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H_∞ CONTROL FOR SAMPLED-DATA SYSTEMS WITH INFORMATION CONSTRAINTS

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Abstract. *This paper proposes the H_∞ control approach for sampled-data systems with information constraints. The signals at the sensor-controller side and the ones at the controller-actor side are quantized, respectively. Within the recently reported sampled-data systems' framework, considering logarithmic quantizer, a unified framework of the closed-loop model is given. By Lyapunov functional approach, the H_∞ performance analysis and H_∞ controller design conditions are derived, which are in terms of LMI. An example is used to show the advantages of the proposed methods. The simulation results show that the proposed approach can maintain the corresponding closed-loop system asymptotically stable and obtain a given disturbance attenuation level.*
Keywords: Sampled-data control systems, Quantization, Time-varying sampling, H_∞ control

1. **Introduction.** The sampled-data control systems simultaneously contain continuous-time and discrete-time signals, which make the closed-loop systems hybrid. A typical sampled data system is networked control system. Because of the rapid growth of the digital hardware technologies, the sampled-data control method has been more important than other control approaches. Some researchers have applied the sampled-data control scheme to solving control problems in various systems such as delay systems [1, 2], chaotic systems [3], dynamical networks [4], and networked control systems [5]. Recently, some research focuses on the sampled-data control problems of linear systems [6, 7]. Moreover, a series of work on stability, robust control, H_2 control, H_∞ control and filtering problems for the sampled-data control systems have been investigated. Recently, [8] investigated sampled-data state-feedback stabilization and sampled-data output-feedback H_∞ control problem of linear systems via Lyapunov-Krasovskii functionals and descriptor approach. [9] investigated sampled-data H_∞ control and filtering problem of linear systems based on descriptor approach. [10] gave a refined sampled-data control approach. However, all the above literature considered that the information of sampled-data systems is not limited. In fact, in many sampled-data systems, when measurements to be used for feedback are transmitted by a digital communication channel, data are quantized before transmission. Therefore, to achieve better performance of the considered systems, the effect of data quantization on the systems should be taken into consideration. This motivates the present study. To the best of the author's knowledge, the sampled-data control systems with information constraints have not been fully investigated and still remain challenging.

The rest of the paper is organized as follows. In Section 2, we first introduce the characteristics of sampled-data systems with information constraints, and then develop the model of sampled-data system to describe both time-varying sampling and signal quantization in a unified framework. Sections 3 deals with the stability analysis and controller design of the sampled-data systems, respectively. The proposed approach is illustrated in Section 4 through a numerical example. Section 5 concludes the paper.

2. **Problem Formulation.**

Consider the following system:

$$\dot{x}(t) = Ax(t) + Bu(t) + B_\omega \omega(t), \tag{1}$$

$$z(t) = Cx(t) + Du(t), \tag{2}$$

where $x(t) \in \mathbf{R}^n$ is the system state, $\omega(t) \in \mathbf{R}^m$ is the disturbance, $u(t) \in \mathbf{R}^p$ is the control input, and $z(t) \in \mathbf{R}^q$ is the controlled output. A, B, B_ω, C, D are some constant matrices of appropriate dimensions.

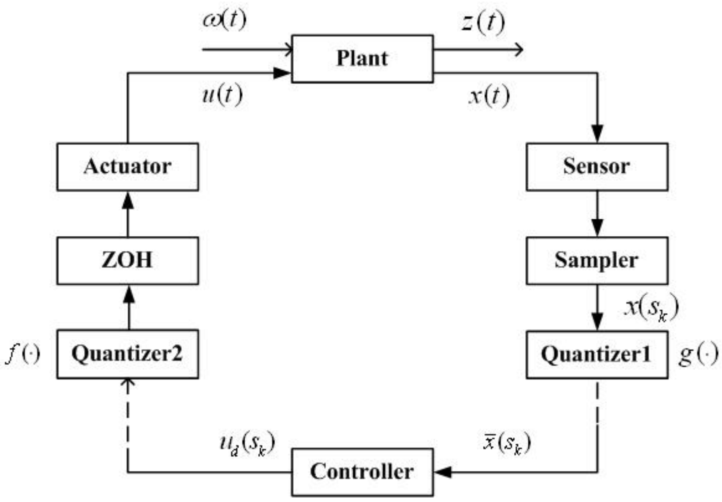

FIGURE 1. Structure of the sampled-data control systems with two quantizers

In this paper, both the state and control input signals are quantized before they are sent to the controller and actuator respectively. Figure 1 shows the structure of sampled-data control system with two quantizers. At the same time, it is assumed that the sensor and sampler is clock-driven, while the controller, ZOH (zero-order holder), actuator and quantizers are event-driven. The sampling intervals are time-varying and the sampling instants are denoted as s_k, $k = 1, \cdots, \infty$. The control input $\bar{x}(s_k)$ satisfies that

