6960, 80, 40)$	$(1000, 1760, 10, 50)$	$(1000, 2580, 30, 30)$
3	$(1240, 1600, 90, 30)$	$(620, 1800, 30, 30)$	$(1220, 3000, 10, 50)$

Table 3. The possibility distributions of freight rate from iron ore suppliers to factories

		factories		
		1	2	3
suppliers	1	$(300, 570, 20, 70)$	$(300, 600, 30, 30)$	$(360, 720, 30, 30)$
	2	$(210, 420, 50, 40)$	$(400, 670, 50, 40)$	$(500, 650, 70, 20)$
	3	$(300, 620, 80, 40)$	$(320, 640, 80, 40)$	$(500, 700, 30, 30)$

Table 4. The possibility distributions of freight rate from factories to distribution centers

		distribution centers		
		1	2	3
factories	1	$(130, 300, 10, 20)$	$(190, 300, 20, 10)$	$(300, 580, 30, 30)$
	2	$(130, 300, 10, 20)$	$(330, 500, 20, 10)$	$(200, 420, 50, 10)$
	3	$(300, 470, 10, 20)$	$(200, 490, 20, 10)$	$(260, 600, 40, 20)$

CPLEX 12.6 is used to solve this model. Command of CPLEX, i.e., cplexmiqcp is used. The value of exitflag is 1, which means the most optimum solution is obtained. $w_1^{\alpha-PIS}$ and $w_1^{\alpha-NIS}$ are 85397751250, 112847206050, respectively. $w_2^{\alpha-PIS}$ and $w_2^{\alpha-NIS}$ are 1180907, 1372733, respectively. The results show that the first and second suppliers and factories should be opened. The first, second and third distribution centers should be opened. The optimum lower semivariance of fuzzy total costs is 95749690423. The optimum total costs is 1253250. The value of L is 0.6229. The flows among facilities are shown from Table 6 to Table 8.

6. **Conclusions and Future Research.** The optimization of iron and steel supply chain network is an important issue of supply chain management. The bi-objective model of iron and steel supply chain network is put forward. Different from previous studies, this paper considers and minimizes the risks and costs of iron and steel supply chain network. The proposed model includes variable costs, fixed costs, and transportation costs. It is an NP (non-deterministic polynomial) problem and a mixed integer nonlinear programming model. However, this bi-objective problem is transformed into a single objective problem by introducing an auxiliary variable and solved perfectly.

There is some guidance for future research. First, the optimization of multi-period and multi-product iron and steel supply chain network should be studied. The production of iron and steel is a repetitive process. Furthermore, one factory can produce many kinds of products. Therefore, it is necessary to study the optimization of iron and steel supply chain network with multi-period and multi-product. It makes the study more realistic.

Table 5. The possibility distributions of freight rate from distribution centers to customer zones

		distribution centers		
		1	2	3
customer zones	1	(350, 700, 20, 10)	(400, 680, 30, 30)	(240, 750, 60, 30)
	2	(200, 490, 20, 10)	(400, 630, 20, 10)	(550, 700, 70, 20)
	3	(400, 560, 30, 30)	(300, 640, 30, 30)	(340, 600, 80, 40)
	4	(350, 700, 20, 10)	(300, 520, 40, 20)	(300, 680, 70, 20)
	5	(300, 590, 20, 10)	(300, 640, 40, 20)	(510, 600, 70, 20)
	6	(300, 800, 20, 10)	(250, 700, 60, 30)	(600, 750, 50, 40)

Table 6. The quantity of iron ore from suppliers to factories

		factories		
		1	2	3
suppliers	1	0	300	0
	2	600	0	0
	3	0	0	0

Table 7. The quantity of iron and steel from factories to distribution centers

		distribution centers		
		1	2	3
factories	1	300	300	0
	2	69.25	0	230.75
	3	0	0	0

Table 8. The quantity of iron and steel from distribution centers to customer zones

		distribution centers		
		1	2	3
customer zones	1	0	150	0
	2	150	0	0
	3	150	0	0
	4	0	150	0
	5	69.25	0	80.75
	6	0	0	150

Second, intelligent algorithms can be used to solve more complex models. CPLEX cannot solve the general optimization problem. Hence, it is necessary to use multi-objective evolutionary algorithm to solve nonlinear optimization problem of iron and steel supply chain network, such as genetic algorithm, particle swarm algorithm, and simulation annealing algorithm. Third, closed-loop supply chain and green supply chain of iron and steel supply network can be studied. Since governments and society pay more attention to environmental problems, iron and steel enterprises not only consider the economic benefits and risks, but also take the environmental responsibilities. Thus, it is significant to regard the reduction of emissions and waste as targets.

REFERENCES

[1] A. S. Zadeh, R. Sahraeian and S. M. Homayouni, A dynamic multi-commodity inventory and facility location problem in steel supply chain network design, *The International Journal of Advanced Manufacturing Technology*, vol.70, pp.1267-1282, 2014.

[2] Z. Wang, *Research on Modern Iron and Steel External Logistics Network*, Anhui University of Technology, 2011.

[3] H. Zheng, *Research on the Distribution Logistics Network Nodes Layout of Iron and Steel Enterprise*, Beijing Jiaotong University, 2012.

[4] A. Alawneh, M. Alrefaei, A. Diabat, R. Al-Aomar and M. N. Faisal, An LP model for optimizing a supply chain management system for steel company, *Proc. of the International MultiConference of Engineers and Computer Scientists*, vol.2, 2014.

[5] W. G. Zhang, Y. J. Liu and W. J. Xu, A new fuzzy programming approach for multi-period portfolio optimization with return demand and risk control, *Fuzzy Sets and Systems*, vol.246, pp.107-126, 2014.

[6] Y. J. Liu and W. G. Zhang, A multi-period fuzzy portfolio optimization model with minimum transaction lots, *European Journal of Operational Research*, vol.242, pp.933-941, 2015.

[7] W. G. Zhang, Y. J. Liu and W. J. Xu, A possibilistic mean-semivariance-entropy model for multi-period portfolio selection with transaction costs, *European Journal of Operational Research*, vol.222, pp.341-349, 2012.