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MATHEMATICAL MODEL FOR AGGREGATE COLLABORATION PLANNING WITH SLOT CHARTERING IN LINER SHIPPING

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Abstract. *Strategic alliance has been recognized as an effective principle for obtaining a larger service market with limited resources in industry fields. In particular, it was earlier applied to liner shipping with several types of collaborations such as joint fleet, slot exchange, slot chartering, slot purchase, and share of port usage for avoiding excessive investment and excessive competition. This study proposes a mathematical model for minimizing the total cost required to transport all the container shipping demands under a slot chartering agreement with other liner shipping companies, which was formulated as a linear model. A slot chartering space level is added to a decision variable list. An illustrative example is provided to verify the appropriateness of the suggested problem.*
Keywords: Liner shipping, Strategic alliance, Aggregate collaboration planning, Slot chartering, Linear model

1. **Introduction.** Strategic alliance among liner shipping companies is very important for avoiding excessive competition and for cost saving. There are various types of strategic alliances for liner shipping companies: joint fleet, slot exchange, slot chartering, slot purchase, and share of port usage [6], of which slot chartering is the most widely used. Slot chartering means that a liner shipping company rents some shipping space on another company's ship for container transportation.

In this study, we present a mathematical model which minimizes the total cost required to transport all the container shipping demand under a slot chartering agreement with other liner shipping companies. Chen and Zhen [3] developed a mathematical model similar to the problem in which there were still some limitations to apply their model in the real world due to the non-linear programming model. The model considered in this study is a linear model and so may be an advanced model compared to Chen and Zhen's model. In addition, this study attempts to enhance the applicability of the proposed model by considering the determination of the chartering space level which has never been dealt with in previous studies.

There have been some studies related to the container slot chartering problem. Ting and Tzeng [8] proposed a conceptual model for liner shipping revenue management and mathematical formulation for slot allocation. Lei *et al.* [5] suggested two strategies for collaboration between two liner shipping companies and compared them to the non-collaboration case. Chen and Zhen [4] proposed a container slot exchange model which was more advanced than slot chartering, but it belongs to the non-linear category. Lu *et al.* [6] found through the use of the Delphi method that strategic alliance became an essential tool for carriers to extend their service ranges in the global market. Mutual trust between partners serves as a corner stone to ensure the success of alliances. In a study by Shi *et al.* [7], the liner carriers who were involved in slot chartering agreements were regarded as the players, and the pay-off of the games should be win-win games rather than zero-sum games. Its main idea is to explain the negotiation stages as well as to design an efficient mechanism for balancing the slot requirements and the equilibrium prices under different circumstances in slot chartering agreements. For the design of the service network in liner shipping, Agarwal and Ergun [1] presented an integrated model, a mixed-integer linear program, to solve ship scheduling and cargo routing problems, simultaneously. A greedy heuristic, a column generation-based algorithm, and a two-phase Benders decomposition-based algorithm were developed. In addition, they studied transportation networks that operate as an alliance among different carriers, especially alliance formation among carriers in liner shipping. They addressed tactical problems such as the design of large-scale networks and operational problems such as the allocation of limited capacity in a transportation network among the carriers in the alliance [2]. Recently, Wang *et al.* developed game-theoretical models to analyze shipping competition between two carriers in a new emerging linear shipping market [9]. We present a mixed integer linear programming model and propose a solution procedure based on Excel Solver. In order to verify the applicability of the proposed model, we apply it to a numerical example.

The remainder of the paper is as follows. Section 2 describes the definition of the problem and a mathematical model. A solution procedure is introduced through a numerical example in Section 3. The conclusions and further research areas appear in Section 4.

2. **Model Design.** This section describes a mathematical model which minimizes the total transportation cost. The underlying assumptions of the suggested model are as follows.

(1) The planning time period is given (for example, 3 months, 6 months, or 1 year).

(2) The routes for company's ships voyage are known.

(3) The demand for the containers transported is given for each route per planning time period.

(4) A penalty cost is incurred for containers that are not delivered during the planning time period.

(5) The company charters container loading space from the cooperating company's ship.

(6) There are some space levels that can be chartered from the cooperating ship. Each level is different regarding container loading capacity. The company may choose one of different space levels from cooperating ship.

(7) The company pays a fixed cost for chartering space level, regardless of the number of containers loaded.