$$\bar{x}(s_k) = g(x(s_k)). \tag{3}$$

Define the zero-order holder control action

$$u_c(t) = f(u_d(s_k)) = f(K\bar{x}(s_k)), \quad s_k \le t < s_{k+1}, \tag{4}$$

where K is the feedback gain to be determined. $f(\cdot)$ and $g(\cdot)$ are two logarithmic quantizers. The definition of logarithmic quantizers has been described by [11]. u_d is a discrete-time control signal and the time s_k is the sampling instant satisfying $0 = s_0 < s_1 < \cdots < s_k < \cdots$. The sampling interval $T_k = s_{k+1} - s_k$ may vary but it is bounded.

$$0 < T_m \le T_k \le T_M. \tag{5}$$

The digital control law may be represented as follows by using input delay approach [8]:

$$u_c(t) = f(Kg(x(s_k)))f(Kg(x(t-\tau(t)))), \quad s_k \le t < s_{k+1}, \tag{6}$$

where $\tau(t) = t - s_k$ is piecewise linear with the derivative $\dot{\tau}(t) = 1$ for $t \ne s_k$. The piecewise-constant control law (6) can be represented as a continuous-time controller with a time-varying piecewise continuous delay:

$$u(t) = f(Kg(x(t-d(t)))), \ s_k \le t < s_{k+1}, \tag{7}$$

where $d(t) = t - s_k$ is piecewise linear with the derivative $\dot{d}(t) = 1$ for $t \neq s_k$. It is assumed that $d(t)$ is bounded that satisfies $T_m \leq d(t) \leq T_M$.

In terms of the method given in [11]. Define $\Delta_f = diag(\Delta_{f1}, \Delta_{f2}, \ldots, \Delta_{fm})$, $\Delta_g = diag(\Delta_{g1}, \Delta_{g2}, \ldots, \Delta_{gn})$. Then, $f(\cdot)$ and $g(\cdot)$ can be written as

$$f(u_d) = (I + \Delta_f)u_d, \tag{8}$$

$$g(x) = (I + \Delta_g)x, \tag{9}$$

where I denotes the identity matrix of appropriate dimensions. For simplicity, it is assumed that $\Delta_{f_i} \in [-\delta_f, \delta_f]$ and $\Delta_{g_j} \in [-\delta_g, \delta_g]$. Combining (8) and (9), and substituting the controller (7) into systems (1) and (2), we can obtain the closed-loop sampled-data control system:

$$\dot{x}(t) = Ax(t) + B(K + \Delta(K))x(t - d(t)) + B_\omega \omega(t), \quad s_k \leq t < s_{k+1}, \tag{10}$$

$$z(t) = Cx(t) + D(K + \Delta(K))x(t - d(t)), \quad s_k \leq t < s_{k+1}, \tag{11}$$

where $\Delta(K) = \Delta_f K + K\Delta_g + \Delta_f K \Delta_g$.

Our objective is to find a state-feedback controller K, which stabilizes systems (1) and (2) with $\omega(t) = 0$ and makes it satisfy

$$J = \int_0^\infty \left(z^T(t)z(t) - \gamma^2 \omega^T(t)\omega(t)\right) dt < 0, \tag{12}$$

for $x(0) = 0$ and all $\omega(t) \neq 0$. The scalar γ is a prescribed positive scalar and indicates an H_∞ disturbance attenuation level.

3. Main Results.

3.1. Robust H_∞ performance analysis for the sampled-data control systems with information constraints.

Theorem 3.1. *For given scalars T_m, T_M, $\delta_f > 0$, $\delta_g > 0$, $0 \leq \alpha < 1$, $\gamma > 0$, and a matrix K, the closed-loop sampled-data system (10) and (11) is asymptotically stable with an H_∞ disturbance attenuation level γ if there exist matrices $P = P^T > 0$, $Q_m = Q_m^T > 0$ ($m = 1, 2, 3$), $Z_j = Z_j^T > 0$ ($j = 1, 2, 3$), and N_i, T_i, M_i, E_i, L_i ($i = 1, 2, 3, 4, 5, 6$) such that*

$$\Gamma = \begin{bmatrix} \Gamma_1 & \Gamma_2 & \mathcal{B}^T Z_1 & \mathcal{B}^T Z_2 & \mathcal{B}^T Z_3 & \mathcal{C}^T \\ * & \Gamma_3 & 0 & 0 & 0 & 0 \\ * & * & -\frac{1}{T_M - T_m} Z_1 & 0 & 0 & 0 \\ * & * & * & -\frac{1}{T_M} Z_2 & 0 & 0 \\ * & * & * & * & -\frac{1}{T_M} Z_3 & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{13}$$

where

$$\Gamma_1 = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \Gamma_{15} & \Gamma_{16} \\ * & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25} & \Gamma_{26} \\ * & * & \Gamma_{33} & \Gamma_{34} & \Gamma_{35} & \Gamma_{36} \\ * & * & * & \Gamma_{44} & \Gamma_{45} & \Gamma_{46} \\ * & * & * & * & \Gamma_{55} & \Gamma_{56} \\ * & * & * & * & * & \Gamma_{66} \end{bmatrix}, \ \Gamma_2 = \begin{bmatrix} N_1 & T_1 & M_1 & E_1 & L_1 \\ N_2 & T_2 & M_2 & E_2 & L_2 \\ N_3 & T_3 & M_3 & E_3 & L_3 \\ N_4 & T_4 & M_4 & E_4 & L_4 \\ N_5 & T_5 & M_5 & E_5 & L_5 \\ N_6 & T_6 & M_6 & E_6 & L_6 \end{bmatrix},$$

$$\Gamma_3 = diag\left\{-\frac{1}{\alpha T_M} Z_1 - \frac{1}{(1-\alpha)T_M} Z_1 - \frac{1}{T_M - T_m}(Z_1 + Z_2) - \frac{1}{T_M} Z_2 - \frac{1}{T_M} Z_3\right\},$$

$$\Gamma_{11} = PA + A^T P + \sum_{i=1}^{3} Q_i + N_1 + N_1^T + L_1 + L_1^T,$$

$$\Gamma_{12} = P(BK + \Delta(K)) + N_2^T - T_1 + M_1 - E_1 + L_2^T,$$

$\Gamma_{13} = E_1 + N_3^T + L_3^T,\ \Gamma_{14} = -M_1 + N_4^T + L_4^T,\ \Gamma_{15} = T_1 - N_1 + N_5^T + L_5^T,$
$\Gamma_{16} = N_6^T + L_6^T,\ \Gamma_{22} = M_2 + M_2^T - T_2 - T_2^T - E_2 - E_2^T,\ \Gamma_{23} = E_2 + M_3^T - T_3^T - E_3^T,$
$\Gamma_{24} = - M_2 + M_4^T - T_4^T - E_4^T,\ \Gamma_{25} = T_2 + N_2 + M_6^T - T_6^T - E_6^T,$
$\Gamma_{26} = M_5^T - T_5^T - E_5^T,\ \Gamma_{33} = -Q_1 + E_3 + E_3^T,\ \Gamma_{34} = -M_3 + E_4^T - L_3,$
$\Gamma_{35} = T_3 - N_3 + E_5^T,\ \Gamma_{36} = E_6^T,\ \Gamma_{44} = -Q_2 - M_4 - M_4^T,\ \Gamma_{45} = T_4 - N_4 - M_5^T - L_5^T,$
$\Gamma_{46} = - M_6^T - L_6^T,\ \Gamma_{55} = -(1-\alpha)Q_3 + T_5 - N_5 + T_5^T - N_5^T,$
$\Gamma_{56} = T_6^T - N_6^T,\ \Gamma_{66} = -\gamma^2 I,$
$\mathcal{B} = \begin{bmatrix} A & B(K+\Delta(K)) & 0 & 0 & 0 & B_\omega \end{bmatrix},\ \mathcal{C} = \begin{bmatrix} C & D(K+\Delta(K)) & 0 & 0 & 0 & 0 \end{bmatrix}.$