The following parameters and variables are introduced to formulate the above mentioned problem:

(Parameters)

I: Set of ship types owned by our company

K: Set of routes

r_i: Number of type i ships owned by our company, $i \in I$

J_k: Set of cooperating ships that voyage in route k, $k \in K$

T_j: Set of space levels chartered from cooperating ship j, $j \in J_k$

D_k: Transportation demand for route k in planning period, $k \in K$

c_{ik}: Transportation cost of type i ship on the route k for one time voyage, $i \in I$, $k \in K$

f_{tjk}: Fixed cost for chartering space level t from cooperating ship j that voyages route k, $t \in T_j$, $j \in J_k$, $k \in K$

m_{ik}: Maximum number of consecutive voyages for ship type i on route k in the planning period, $i \in I$, $k \in K$

p_k: Per-unit penalty cost for un-delivered container for route k, $k \in K$

u_k: Maximum tolerable limit for un-delivered container for route k, $k \in K$

r_i: Number of type i ship owned by our company, $i \in I$

Q_i: Capacity of type i ship, $i \in I$

q_{tjk}: Maximum amount loaded on chartering space level t of cooperating ship j on route k, $t \in T_j$, $j \in J_k$, $k \in K$

(Decision Variables)

x_{ik}: Number of consecutive voyages for type i ship on route k in the planning period

y_{tjk}: 0-1 variable which has 1 if space level t is chartered from cooperating ship j on route k, 0 otherwise, $t \in T_j$, $j \in J_k$, $k \in K$

z_k: Number of undelivered container on route k in the planning period

Using the above parameters and variables, a slot chartering problem can be formulated as the following mixed integer linear programming model:

(P)

$$\text{Min Z} = \sum_{i \in I} \sum_{k \in K} c_{ik} x_{ik} + \sum_{t \in T_j} \sum_{j \in J_k} \sum_{k \in K} f_{tjk} y_{tjk} + \sum_{k \in K} p_k z_k \tag{1}$$

s.t.

$$\sum_{k \in K} \frac{1}{m_{ik}} x_{ik} \le r_i, \qquad i \in I \tag{2}$$

$$\sum_{i \in I} Q_i x_{ik} + \sum_{t \in T_j} \sum_{j \in J_k} q_{tjk} y_{tjk} + z_k \ge D_k, \qquad k \in K \tag{3}$$

$$\sum_{t \in T_j} y_{tjk} \le 1, \qquad j \in J_k,\ k \in K \tag{4}$$

$$z_k \le u_k, \qquad k \in K \tag{5}$$

$$x_{ik} \ge 0,\ x_{ik}\text{: integer}, \qquad i \in I,\ k \in K \tag{6}$$

$$z_k \ge 0, \qquad k \in K \tag{7}$$

$$y_{tjk} \in \{0, 1\}, \qquad t \in T_j,\ j \in J_k,\ k \in K \tag{8}$$

Objective function (1) expresses total cost, which consists of the transportation cost for the company-owned ship, the fixed cost for the cooperating ship, and the penalty cost for un-delivered container. Constraint (2) means that each company-owned ship has a limit for voyage frequencies. Constraint (3) shows that the container shipping demand for each route can be transported by the company-owned ship as well as the cooperating ship, after which left-over containers are considered in the un-delivered amount. The reason why inequality "$\ge$" is used is because total transshipment amounts in a route may exceed the demands. This means that the transportation capacities of ships can be greater than the demands because batch sized space is chartered. Constraint (4) describes that in the case of chartering a slot from cooperating ship at most one level of space can be chartered. Constraint (5) shows that the number of un-delivered containers should not be greater than the predetermined amount on each route in the planning period. Constraints (6)-(8) are variable constraints. In particular, constraint (8) represents a binary constraint which expresses whether or not the container slot is chartered from the cooperating ship, and which type of space level is selected if a slot is chartered.

3. **Solution Procedure.**

3.1. **Numerical example.** An illustrative example is carried out to verify the approapriateness of the proposed model with experimental data similar to Chen and Zhen's model [3,4]. We assume that our company cooperates with another liner shipping company in the form of slot chartering. Our company has 3 types of ships: type 1 (6,000TEU), type 2 (3,400TEU), and type 3 (2,000TEU). There are 4 liner shipping routes on which our company has to transport containers within the planning time period. Our company has 6 6,000TEU ships, 8 3,400TEU ships, and 8 2,000TEU ships. Five cooperating company's ships are available for each route. The maximum number of voyages for each type of ship on each route within the planning time period is given in Table 1.