Proof: Consider the following Lyapunov-Krasovskii functional

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t) + V_7(t), \tag{14}$$

where

$$V_1(t) = x^T(t)Px(t),\ V_2(t) = \int_{t-T_m}^{t} x(s)^T Q_1 x(s)ds,\ V_3(t) = \int_{t-T_M}^{t} x(s)^T Q_2 x(s)ds,$$
$$V_4(t) = \int_{t-\alpha d(t)}^{t} x(s)^T Q_3 x(s)ds,\ V_5(t) = \int_{-T_M}^{0}\int_{t+\beta}^{t} \dot{x}^T(s) Z_1 \dot{x}(s)dsd\beta,$$
$$V_6(t) = \int_{-T_M}^{-T_m}\int_{t+\beta}^{t} \dot{x}^T(s) Z_2 \dot{x}(s)dsd\beta,\ V_7(t) = \int_{-T_M}^{0}\int_{t+\beta}^{t} \dot{x}^T(s) Z_3 \dot{x}(s)dsd\beta,$$
$$P = P^T > 0,\ Q_m = {Q_m}^T > 0\ (m = 1,2,3),\ Z_j = {Z_j}^T > 0\ (j = 1,2,3).$$

1) For $t_k < t < t_{k+1}$, calculating the derivative of $V(t)$ with respect to t along the solutions of the system (10) and (11) and using Leibniz-Newton formula, it yields that

$$\begin{aligned}
&\dot{V}(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) \\
=& 2x^T(t)P\dot{x}(t) + \sum_{i=1}^{3} x^T(t)Q_i x(t) - x^T(t-T_m)Q_1 x(t-T_m) - x^T(t-T_M)Q_2 x(t-T_M) \\
&- \left(1-\alpha\dot{d}(t)\right) x^T(t-\alpha d(t))Q_3 x(t-\alpha d(t)) + \dot{x}^T(t)(T_M Z_1 + h_{12}Z_2 + T_M Z_3)\dot{x}(t) \\
&- \int_{t-\alpha d(t)}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-d(t)}^{t-\alpha d(t)} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-T_M}^{t-d(t)} \dot{x}^T(s)(Z_1+Z_2)\dot{x}(s)ds \\
&- \int_{t-d(t)}^{t-T_m} \dot{x}^T(s)Z_2\dot{x}(s)ds - \int_{t-T_M}^{t} \dot{x}^T(s)Z_3\dot{x}(s)ds + z(t)^T z(t) - \gamma^2\omega^T(t)\omega(t) \\
\le& 2x^T(t)P(Ax(t) + BKx(t-d(t)) + B_\omega\omega(t)) + \sum_{i=1}^{3} x^T(t)Q_i x(t) \\
&- x^T(t-T_m)Q_1 x(t-T_m) - x^T(t-T_M)Q_2 x(t-T_M) \\
&- (1-\alpha)x^T(t-\alpha d(t))Q_3 x(t-\alpha d(t)) + \dot{x}^T(t)(T_M Z_1 + (T_M - T_m)Z_2 + T_M Z_3)\dot{x}(t) \\
&- \int_{t-\alpha d(t)}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-d(t)}^{t-\alpha d(t)} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-T_M}^{t-d(t)} \dot{x}^T(s)(Z_1+Z_2)\dot{x}(s)ds \\
&- \int_{t-d(t)}^{t-T_m} \dot{x}^T(s)Z_2\dot{x}(s)ds - \int_{t-T_M}^{t} \dot{x}^T(s)Z_3\dot{x}(s)ds \\
&+ 2\zeta^T(t)N\left[x(t) - x(t-\alpha d(t)) - \int_{t-\alpha d(t)}^{t} \dot{x}(s)ds\right]
\end{aligned}$$

$$
\begin{aligned}
&+ 2\zeta^T(t)T\left[x(t-\alpha d(t)) - x(t-d(t)) - \int_{t-d(t)}^{t-\alpha d(t)} \dot{x}(s)ds\right] \\
&+ 2\zeta^T(t)M\left[x(t-d(t)) - x(t-T_M) - \int_{t-T_M}^{t-d(t)} \dot{x}(s)ds\right] \\
&+ 2\zeta^T(t)E\left[x(t-T_m) - x(t-d(t)) - \int_{t-d(t)}^{t-T_m} \dot{x}(s)ds\right] \\
&+ 2\zeta^T(t)L\left[x(t) - x(t-T_M) - \int_{t-T_M}^{t} \dot{x}(s)ds\right] \\
&+ (Cx(t) + DKx(t-d(t)))^T(Cx(t) + DKx(t-d(t))) - \gamma^2\omega^T(t)\omega(t) \\
\le{}& \zeta^T(t)\left(\Gamma_1 + \alpha T_M N Z_1^{-1} N^T + (1-\alpha)T_M T Z_1^{-1} T^T + h_{12} M(Z_1+Z_2)^{-1} M^T\right. \\
&\left. + h_{12} E Z_2^{-1} E^T + T_M L {Z_3}^{-1} L^T + \mathcal{B}^T(T_M Z_1 + (T_M - T_m)Z_2 + T_M Z_3)\mathcal{B} + \mathcal{C}^T\mathcal{C}\right)\zeta(t) \\
&- \int_{t-\alpha d(t)}^{t} \mathcal{H}_1 Z_1^{-1} {\mathcal{H}_1}^T ds - \int_{t-d(t)}^{t-\alpha d(t)} \mathcal{H}_2 Z_1^{-1} {\mathcal{H}_2}^T ds - \int_{t-T_M}^{t-d(t)} \mathcal{H}_3 (Z_1+Z_2)^{-1} {\mathcal{H}_3}^T ds \\
&- \int_{t-d(t)}^{t-T_m} \mathcal{H}_4 Z_2^{-1} {\mathcal{H}_4}^T ds - \int_{t-T_M}^{t} \mathcal{H}_5 Z_3^{-1} {\mathcal{H}_5}^T ds,
\end{aligned}
$$

where

$$
\begin{aligned}
&\mathcal{H}_1 = \zeta^T(t)N + \dot{x}^T(s)Z_1,\ \mathcal{H}_2 = \zeta^T(t)T + \dot{x}^T(s)Z_1, \\
&\mathcal{H}_3 = \zeta^T(t)M + \dot{x}^T(s)(Z_1+Z_2),\ \mathcal{H}_4 = \zeta^T(t)E + \dot{x}^T(s)Z_2,\ \mathcal{H}_5 = \zeta^T(t)L + \dot{x}^T(s)Z_3, \\
&\zeta(t) = \begin{bmatrix} x^T(t) & x^T(t-d(t)) & x^T(t-T_m) & x^T(t-T_M) & x^T(t-\alpha d(t)) & \omega^T(t) \end{bmatrix}^{\mathrm{T}}, \\
&N = \begin{bmatrix} N_1^T & N_2^T & N_3^T & N_4^T & N_5^T & N_6^T \end{bmatrix}^T, \\
&T = \begin{bmatrix} T_1^T & T_2^T & T_3^T & T_4^T & T_5^T & T_6^T \end{bmatrix}^T,\ M = \begin{bmatrix} M_1^T & M_2^T & M_3^T & M_4^T & M_5^T & M_6^T \end{bmatrix}^T, \\
&E = \begin{bmatrix} E_1^T & E_2^T & E_3^T & E_4^T & E_5^T & E_6^T \end{bmatrix}^T,\ L = \begin{bmatrix} L_1^T & L_2^T & L_3^T & L_4^T & L_5^T & L_6^T \end{bmatrix}^T.
\end{aligned}
$$

By the Schur complements, combine (13) to obtain $\dot{V}(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) < 0$ for all $t_k < t < t_{k+1}$. On the other hand, we prove the asymptotic stability of system (10). In this case, the external perturbation $\omega(t)$ is assumed to be zero. Then, using the Lyapunov functional (14), by Schur complements, it follows from (13) that there exists a scalar $\varepsilon > 0$ such that $\dot{V}(t) \le -\varepsilon\|x(t)\|^2$. This implies that system (10) with $\omega = 0$ is asymptotically stable for $t_k < t < t_{k+1}$.

2) It is noting that $d(t_k) = t_k - s_k$, $\forall k \in N$, $d(t_k^-) = t_k - s_{k-1}$, $\forall k \in N$.