TABLE 1. Maximum number of voyages for each type of ship on each route

Type \ Route	1	2	3	4
1	3	2	3	1
2	4	3	3	2
3	5	5	5	2

TABLE 2. Transportation cost c_{ik} (unit: thousand USD)

i \ k	1	2	3	4
1	1,000	1,100	1,200	1,400
2	800	900	1,000	1,000
3	600	800	800	900

TABLE 3. Chartering cost and penalty cost (unit: thousand USD)

Cost	Route			
	1	2	3	4
f_{1jk}	500	600	600	800
f_{2jk}	800	900	1,000	1,200
f_{3jk}	1,200	1,400	1,500	1,800
p_k	3	1.25	2	1.75

TABLE 4. Demand and maximum tolerable limit of un-delivered containers

	Route			
	1	2	3	4
D_k	40,000	80,000	21,000	80,000
u_k	2,000	4,000	1,050	4,000

The transportation cost for one consecutive voyage of the type i ship on route k is given in Table 2.

It is assumed that there are 3 types of space levels for slot chartering on each cooperating ship: 1,000TEU, 1,500TEU, and 2,000TEU. The chartering cost for each type of space level and per-unit penalty cost for un-delivered TEU are shown in Table 3.

Table 4 shows the transportation demand for containers for each route and the maximum tolerable limit of un-delivered containers for each route.

3.2. **Solution method and results.** The mathematical model proposed by Chen and Zhen [3] has a limitation when applying it to real-world problems because it is a nonlinear programming model. On the other hand, the model presented in this paper is a linear integer programming model, so it is easier to apply and more useful. In order to solve the numerical example, the Premium Solver Platform, which is an Excel Solver add-in program, is used.

The optimal solutions obtained by executing Excel Solver are shown in Tables 5, 6 and 7.

Table 6 shows which type of space level is chartered from cooperating ships for each route. For route 1, type 1 space levels (1,000TEU) are chartered from cooperating ship 1, 3, and 5, and for route 2, type 2 (1,500TEU) levels are chartered from all the 5 cooperating ships. The space is not chartered from any cooperating ship for route 3. For route 4, type 3 (2,000TEU) levels are chartered from cooperating ships 1, 2, and 4 and type 2 (1,500TEU)

Table 5. Number of consecutive voyages x_{ik}

i \ k	1	2	3	4
1	6	4	3	1
2	0	0	0	16
3	1	24	2	5

Table 6. Type of chartering space

j \ k	1	2	3	4
1	1	2	0	3
2	0	2	0	3
3	1	2	0	2
4	0	2	0	3
5	1	2	0	2

Table 7. Number of un-delivered containers

k	1	2	3	4
z_k	0	500	0	600

Table 8. Optimal assignment of demand

Route	1	2	3	4
Demand	40,000	80,000	21,000	80,000
Our ship	38,000	72,000	22,000	70,400
Cooperating	3,000	7,500	0	9,000
Un-delivered	0	500	0	600
Total	41,000	80,000	22,000	80,000

levels are chartered from cooperating ships 3 and 5. The transportation demand for each route is satisfied by the way shown in Table 8.

In routes 2 and 4, 500 and 600 TEUs are not transported, respectively. However, the total transshipment amounts in routes 1 and 3 exceed the demands. This means that transportation capacities of ships can be greater than the demands because the container loading space is chartered by batch size from cooperating ship.

4. **Conclusions.** There have been many different types of alliances with competitors in the liner shipping industry since 1990. Among them, container slot chartering has been the most widely used. If liner shipping companies which voyage the same routes make an alliance for slot chartering, they can reduce transportation costs and enhance customer service. This paper presents a mixed integer linear programming model for container slot chartering. This model determines the voyage frequency of our ship and also whether or not a certain level of slot space of cooperating ship is chartered. We use Excel Solver as a solution method. In order to verify the applicability of the proposed model, we apply presented model to the numerical example using the same data as in previous research. Our model can easily solve the problem. As a result, it is clear that our model can be successfully applied to real-world problems and is superior to the previous nonlinear model. The limitation of this paper is that we consider only a one-way slot chartering model. If we consider the problem in which two companies charter container slots from

each other, it may be a more realistic problem and indicates highly meaningful future research.

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