The value of x before and after t_k pionts remains unchanged (since $x(t)$ is continuous). Then, we have $V_i(t_k^-) = V_i(t_k)$ $(i = 1, 2, 3, 5, 6, 7)$ in Lyapunov-Krasovskii functional (14). Moreover, for $V_4(t)$, there exists $V_4(t_k^-) \ge V_4(t_k)$. Thus, $V(t_k^-) \ge V(t_k)$ for $k = 0, 1, 2, 3, \cdots$.

For $t \in [\, t_k, \ \ t_{k+1} \,)$, we have

$$
V(t) - V(t_k) \le \int_{t_k}^{t} -z^T(s)z(s) + \gamma^2\omega^T(s)\omega(s)ds.
$$

It follows that $\|z(t)\|_2 \le \gamma\|\omega(t)\|_2$. This completes the proof.

3.2. **H_∞ controller design for the sampled-data control systems with information constraints.** This section is devoted to solving the problem of H_∞ controller design for sampled-data systems (10) and (11). The following theorem presents the conditions for existence of the desired controller.

Theorem 3.2. *Consider the sampled-data in Figure 1. For given scalars T_m, T_M, $\delta_g > 0$, $\delta_f > 0$, $0 \leq \alpha < 1$, $0 \leq \varepsilon_i < 1$ $(i = 1, 2, 3)$ the closed-loop system (10) and (11) is asymptotically stable with an H_∞ disturbance attenuation level γ if there exist matrices $X = X^T > 0$, $\tilde{Q}_i = \tilde{Q}_i^T > 0$ $(i = 1, 2, 3)$, $\tilde{Z}_j = \tilde{Z}_j^T > 0$ $(j = 1, 2, 3)$, $\tilde{N}_l$, $\tilde{T}_l$, $\tilde{M}_l$, $\tilde{E}_l$, $\tilde{L}_l$ $(l = 1, 2, 3, 4, 5, 6)$, and Y of appropriate dimensions, such that*

$$\begin{bmatrix} \Xi & \tilde{B} & \tilde{Y} \\ * & -\frac{1}{\gamma_1} I & 0 \\ * & * & -\frac{1}{\gamma_2} I \end{bmatrix} < 0, \tag{15}$$

where

$$\Xi = \begin{bmatrix} \Xi_1 & \Xi_2 & A_L & A_L & A_L & C_L \\ * & \Xi_3 & 0 & 0 & 0 & 0 \\ * & * & \frac{1}{\sigma}(\tilde{Z}_1 - 2X) & 0 & 0 & 0 \\ * & * & * & \frac{1}{\sigma}(\tilde{Z}_2 - 2X) & 0 & 0 \\ * & * & * & * & \frac{1}{\sigma}(\tilde{Z}_3 - 2X) & 0 \\ * & * & * & * & * & -I \end{bmatrix}, \tag{16}$$

$$\Xi_1 = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} \\ * & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} \\ * & * & * & * & \Xi_{55} & \Xi_{56} \\ * & * & * & * & * & \Xi_{66} \end{bmatrix}, \ \Xi_2 = \begin{bmatrix} \tilde{N}_1 & \tilde{T}_1 & \tilde{M}_1 & \tilde{E}_1 & \tilde{L}_1 \\ \tilde{N}_2 & \tilde{T}_2 & \tilde{M}_2 & \tilde{E}_2 & \tilde{L}_2 \\ \tilde{N}_3 & \tilde{T}_3 & \tilde{M}_3 & \tilde{E}_3 & \tilde{L}_3 \\ \tilde{N}_4 & \tilde{T}_4 & \tilde{M}_4 & \tilde{E}_4 & \tilde{L}_4 \\ \tilde{N}_5 & \tilde{T}_5 & \tilde{M}_5 & \tilde{E}_5 & \tilde{L}_5 \\ \tilde{N}_6 & \tilde{T}_6 & \tilde{M}_6 & \tilde{E}_6 & \tilde{L}_6 \end{bmatrix},$$

$$\begin{aligned}
\Xi_{11} &= AX + XA^T + \sum_{i=1}^{3} \tilde{Q}_i + \tilde{N}_1 + \tilde{N}_1^T + \tilde{L}_1 + \tilde{L}_1^T, \\
\Xi_{12} &= BY + \tilde{N}_2^T - \tilde{T}_1 + \tilde{M}_1 - \tilde{E}_1 + \tilde{L}_2^T, \ \Xi_{13} = \tilde{E}_1 + \tilde{N}_3^T + \tilde{L}_3^T, \\
\Xi_{14} &= -\tilde{M}_1 + \tilde{N}_4^T + \tilde{L}_4^T, \ \Xi_{15} = \tilde{T}_1 - \tilde{N}_1 + \tilde{N}_5^T + \tilde{N}_5^T, \ \Xi_{16} = \tilde{N}_6^T + \tilde{L}_6^T \\
\Xi_{22} &= \tilde{M}_2 + \tilde{M}_2^T - \tilde{T}_2 - \tilde{T}_2^T - \tilde{E}_2 - \tilde{E}_2^T, \ \Xi_{23} = \tilde{E}_2 + \tilde{M}_3^T - \tilde{T}_3^T - \tilde{E}_3^T, \\
\Xi_{24} &= -\tilde{M}_2 - \tilde{L}_2 + \tilde{M}_4^T - \tilde{T}_4^T - \tilde{E}_4^T, \ \Xi_{25} = \tilde{T}_2 - \tilde{N}_2 + \tilde{M}_6^T - \tilde{T}_6^T - \tilde{E}_6^T, \\
\Xi_{26} &= \tilde{M}_6^T - \tilde{T}_6^T - \tilde{E}_6^T, \ \Xi_{33} = -\tilde{Q}_1 + \tilde{E}_3 + \tilde{E}_3^T, \ \Xi_{34} = -\tilde{M}_3 + \tilde{E}_4^T - \tilde{L}_3, \\
\Xi_{35} &= \tilde{T}_3 - \tilde{N}_3 + \tilde{E}_5^T, \ \Xi_{36} = \tilde{E}_6^T, \ \Xi_{44} = -\tilde{Q}_2 - \tilde{M}_4 - \tilde{M}_4^T, \ \Xi_{45} = \tilde{T}_4 - \tilde{N}_4 - \tilde{M}_5^T, \\
\Xi_{46} &= -\tilde{M}_6^T - \tilde{L}_6^T, \ \Xi_{55} = -(1-\alpha)Q_3 + \tilde{T}_5 - \tilde{N}_5 + \tilde{T}_5^T - \tilde{N}_5^T, \\
\Xi_{56} &= \tilde{T}_6^T - \tilde{N}_6^T, \ \Xi_{66} = -\gamma^2 I, \\
A_L &= \begin{bmatrix} AX^T & BY & 0 & 0 & 0 & B_\omega \end{bmatrix}^T, \ C_L = \begin{bmatrix} CX^T & DY & 0 & 0 & 0 & B_\omega \end{bmatrix}^T, \\
\Xi_3 &= diag\left\{ -\frac{1}{\alpha T_M}\tilde{Z}_1 \ -\frac{1}{(1-\alpha)T_M}\tilde{Z}_1 - \frac{1}{T_M - T_m}(\tilde{Z}_1 + \tilde{Z}_2) \ -\frac{1}{T_M - T_m}\tilde{Z}_2 \ -\frac{1}{T_M}\tilde{Z}_3 \right\}, \\
\tilde{B} &= \begin{bmatrix} B^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & B^T & B^T & B^T & D^T \end{bmatrix}^T, \\
\tilde{Y} &= \begin{bmatrix} 0 & Y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
\gamma_2 &= \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \delta_g^2, \ \gamma_1 = \frac{1}{\varepsilon_1}\delta_f^2 + \frac{1}{\varepsilon_2}\delta_g^2 + \frac{1}{\varepsilon_3}\delta_f^2.
\end{aligned} \tag{17}$$

In this case, the state-feedback gain is given by

$$K = YX^{-1}.$$

4. **Numerical Example.** Consider the following system:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix} u(t) + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \omega(t), \tag{18}$$

$$z(t) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix} x(t). \tag{19}$$

Choosing the bound of sampling intervals is $T_M = 1.1s$, quantizer parameters are $\delta_g = 0.1$ and $\delta_f = 0.1$. By using Theorem 3.2, solving the LMI problem (15), one can obtain the state feedback controller gain $K = \begin{bmatrix} -1.5964 & -2.4346 \end{bmatrix}$, and the H_∞ disturbance attenuation level $\gamma = 0.4291$. The disturbance signal $\omega(t)$ is given as

$$\omega(t) = \begin{cases} 2\sin 2t, & \text{for } 5 \le t \le 15 \\ 0, & \text{for } t < 5 \text{ or } t > 15 \end{cases} \tag{20}$$

Figure 2 and Figure 3 show the control input and the state response under the proposed control scheme, respectively, where the initial state of the system is $x_0 = [0,\ 0]^T$. When the maximum allowable upper bound of the sampling interval is $1.1s$, the quantizer parameters are $\delta_g = 0.1$ and $\delta_f = 0.1$, and the closed-loop system is asymptotically stable with above obtained control gain. It shows the effectiveness of quantized controller design method proposed in this work.

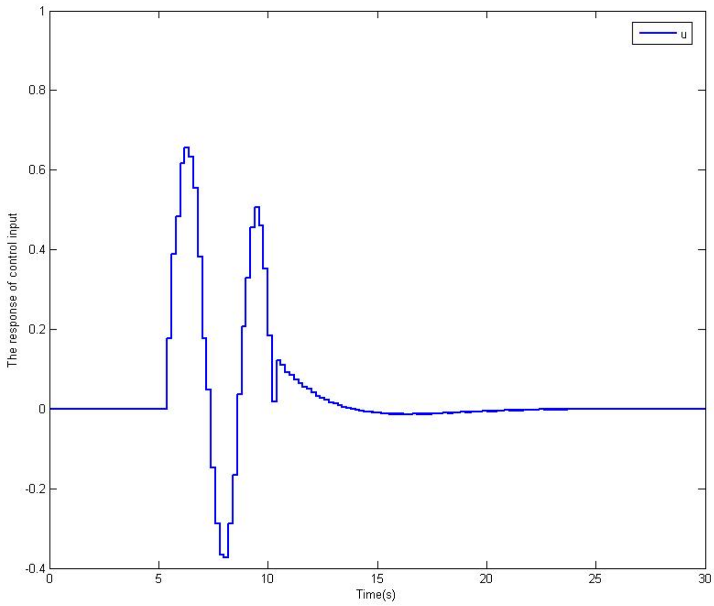

FIGURE 2. The control input of the sampled-data system with two quantizers

5. **Conclusions.** This paper investigates the problem of quantized H_∞ control for a sampled-data system with information constraints whereby the system is continuous-time and the controller is discrete-time signal. By using Lyapunov functional approach, the relation of quantization parameter and H_∞ performance index of the systems is explored. Moreover, the proposed H_∞ performance analysis and H_∞ controller design conditions can be presented in terms of linear matrix inequalities (LMIs). A numerical example demonstrates the effectiveness of the proposed methods. In the future work, the case of

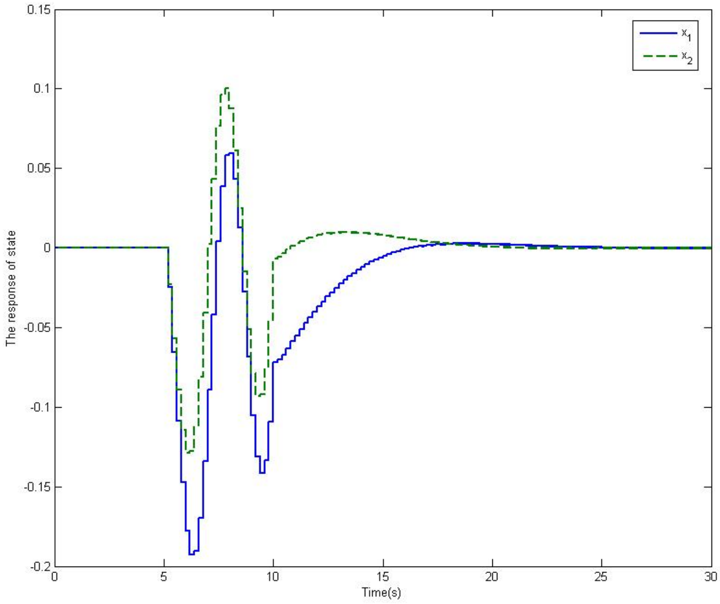

Figure 3. State response of the sampled-data system with two quantizers

the nonlinear sampled-data systems with quantization and variable sampling will be considered. We will investigate the control problem for the nonlinear sampled-data systems with information constraints.

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